

# Proving with Computer Assistance, 2IMF15

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## Exercises on First order dependent type theory, $\lambda P$ , some answers

NB  $\rightarrow$  binds strongest.

1. Give a precise derivation of the following judgment. (This is example 3 on the slides of the course.)

$$A : *, P : A \rightarrow *, a : A \vdash (P a) \rightarrow * : \square$$

(Advise: give the derivation in “flag style”, as it was shown in the lecture.)

2. Find a term of the following type and write down the context in which this term is typed. (This is example 5 on the slides of the course.)

$$(\Pi x:A.P x \rightarrow Q x) \rightarrow (\Pi x:A.P x) \rightarrow \Pi x:A.Q x$$

Do this by giving a derivation in “flag style”, where you may omit derivations of the well-formedness of types.

3. Find a term of the following type and write down the context in which this term is typed.

$$(\Pi x:A.P x \rightarrow \Pi z:A.R x z) \rightarrow (\Pi x:A.P x) \rightarrow \Pi z:A.R z z).$$

(NB. Read this type in the proper way:  $\rightarrow$  binds stronger than  $\Pi$ !)

4. Give a term  $M$  of type  $\Pi x:A.P(f(f x))$  in the context

$$\begin{aligned} \Gamma \quad & := \quad A : *, P : A \rightarrow *, f : A \rightarrow A, g : A \rightarrow A, \\ & h : \Pi x:A.P(f x) \rightarrow P(g x), k : \Pi x, y:A.(P x \rightarrow P y) \rightarrow P(f x). \end{aligned}$$

Also give a derivation of  $\Gamma \vdash M : \Pi x:A.P(f(f x))$  in ‘short form’, so you don’t have to show the well-formedness of the types.

5. Find a term of the following type and write down the context in which this term is typed.

$$(\Pi x:A.P x \rightarrow Q) \rightarrow (\Pi x:A.P x) \rightarrow Q$$

What is special about your context? It should somehow explicitly state that the type  $A$  is not empty. How? Why?

6. Find a term from the given hypotheses of the following type and write down the context in which this term is typed.

$$\begin{aligned} & \forall x.(P(x) \rightarrow R(x, f(x))), \\ & \forall x, y.(R(x, y) \rightarrow R(y, x)), \\ & \forall x, y.(R(x, y) \rightarrow R(f(y), x)) \quad \vdash \quad \forall x.(P(x) \rightarrow R(f(x), f(x))) \end{aligned}$$