Proving with Computer Assistance, 2IF65

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Exercises on Lecture 3

On simple type theory à la Church

1. Find a term of type $(\delta\to\delta\to\alpha)\to(\alpha\to\beta\to\gamma)\to(\delta\to\beta)\to\delta\to\gamma$
2. Find two terms of type $(\delta\to\delta\to\alpha)\to(\gamma\to\alpha)\to(\alpha\to\beta)\to\delta\to\gamma\to\beta$
3. Find a term of type $((\alpha\to\beta)\to\alpha)\to(\alpha\to\alpha\to\beta)\to\alpha$
4. Find a term of type $((\alpha\to\beta)\to\alpha)\to(\alpha\to\alpha\to\beta)\to\beta$ (Hint: use the previous exercise.)

On simple type theory à la Curry

1. Determine the most general unifiers of
   (a) $(\alpha\to\beta)\to\gamma$ and $\alpha\to\beta\to\gamma$
   (b) $(\alpha\to\beta)\to\gamma$ and $\gamma\to\alpha\to\beta$
2. Compute the principal type of $\mathbf{S} := \lambda x.\lambda y.\lambda z.x\,z(y\,z)$.
3. Which of the following terms is typable? If it is, determine the *principal type*; if it isn't, show that the typing algorithm fails.
   (a) $\lambda z\,x.z(x(\lambda y.y\,x))$
   (b) $\lambda z\,x.z(x(\lambda y.y\,z))$
4. Compute the principal type of $M := \lambda x.\lambda y.x(y(\lambda z.x\,z\,z))(y(\lambda z.x\,z\,z))$.