# Proving with Computer Assistance, 2IMF15

## Herman Geuvers, TUE

## Exercises on Simple Type Theory à la Curry: assigning types to untyped terms, principal type algorithm

### On simple type theory à la Church

1. Consider the following types

$$\begin{array}{rcl} A_1 & = & ((\alpha{\to}\beta){\to}\alpha){\to}\alpha \\ A_2 & = & ((\alpha{\to}\alpha){\to}\beta){\to}\beta \\ A_3 & = & ((\alpha{\to}\beta){\to}\beta){\to}\beta \end{array}$$

   Pick one of these types $A_i$ and construct a closed term $M$ of type $A_i$. Give a typing derivation of $M : A_i$.

2. In case you haven't done these yet:

   (a) Find a term of type $(\delta{\to}\delta{\to}\alpha){\to}(\alpha{\to}\beta{\to}\gamma){\to}(\delta{\to}\beta){\to}\delta{\to}\gamma$

   (b) Find two terms of type $(\delta{\to}\delta{\to}\alpha){\to}(\gamma{\to}\alpha){\to}(\alpha{\to}\beta){\to}\delta{\to}\gamma{\to}\beta$

   (c) Find a term of type $((\alpha{\to}\beta){\to}\alpha){\to}(\alpha{\to}\alpha{\to}\beta){\to}\alpha$

   (d) Find a term of type $((\alpha{\to}\beta){\to}\alpha){\to}(\alpha{\to}\alpha{\to}\beta){\to}\beta$ (Hint: use the previous exercise.)

### On simple type theory à la Curry

1. Determine the most general unifiers of

   (a) $(\alpha{\to}\beta){\to}\gamma$ and $\alpha{\to}\beta{\to}\gamma$

   (b) $(\alpha{\to}\beta){\to}\gamma$ and $\gamma{\to}\alpha{\to}\beta$

2. Compute the principal type of $\mathbf{S} := \lambda x.\lambda y.\lambda z.x\,z(y\,z)$.

3. Which of the following terms is typable? If it is, determine the *principal type*; if it isn't, show that the typing algorithm fails.

   (a) $\lambda z\,x.z(x(\lambda y.y\,x))$

   (b) $\lambda z\,x.z(x(\lambda y.y\,z))$

4. Consider the following two terms

   - $\lambda x.x\,(\lambda y.y\,(\lambda z.x))$
   - $\lambda x.x\,(\lambda y.x\,(\lambda z.z))$

   For each of these terms, compute its principal type, if it exists. (Give the end result and show your computation; if the term has no principal type, show how your computation yields 'fail'.)

5. Compute the principal type of $M := \lambda x.\lambda y.x(y(\lambda z.x\,z\,z))(y(\lambda z.x\,z\,z))$.

6. For each of the following two terms, compute its principal type, if it exists.

   - $\lambda x.(\lambda y.x\,(x\,y))\,(\lambda u\,v.u)$
   - $\lambda y.(\lambda x.x\,(x\,y))\,(\lambda u\,v.u)$

   Give the end result and show your computation; if the term has no principal type, show how your computation yields 'fail'.