Test NWI-MOL090 31/5/2013

Ex 1

(10)

$$e = (a\cup b)^*\big(aba\,(a\cup b)^*abba \cup ababba \cup abbaba \cup abba\,(a\cup b)^*aba\big)(a\cup b)^*$$

Evt vervang $(a\cup b)^*$ door $(a^*b^*)^*$

Evt $(a\cup b)^*\big(\,aba\,((a\cup b)^*abba \cup bba) \cup (abba\,((a\cup b)^*aba \cup ba)\,\big)(a\cup b)^*$

Evt haakjes wegwerken ⇝ 4 summanden

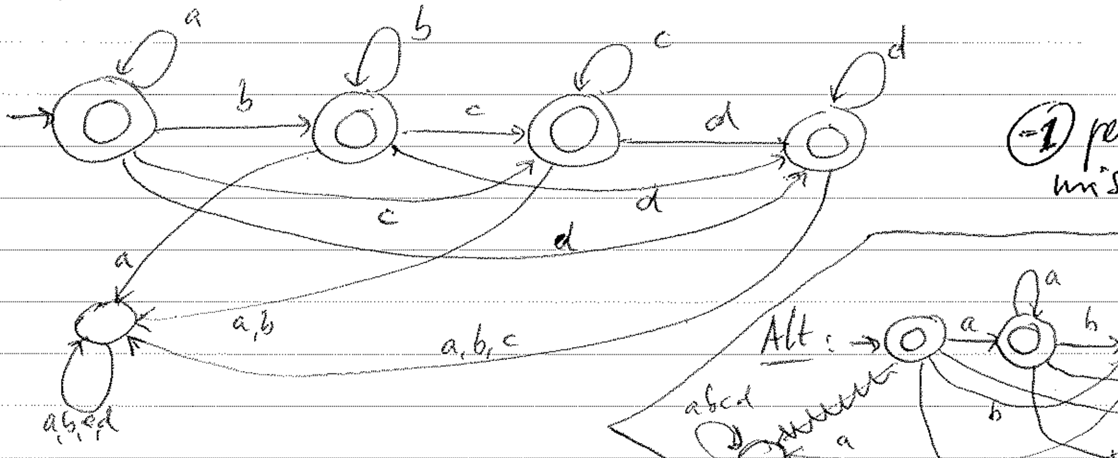
1 x overlap ababba of abbaba vergeten (8) −2

2 x ——— (6) −2!

volgorde aba/abba vergeten (8) −2

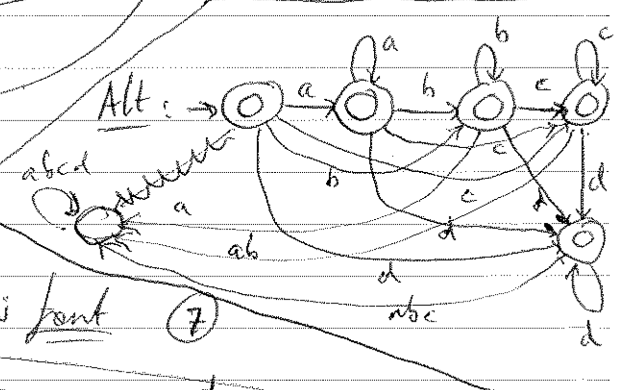
Ex 2

(10)



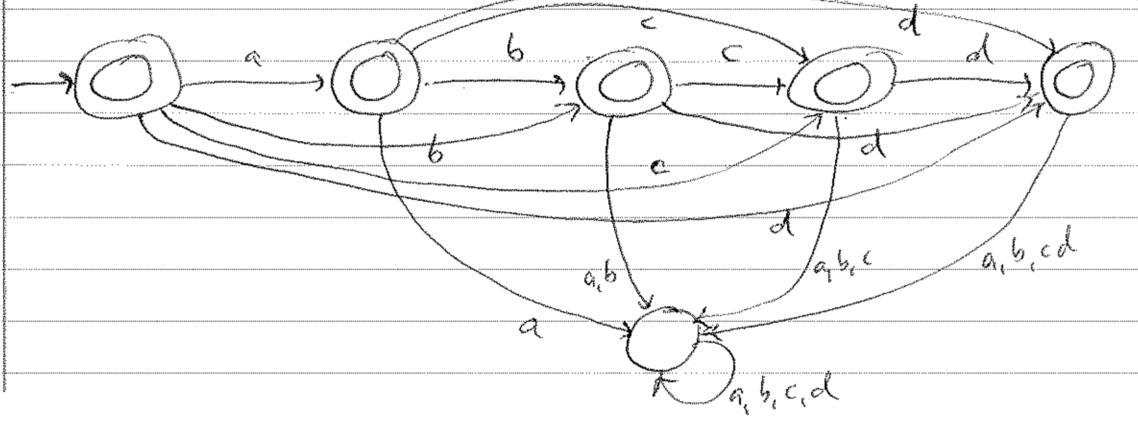
−1 pen missende pijl

Alt:



NB

Deze uitwerking die dubbele niet accepteert is fout (7)



