Eindhoven University of Technology

Faculty of Mathematics and Computer Science

Written Exam **Proving with Computer Assistance 2IF65**
Friday June 27 2008, 14.00–17.00
The maximum number of points per exercise is indicated in the margin. (Maxium 100 points in total.) **NB** Typing derivations may be given in "flag style" or in "sequent style".

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(10) 1. Show that the following term P is typable in simple type theory ($\lambda\!\to$) à la Church.

$$\lambda x : (\alpha \to (\alpha \to \alpha) \to \alpha) \to \alpha . x(\lambda y : \alpha . \lambda z : \alpha \to \alpha . z\, y)$$

Give a typing derivation that gives the type of P.

(15) 2. Construct a term M of type $(\alpha \to \beta) \to \alpha \to (\beta \to \gamma) \to \gamma$ in simple type theory ($\lambda\!\to$) à la Church and give a typing derivation of

$$M : (\alpha \to \beta) \to \alpha \to (\beta \to \gamma) \to \gamma.$$

(15) 3. Compute the principal type of

$$\lambda x . \lambda y . \lambda z . y (x\, z) x$$

in simple type theory ($\lambda\!\to$) à la Curry. (Give the end result and show your computation.)

(10) 4. Give a type to the term

$$\lambda x . x\, x\, (\lambda y . y\, y)$$

in $\lambda 2$ à la Curry. Also give the typing derivation of your result.

Continue on the other side

5. In $\lambda 2$ à la Church we define the following type:

$$\mathsf{Three} := \forall \alpha{:}*.\alpha \to \alpha \to \alpha \to \alpha.$$

(10) (a) Give three different closed inhabitants of the type Three in $\lambda 2$ à la Church: $\mathsf{one}, \mathsf{two}, \mathsf{three} : \mathsf{Three}$.

(10) (b) Define a function $\mathsf{Shift} : \mathsf{Three} \to \mathsf{Three}$ that does the following

$$\begin{aligned} \mathsf{Shift}\,\mathsf{one} &=_\beta \mathsf{two} \\ \mathsf{Shift}\,\mathsf{two} &=_\beta \mathsf{three} \\ \mathsf{Shift}\,\mathsf{three} &=_\beta \mathsf{one} \end{aligned}$$

(15) 6. In the system λP, give a term of type $\Pi x{:}A.P(f\,x)$ in the context

$$\Gamma := A : *, P : A \to *, h : \Pi x{:}A.P(f\,x) \to P\,x, g : \Pi x, y{:}A.(P\,y \to P\,x) \to P\,y.$$

That is, solve the following type inhabitation problem in λP:

$$\Gamma \vdash ? : \Pi x{:}A.P(f\,x)$$

You don't have to give a typing derivation .

(15) 7. We define, in the Calculus of Constructions λC, the existential quantifier by

$$\exists x{:}A.\varphi := \Pi \alpha{:}*.(\Pi x{:}A.\varphi \to \alpha) \to \alpha$$

As usual, $\bot$ is defined as $\Pi \beta{:}*.\beta$.
Give a term of type

$$(\exists x{:}A.\varphi) \to (\Pi x{:}A.\varphi \to \bot) \to \bot$$

You don't have to give a typing derivation .

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