

Proving with Computer Assistance, 2IF65

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Exercises on Lecture 5

NB \rightarrow binds strongest.

1. Give a precise derivation of the following judgment. (This is example 3 on the slides of the course.)

$$A : *, P : A \rightarrow *, a : A \vdash (P a) \rightarrow * : \square$$

(Advise: give the derivation in “flag style”, as it was shown in the lecture.)

2. Find a term of the following type and write down the context in which this term is typed. (This is example 5 on the slides of the course.)

$$(\Pi x:A.P x \rightarrow Q x) \rightarrow (\Pi x:A.P x) \rightarrow \Pi x:A.Q x$$

Do this by giving a derivation in “flag style”, where you may omit derivations of the well-formedness of types.

3. Find a term of the following type and write down the context in which this term is typed.

$$(\Pi x:A.P x \rightarrow \Pi z:A.R x z) \rightarrow (\Pi x:A.P x) \rightarrow \Pi z:A.R z z).$$

(NB. Read this type in the proper way: \rightarrow binds stronger than Π !)

4. Construct a term of type $\top(A \Rightarrow (B \Rightarrow A))$ in the context for propositional logic in LF. (See the slides of the course.)

5. Find a term of the following type and write down the context in which this term is typed.

$$(\Pi x:A.P x \rightarrow Q) \rightarrow (\Pi x:A.P x) \rightarrow Q$$

What is special about your context? It should somehow explicitly state that the type A is not empty. How? Why?