Proving with Computer Assistance

Representative exam questions

Apart from the exercises that have already been presented at the lecture, here are some more representative questions related to Normalization and Church-Rosser that could be at the exam. See the Slides and the Course Notes by Herman Geuvers for the definitions. We consider the term

$$M := (\lambda x.\mathbf{I}_1\,(x\,(x\,(\mathbf{I}_2\,\mathbf{I}_3))))\,(\mathbf{I}_4\,\mathbf{I}_5)$$

where each of the $\mathbf{I}_i$ is just a different copy of the identity, $\lambda y.y$. (I have labelled each occurrence of $\mathbf{I}$ with a different index for clarity.) We also abbreviate $P := \lambda x.\mathbf{I}_1\,(x\,(x\,(\mathbf{I}_2\,\mathbf{I}_3)))$.

We let $\mathbf{I}_3$ be of type $\alpha{\to}\alpha$ (for α a type variable) and wrtite A for $\alpha{\to}\alpha$.

1. Give a type to M in simple type theory, using $\mathbf{I}_3 : A$. You don't have to give a derivation, but give the types of x, $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_4$ and $\mathbf{I}_5$ in M.

2. What is $m(M)$? Here m is the *measure* given to a term in the WN (weak normalization) proof. Explain your answer.

3. In the proof of WN, we contract a specific redex in M, to arrive at a new term N, so $M \longrightarrow_\beta N$. What is N? Explain your answer.

4. In the proof of CR (Church-Rosser), we use the function $(-)^*$. What are M^* and $(M^*)^*$?

5. Give the reduction graph of M.