# Proving with Computer Assistance, 2IF65


## Herman Geuvers, TUE


## Exercises on Lecture 4

1. Recall: $\bot := \forall\alpha.\alpha$, $\top := \forall\alpha.\alpha{\to}\alpha$.

   (a) Verify that in Church $\lambda 2$: $\lambda x{:}\top.x\top x : \top{\to}\top$.

   (b) Verify that in Curry $\lambda 2$: $\lambda x.xx : \top{\to}\top$

   (c) Find a type in Curry $\lambda 2$ for $\lambda x.x\,x\,x$

   (d) Find a type in Curry $\lambda 2$ for $\lambda x.(x\,x)(x\,x)$

   (e) Find a type in Curry $\lambda 2$ for $\lambda z.z(\lambda x.x\,x)$

2. Define: $\sigma \times \tau := \forall\alpha.(\sigma{\to}\tau{\to}\alpha){\to}\alpha$,
   $\sigma + \tau := \forall\alpha.(\sigma{\to}\alpha){\to}(\tau{\to}\alpha){\to}\alpha$
   $\mathrm{Tree}_{A,B} := \forall\alpha.(B{\to}\alpha){\to}(A{\to}\alpha{\to}\alpha{\to}\alpha){\to}\alpha$

   (a) Define $\mathrm{inl} : \sigma \to \sigma + \tau$

   (b) Define pairing : $[-,-] : \sigma \to \tau \to \sigma \times \tau$

   (c) Show that the addition function (as defined on the slides) behaves as expected.

   (d) Define join : $\mathrm{Tree}_{A,B} \to \mathrm{Tree}_{A,B} \to A \to \mathrm{Tree}_{A,B}$