8. Pushdown automata and context-free languages: various exercises

8.1. Consider the following grammars

\[
\begin{align*}
G_1 & : S \rightarrow a \mid aSbb \\
G_2 & : S \rightarrow a \mid ASB \\
& \quad A \rightarrow aA \mid \lambda \\
& \quad B \rightarrow bB \mid \lambda 
\end{align*}
\]

(i) Give set theoretic expressions for \( L_1 = L(G_1) \), \( L_2 = L(G_2) \). Motivate your answers.
(ii) Construct a push-down automaton \( M_1 \) with \( L(M_1) = L(G_1) \).
(iii) Construct a push-down automaton \( M_2 \) with \( L(M_2) = L(G_2) \).
(iv) One of these two languages is regular. Show this by providing a regular expression for it.
(v) Construct a regular grammar for the regular language among \( L_1, L_2 \).
(vi) Construct also a DFA for the regular language.
(vii) Show that the other language is not regular. (Use the pumping lemma.)

8.2. Let \( M \) be the PDA with

\[
\begin{align*}
Q & = \{q_0, q_1, q_2\} \\
\Sigma & = \{a, b\} \\
\Gamma & = \{A, E\} \\
F & = \{q_0\} \\
\delta(q_0, \lambda, \lambda) & = \{[q_1, E]\} \\
\delta(q_1, a, E) & = \{[q_0, \lambda]\} \\
\delta(q_1, a, \lambda) & = \{[q_1, A]\} \\
\delta(q_1, b, \lambda) & = \{[q_2, \lambda]\} \\
\delta(q_2, b, \lambda) & = \{[q_1, \lambda]\} \\
\delta(q_2, a, A) & = \{[q_2, \lambda]\} \\
\end{align*}
\]

(i) Draw a state diagram for \( M \).
(ii) Check which of the following words is in \( L(M) \) and explain your answer: \( aba, ababa, abbba \) and \( abbbbba \).
(iii) Is \( L(b^*a) \subseteq L(M) \)? Explain your answer.
(iv) Is \( \{a^nba^nba^n \mid n \geq 0\} \subseteq L(M) \)? Explain your answer.
(v) Is \( \{a^nba^mba^m \mid n, m \geq 0\} \subseteq L(M) \)? Explain your answer.
(vi) (*) Can you describe \( L(M) \) using set-notation?