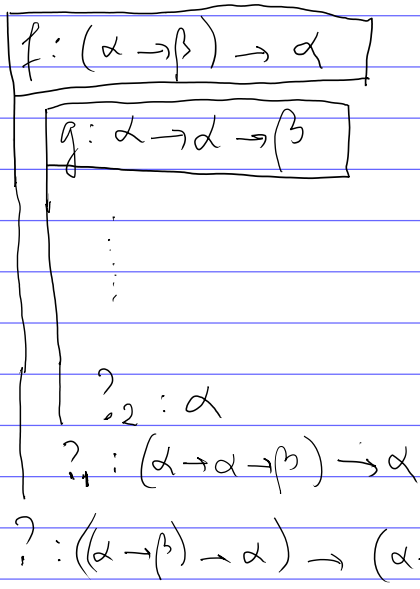


Exercises on Simple Type Theory

(5c) Find a term of type $((\alpha \rightarrow \beta) \rightarrow \alpha) \rightarrow (\alpha \rightarrow \alpha \rightarrow \beta) \rightarrow \alpha$

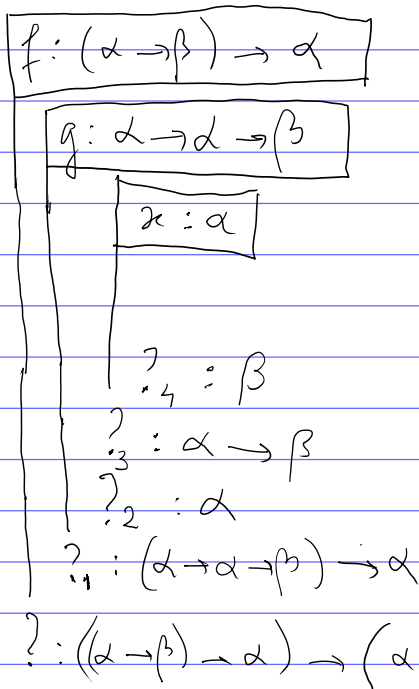
We construct a flag style derivation and start as follows



where we have to fill in the ?
 it should be clear how
 to get ? from ?₁ and
 ?₁ from ?₂

The only way to get ? : α is by applying f.

Then we get a new goal ? : $\alpha \rightarrow \beta$, which doesn't look very hopeful, but the point is to just keep going



Now it is clear we can take ?₄ := g x x and you can redo the full derivation, filling in the ?

$$?_2 = f ?_3$$

Term: ? = $\lambda f: (\alpha \rightarrow \beta) \rightarrow \alpha. \lambda g: \alpha \rightarrow \alpha \rightarrow \beta. f(\lambda x: \alpha. g x x)$