Homework Complexity Theory

March 10, 2020

To be handed in on **March 20**, 2020, in the delivery box of your TA in front of room M1.07A. **Deadline: 11:00 AM**.

Exercise 1. Give an unsatisfiable CNF over three variables p, q, r in which every clause consists of three distinct literals.

Exercise 2. Prove or disprove, for $L_0, L_1, L_2 \subseteq \{0,1\}^*$:

$$\text{If } L_1 \leq_P L_2 \text{ and } L_0 \subseteq L_1, \text{ then } L_0 \leq_P L_2.$$

Exercise 3. Given an undirected graph $G = (V, E)$, the set of nodes $V' \subseteq V$ is called *independent* if no two nodes of V' are connected by an edge. $(\forall u, v \in V'((u, v) \notin E))$. $\mathsf{Indep}(G, k)$ is the problem to decide whether for graph G and number k, the graph G contains an independent set of k nodes.

Prove that $\mathsf{Indep}(G, k)$ is NP-complete.

Exercise 4. A graph (V_1, E_1) is a *subgraph* of a graph (V_2, E_2) if there is an injective mapping $h : V_1 \to V_2$ such that

$$\forall (u, v) \in E_1 \, (h(u), h(v)) \in E_2.$$

The decision problem $\mathsf{Subgraph}(G_1, G_2)$ is: given two graphs $G_1 = (V_1, E_1)$, $G_2 = (V_2, E_2)$, is G_1 a subgraph of G_2?

Prove that $\mathsf{Subgraph}$ is NP-complete, using the fact that Clique is NP-complete. (First describe precisely what has to be proven.)

Exercise 5. A CNF φ is satisfiable if there is an assignment $v : \text{Atoms}(\varphi) \to \{0, 1\}$ for which every clause in φ is true. A CNF φ is called *pre-satisfiable* if there is an assignment $v : \text{Atoms}(\varphi) \to \{0, 1\}$ for which all clauses in φ are true, except for at most one.

Prove that pre-satisfiability of CNFs $\mathsf{CNF\text{-}preSAT}$ is NP-complete. (First describe precisley what has to be proven for this.)