Exercises week 8

Languages and Automata

June 21, 2013

8. Pushdown automata and context-free languages: various exercises

8.1. Consider the following grammars

G_1

S	$\rightarrow$	a | aSbb

G_2

S	$\rightarrow$	a | ASB
A	$\rightarrow$	aA | λ
B	$\rightarrow$	bB | λ

(i) Give set theoretic expressions for $L_1 = L(G_1), L_2 = L(G_2)$. Motivate your answers.
(ii) Construct a push-down automaton M_1 with $L(M_1) = L(G_1)$.
(iii) Construct a push-down automaton M_2 with $L(M_2) = L(G_2)$.
(iv) One of these two languages is regular. Show this by providing a regular expression for it.
(v) Construct a regular grammar for the regular language among L_1, L_2.
(vi) Construct also a DFA for the regular language.
(vii) Show that the other language is not regular. (Use the pumping lemma.)

8.2. Let M be the PDA with

$$\begin{array}{ll}
Q = \{q_0, q_1, q_2\} & \delta(q_0, \lambda, \lambda) = \{[q_1, E]\} \\
\Sigma = \{a, b\} & \delta(q_1, a, E) = \{[q_0, \lambda]\} \\
\Gamma = \{A, E\} & \delta(q_1, a, \lambda) = \{[q_1, A]\} \\
F = \{q_0\} & \delta(q_1, b, \lambda) = \{[q_2, \lambda]\} \\
 & \delta(q_2, b, \lambda) = \{[q_1, \lambda]\} \\
 & \delta(q_2, a, A) = \{[q_2, \lambda]\}
\end{array}$$

(i) Draw a state diagram for M.
(ii) Check which of the following words is in $L(M)$ and explain your answer: aba, $ababa$, $abbba$ and $abbbbba$.
(iii) Is $L(b^*a) \subseteq L(M)$? Explain your answer.
(iv) Is $\{a^nba^nba \mid n \geq 0\} \subseteq L(M)$? Explain your answer.
(v) Is $\{a^nba^mba^mba^nba \mid n, m \geq 0\} \subseteq L(M)$? Explain your answer.
(vi) (*) Can you describe $L(M)$ using set-notation?