Ex3

(a) abba geaccepteerd door $q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{b} q_0 \xrightarrow{a} q_2$

(6) ababa geaccepteerd door $q_0 \xrightarrow{a} q_2 \xrightarrow{b} q_0 \xrightarrow{a} q_2 \xrightarrow{b} q_0 \xrightarrow{a} q_2$

abab geaccepteerd door $q_0 \xrightarrow{a} q_2 \xrightarrow{b} q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2$

(2) pt. per goed woord ~~(+ goede uitleg)~~

(b) Ja $L(ab(ab)^*) \subseteq L(M)$

Uitleg: $(w \in) L(ab(ab)^*)$ iff $w = \underbrace{(ab)(ab) \dots (ab)}_{n+1 \text{ keer}}$ $(n \geq 0)$

(6) Om zo'n w te accepteren met M doe je

- eerst n keer $q_0 \xrightarrow{a} q_2 \xrightarrow{b} q_0$
- dan 1 keer $q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2$

Andere uitleg:
- ab wordt geaccepteerd : $q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2$
- als w wordt geaccepteerd, dan ook abw

want $q_0 \xrightarrow{w}{}^* q_2$ (M accepteert w),

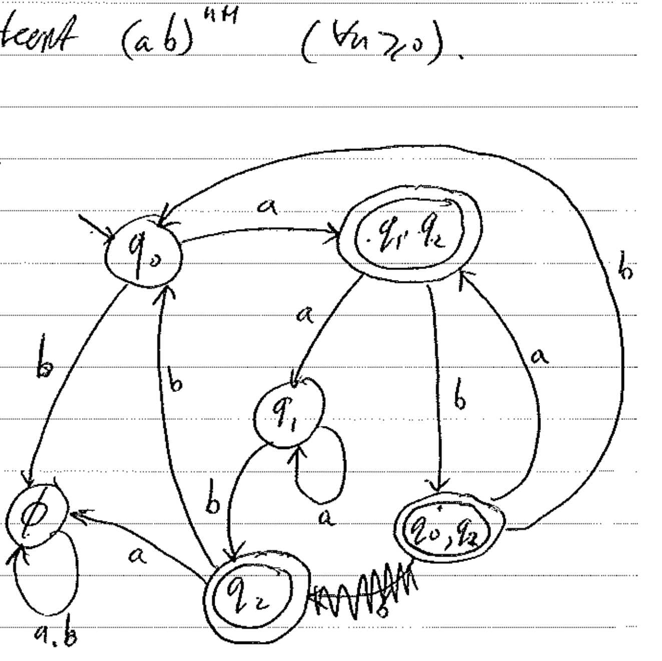
dus $q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{w}{}^* q_2$ (M accepteert abw)

Conclusie: M accepteert $(ab)^{n+1}$ $(\forall n \geq 0)$.

(c)

(7)

	a	b
q_0	q_1, q_2	$\emptyset$
$\emptyset$	$\emptyset$	$\emptyset$
q_1, q_2	q_1	q_0, q_2
q_1	q_1	q_2
q_0, q_2	q_1, q_2	q_0
q_2	$\emptyset$	q_0



Ex 4 Suppose $L_4 = \{ww \mid w \in \Sigma^*\}$ is regular (*)
Let $p \geq 1$ as in the pumping lemma.

(⊗) Take $w = a^p b a^p b \in L_4$, $(|w| \geq p)$

So there are x, y, z such that
- $w = xyz$
- $|xy| \leq p$
- $|y| \geq 1$
- $\forall n \in \mathbb{N}\ (xy^n z \in L_4)$

Because $|xy| \leq p$, we know that $y = a^q$ for some $q \in \mathbb{N}$.
Because $|y| \geq 1$, we know that $q \geq 1$.

(+) Take $n = 0$, then $xy^n z \in L_4$

But: $xy^n z = a^{p-q} b a^p b \notin L_4$

because $q \geq 1$, so $p - q < p$

Contradiction, so (*) is not true

So L_4 is not regular

Evt take $n = 2$ or $n = p$ or .. at (+)

Evt take $w = a^p b^p a^p b^p$ or ...

(NB $w = a^p a^p$ doesn't work!)

Ex5.

Suppose L be regular, say $L = L(M)$ for M a DFA

Change in M each arrow $q_1 \xrightarrow{b} q_2$

into
$$
\begin{tikzcd}
q_1 \arrow[r, "b"] & q \arrow[r, "c"] & q_2
\end{tikzcd}
$$

where q is a new state (for each arrow)

NB: this includes changing
$$
\begin{tikzcd}
q_1 \arrow[loop right, "b"]
\end{tikzcd}
$$
to
$$
\begin{tikzcd}
q_1 \arrow[r, bend left, "b"] & q \arrow[l, bend left, "c"]
\end{tikzcd}
$$

We thus obtain M', an NFA

We have: $L(M') = F(L)$

Alt proof: If $L = L(e)$, e a Reg. expression:

change e into e'

by replacing each b in e by bc

Then $L(e') = F(L)$.