This test consists of 4 exercises + 1 bonus exercise. The maximum number of points per question is given in the margin. (The grade is obtained by dividing the number of points by 5, rounding down to 10 if necessary.)

In this test, \( \Sigma = \{a, b\} \).

1. Give a regular expression \( e \) such that \( L(e) = L_1 \), where

\[
L_1 = \{ w \in \Sigma^* \mid w \text{ contains } aba \text{ and } abba \}
\]

NB. Note that \( aba \) and \( abba \) can overlap.

2. Give a DFA (deterministic finite automaton) that accepts the language \( L_2 \) where

\[
L_2 = \{ w \in \{a, b, c, d\}^* \mid \text{all characters in } w \text{ appear in alphabetic order} \}
\]

So, \( abbcd \in L_2 \) and \( bc \in L_2 \), but \( abad \notin L_2 \) and \( db \notin L_2 \).

3. Consider the following NFA (non-deterministic finite automaton) \( M \)

(a) Indicate for each of the following words whether they are accepted by \( M \): \( abba \), \( ababa \), \( abab \).

(b) Is it true that \( L(ab(ab)^*) \subseteq L(M) \)? Explain your answer. (So, the question is to verify whether all words in \( L(ab(ab)^*) \) are also in \( L(M) \).)

(c) Construct a DFA \( M' \) that accepts the same language as \( M \). (Use the “powerset construction” that was used in the lecture.)
4. Prove that the following language $L_4$ is not regular.

$$L_4 = \{ww \mid w \in \Sigma^*\}$$

So, a word is in $L_4$ if it consists of two copies of the same word. For example, $aabaab \in L_4$, $babaababa \in L_4$, but $abaa \notin L_4$, $aaaaa \notin L_4$.

5. [Bonus] We define the function $F : \{a, b\}^* \rightarrow \{a, b, c\}^*$ that puts a $c$ behind every $b$ in a word. For example, $F(abab) = abcabc$, $F(abba) = abcba$ and $F(aaa) = aaa$.

The function $F$ extends to languages in the obvious way: if $L \subseteq \{a, b\}^*$, then $F(L) = \{F(w) \mid w \in L\}$.

Show that the following holds.

If $L$ is regular, then $F(L)$ is regular.