B Tactics in Fitch style

The green 'H:' labels that occur in these descriptions are not part of the way proofs are written in Huth & Ryan, but are necessary for working in ProofWeb. They are the symbolic equivalents (which stay the same throughout the proof process) of the line numbers (which change all the time).

B.1 Structural tactics

`exact H`

m	H:	A		m	H:	A	
		...					
n		A	$\longrightarrow$	n		A	copy m

`insert H` B

						...
				n	H:	B
		...				...
n		A	$\longrightarrow$	$n+1$		A

B.2 Backward tactics

The tactic names may be prefixed with `b_...` to contrast them to the corresponding forward tactics.

Rules that are not intuitionistically valid are marked with a star. Rules that according to Huth & Ryan are derived rules are marked with a dagger.

conjunction introduction

`con_i`

				...	
			n	A	
				...	
	...		$n+1$	B	
n	$A \wedge B$	$\longrightarrow$	$n+2$	$A \wedge B$	$\wedge$i $n, (n+1)$

conjunction elimination left

`con_e1` B

				...	
	...		n	$A \wedge B$	
n	A	$\longrightarrow$	$n+1$	A	$\wedge e_1$ n

conjunction elimination right

`con_e2` A

				$\dots$	
	$\dots$		n	$A \wedge B$	
n	B	$\longrightarrow$	$n+1$	B	$\wedge e_2\ n$

disjunction introduction left

`dis_i1`

				$\dots$	
	$\dots$		n	A	
n	$A \vee B$	$\longrightarrow$	$n+1$	$A \vee B$	$\vee i_1\ n$

disjunction introduction right

`dis_i2`

				$\dots$	
	$\dots$		n	B	
n	$A \vee B$	$\longrightarrow$	$n+1$	$A \vee B$	$\vee i_1\ n$

disjunction elimination

`dis_e (`A `\/` B`) H1 H2`

					$\dots$	
			n		$A \vee B$	
			$n+1$	`H1:`	A	assumption
					$\dots$	
			$n+2$		C	
			$n+3$	`H2:`	B	assumption
					$\dots$	
	$\dots$		$n+4$		C	
n	C	$\longrightarrow$	$n+5$		C	$\vee e\ n,(n+1)—(n+2),(n+3)—(n+4)$

implication introduction

`imp_i H`

			n	`H:`	A	assumption
					$\dots$	
	$\dots$		$n+1$		B	
n	$A \to B$	$\longrightarrow$	$n+2$		$A \to B$	$\to i\ n—(n+1)$

implication elimination

`imp_e` A

					$\dots$	
			n		$A \to B$	
					$\dots$	
	$\dots$		$n+1$		A	
n	B	$\longrightarrow$	$n+2$		B	$\to$e $n, (n+1)$

negation introduction

`neg_i H`

			n	`H:`	A	assumption
					$\dots$	
	$\dots$		$n+1$		$\bot$	
n	$\neg A$	$\longrightarrow$	$n+2$		$\neg A$	$\neg$i n—$(n+1)$

negation elimination

`neg_e` A

					$\dots$	
			n		$\neg A$	
					$\dots$	
	$\dots$		$n+1$		A	
n	$\bot$	$\longrightarrow$	$n+2$		$\bot$	$\neg$e $n, (n+1)$

falsum elimination

`fls_e`

					$\dots$	
	$\dots$		n		$\bot$	
n	A	$\longrightarrow$	$n+1$		A	$\bot$e n

truth introduction

`tru_i`

	$\dots$					
n	$\top$	$\longrightarrow$	n		$\top$	$\top$i n

double negation introduction†

`negneg_i`

					$\dots$	
	$\dots$		n		A	
n	$\neg\neg A$	$\longrightarrow$	$n+1$		$\neg\neg A$	$\neg\neg$i n

*double negation elimination**

`negneg_e`

				$\dots$	
	$\dots$		n	$\neg\neg A$	
n	A	$\longrightarrow$	$n+1$	A	$\neg\neg$e n

*law of excluded middle**†

`LEM`

	$\dots$				
n	$A \vee \neg A$	$\longrightarrow$	n	$A \vee \neg A$	LEM

*proof by contradiction**†

`PBC H`

			n	`H:`	$\neg A$	assumption
					$\dots$	
	$\dots$		$n+1$		$\bot$	
n	A	$\longrightarrow$	$n+2$		A	PBC n—$(n+1)$

modus tollens†

`MT` B

				$\dots$	
			n	$A \to B$	
				$\dots$	
	$\dots$		$n+1$	$\neg B$	
n	$\neg A$	$\longrightarrow$	$n+2$	$\neg A$	MT $n, (n+1)$

universal introduction

`all_i` y

				y	
				$\dots$	
	$\dots$		n	$A[y/x]$	
n	$\forall x\, A$	$\longrightarrow$	$n+1$	$\forall x\, A$	$\forall$i n—n

universal elimination

`all_e (all x,` A`)`

				$\dots$	
	$\dots$		n	$\forall x\, A$	
n	$A[t/x]$	$\longrightarrow$	$n+1$	$A[t/x]$	$\forall$e n

existential introduction

`exi_i` t

				...	
	...		n	$A[t/x]$	
n	$\exists x\, A$	$\longrightarrow$	$n+1$	$\exists x\, A$	$\exists$i n

existential elimination

`exi_e (exi x,` A`)` y `H`

					...	
			n		$\exists x\, A$	
				y		
			$n+1$	`H:`	$A[y/x]$	
					...	
	...		$n+2$		B	
n	B	$\longrightarrow$	$n+3$		B	$\exists$e n,$(n+1)$—$(n+2)$

