Proving with Computer Assistance, 2IMF15

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Exercises on First order dependent type theory, λP

NB. $\to$ binds strongest.

1. Give a precise derivation of the following judgment. (This is example 3 on the slides of the course.)

$$A : *,\ P : A \to *,\ a : A \vdash (P\, a) \to * : \Box$$

(Advise: give the derivation in "flag style", as it was shown in the lecture.)

2. Find a term of the following type and write down the context in which this term is typed. (This is example 5 on the slides of the course.)

$$(\Pi x{:}A.P\, x{\to}Q\, x){\to}(\Pi x{:}A.P\, x){\to}\Pi x{:}A.Q\, x$$

Do this by giving a derivation in "flag style", where you may omit derivations of the well-formedness of types.

3. Find a term of the following type and write down the context in which this term is typed.

$$(\Pi x{:}A.P\, x{\to}\Pi z{:}A.R\, x\, z){\to}(\Pi x{:}A.P\, x){\to}\Pi z{:}A.R\, z\, z).$$

(**NB**. Read this type in the proper way: $\to$ binds stronger than Π!)

4. Give a term M of type $\Pi x{:}A.P(f(f\, x))$ in the context

$$\begin{aligned}\Gamma \quad := \quad & A : *, P : A{\to}*, f : A{\to}A, g : A{\to}A,\\ & h : \Pi x{:}A.P(f\, x){\to}P(g\, x), k : \Pi x, y{:}A.(P\, x{\to}P\, y){\to}P(f\, x).\end{aligned}$$

Also give a derivation of $\Gamma \vdash M : \Pi x{:}A.P(f(f\, x))$ in 'short form', so you don't have to show he well-formedness of the types.

5. Find a term of the following type and write down the context in which this term is typed.

$$(\Pi x{:}A.P\, x{\to}Q){\to}(\Pi x{:}A.P\, x){\to}Q$$

What is special about your context? It should somehow explicitly state that the type A is not empty. How? Why?

6. Find a term from the given hypotheses of the following type and write down the context in which this term is typed.

$$\begin{aligned}&\forall x.\,(P(x) \to R(x, f(x))),\\ &\forall x, y.\,(R(x, y) \to R(y, x)),\\ &\forall x, y.\,(R(x, y) \to R(f(y), x)) \quad \vdash \quad \forall x.\,(P(x) \to R(f(x), f(x)))\end{aligned}$$