Proving with Computer Assistance, 2IF65

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Exercises on Lecture 2

See the course notes – notably *Introduction to Type Theory* by Herman Geuvers – and the slides on the homepage.

1. Verify in detail (by giving a derivation in STT) that

$$\lambda x^{\alpha\to\beta}.\lambda y^{\beta\to\gamma}.\lambda z^{\alpha}.y(xz) : (\alpha\to\beta)\to(\beta\to\gamma)\to\alpha\to\gamma$$

2. Verify in detail (by giving a derivation in STT) that

$$\lambda x^{\alpha}.\lambda y^{(\beta\to\alpha)\to\alpha}.y(\lambda z^{\beta}.x) : \alpha\to((\beta\to\alpha)\to\alpha)\to\alpha$$

3. "Dress up" the λ-term $\lambda x.\lambda y.y(\lambda z.x)$ with type information in such a way that it is of type $\alpha\to((\beta\to\alpha)\to\gamma)\to\gamma$

4. Give the natural deduction (either in Fitch style or in tree form) that corresponds to

$$\lambda x{:}\gamma\to\epsilon.\lambda y{:}(\gamma\to\epsilon)\to\epsilon.y(\lambda z{:}\gamma.y\,x) : (\gamma\to\epsilon)\to((\gamma\to\epsilon)\to\epsilon)\to\epsilon$$

5. Give another term of the same type

$$(\gamma\to\epsilon)\to((\gamma\to\epsilon)\to\epsilon)\to\epsilon$$

and the natural deduction (either in Fitch style or in tree form) that it corresponds to.