(Lines $n+1$ to $n+2$, together with y, are enclosed in a box.)

equality introduction

`equ_i`

	...				
n	$t = t$	$\longrightarrow$	n	$t = t$	=i

equality elimination, simple version

`equ_e (`t_1 `=` t_2`)`

				...	
			n	$t_1 = t_2$	
				...	
	...		$n+1$	$A[t_1/x]$	
n	$A[t_2/x]$	$\longrightarrow$	$n+2$	$A[t_2/x]$	=e $n, (n+1)$

equality elimination, general version (t_2 *may occur in* A)

`equ_e' (`t_1 `=` t_2`) (fun x =>` A`)`

				...	
	...		n	$t_1 = t_2$	
				...	
	...		$n+1$	$A[t_1/x]$	
n	$A[t_2/x]$	$\longrightarrow$	$n+2$	$A[t_2/x]$	=e $n, (n+1)$

B.3 Forward versus backward tactics

conjunction introduction

`f_con_i H1 H2`

m_1	H1 :	A		m_1	H1 :	A	
m_2	H2 :	B		m_2	H2 :	B	
		$\dots$					
n		$A \wedge B$	$\longrightarrow$	n		$A \wedge B$	$\wedge$i m_1, m_2

conjunction elimination left

`f_con_e1 H`

m	H :	$A \wedge B$		m	H :	$A \wedge B$	
		$\dots$					
n		A	$\longrightarrow$	n		A	$\wedge$e$_1$ m

conjunction elimination right

`f_con_e2 H`

m	H :	$A \wedge B$		m	H :	$A \wedge B$	
		$\dots$					
n		B	$\longrightarrow$	n		B	$\wedge$e$_2$ m

disjunction introduction left

`f_dis_i1 H`

m	H :	A		m	H :	A	
		$\dots$					
n		$A \vee B$	$\longrightarrow$	n		$A \vee B$	$\vee$i$_1$ m

disjunction introduction right

`f_dis_i2 H`

m	H :	B		m	H :	B	
		$\dots$					
n		$A \vee B$	$\longrightarrow$	n		$A \vee B$	$\vee$i$_2$ m

disjunction elimination

`f_dis_e H H1 H2`

m	H:	$A \vee B$		m	H:	$A \vee B$	
				n	H1:	A	assumption
						$\dots$	
				$n+1$		C	
				$n+2$	H2:	B	assumption
		$\dots$				$\dots$	
				$n+3$		C	
n		C	$\longrightarrow$	$n+4$		C	$\vee$e m,n—$(n+1)$,$(n+2)$—$(n+3)$

implication elimination

`f_imp_e H1 H2`

m_1	H1:	$A \rightarrow B$		m_1	H1:	$A \rightarrow B$	
m_2	H2:	A		m_2	H2:	A	
		$\dots$					
n		B	$\longrightarrow$	n		B	$\rightarrow$e m_1, m_2

negation elimination

`f_neg_e H1 H2`

m_1	H1:	$\neg A$		m_1	H1:	$\neg A$	
m_2	H2:	A		m_2	H2:	A	
		$\dots$					
n		$\bot$	$\longrightarrow$	n		$\bot$	$\neg$e m_1, m_2

falsum elimination

`f_fls_e H`

m	H:	$\bot$		m	H:	$\bot$	
		$\dots$					
n		A	$\longrightarrow$	n		A	$\bot$e m

truth introduction

`f_tru_i`

		...				
n		$\top$	$\longrightarrow$	n	$\top$	$\top$i n

double negation introduction†

`f_negneg_i H`

m	H:	A		m	H:	A	
		...					
n		$\neg\neg A$	$\longrightarrow$	n		$\neg\neg A$	$\neg\neg$i m

*double negation elimination**

`f_negneg_e H`

m	H:	$\neg\neg A$		m	H:	$\neg\neg A$	
		...					
n		A	$\longrightarrow$	n		A	$\neg\neg$e m

*law of excluded middle**†

`f_LEM`

		...				
n		$A \vee \neg A$	$\longrightarrow$	n	$A \vee \neg A$	LEM

modus tollens†

`f_MT H1 H2`

m_1	H1:	$A \to B$		m_1	H1:	$A \to B$	
m_2	H2:	$\neg B$		m_2	H2:	$\neg B$	
		...					
n		$\neg A$	$\longrightarrow$	n		$\neg A$	MT m_1, m_2

universal elimination

`f_all_e H`

m	H:	$\forall x\, A$		m	H:	$\forall x\, A$	
		...					
n		$A[t/x]$	$\longrightarrow$	n		$A[t/x]$	$\forall$e m

existential introduction

`f_exi_i H`

m	H:	$A[t/x]$		m	H:	$A[t/x]$	
		$\ldots$					
n		$\exists x\, A$	$\longrightarrow$	n		$\exists x\, A$	$\exists$i m

existential elimination

`f_exi_e H` y `H1`

m	H:	$\exists x\, A$		m	H:	$\exists x\, A$	
					y		
				n	H1:	$A[y/x]$	
						$\ldots$	
		$\ldots$		$n+1$		B	
n		B	$\longrightarrow$	$n+2$		B	$\exists$e m,n—$(n+1)$

equality introduction

`f_equ_i`

		$\ldots$					
n		$t = t$	$\longrightarrow$	n		$t = t$	=i

equality elimination

`f_equ_e H1 H2`

m_1	H1:	$t_1 = t_2$		m_1	H1:	$t_1 = t_2$	
m_2	H2:	$A[t_1/x]$		m_2	H2:	$A[t_1/x]$	
		$\ldots$					
n		$A[t_2/x]$	$\longrightarrow$	n		$A[t_2/x]$	=e m_1, m_2