1. Show that the following term $P$ is typable in simple type theory ($\lambda \to$) à la Church.

$$\lambda x : (\alpha \to (\alpha \to \alpha) \to \alpha) \to \alpha. x(\lambda y : \alpha. \lambda z : \alpha \to \alpha. z y)$$

Give a typing derivation that gives the type of $P$.

2. Construct a term $M$ of type $(\alpha \to \beta) \to \alpha \to (\beta \to \gamma) \to \gamma$ in simple type theory ($\lambda \to$) à la Church and give a typing derivation of

$$M : (\alpha \to \beta) \to \alpha \to (\beta \to \gamma) \to \gamma.$$ 

3. Compute the principal type of

$$\lambda x. \lambda y. \lambda z. y(x z) x$$

in simple type theory ($\lambda \to$) à la Curry. (Give the end result and show your computation.)

4. Give a type to the term

$$\lambda x. x (\lambda y. y y)$$

in $\lambda 2$ à la Curry. Also give the typing derivation of your result.
5. In $\lambda 2$ à la Church we define the following type:

$$\text{Three} := \forall \alpha : * . \alpha \rightarrow \alpha \rightarrow \alpha \rightarrow \alpha.$$  

(10) (a) Give three different closed inhabitants of the type $\text{Three}$ in $\lambda 2$ à la Church: one, two, three : Three.

(10) (b) Define a function $\text{Shift} : \text{Three} \rightarrow \text{Three}$ that does the following

$$\begin{align*}
\text{Shift one} &= \beta \text{ two} \\
\text{Shift two} &= \beta \text{ three} \\
\text{Shift three} &= \beta \text{ one}
\end{align*}$$  

(15) 6. In the system $\lambda P$, give a term of type $\Pi x : A.P(f x)$ in the context

$$\Gamma := A : *, P : A \rightarrow *, h : \Pi x : A.P(f x) \rightarrow P x, g : \Pi x, y : A.(P y \rightarrow P x) \rightarrow P y.$$  

That is, solve the following type inhabitation problem in $\lambda P$:

$$\Gamma \vdash \ ? : \Pi x : A.P(f x)$$  

You don’t have to give a typing derivation.

(15) 7. We define, in the Calculus of Constructions $\lambda C$, the existential quantifier by

$$\exists x : A.\varphi := \Pi \alpha : * . (\Pi x : A.\varphi \rightarrow \alpha) \rightarrow \alpha$$  

As usual, $\bot$ is defined as $\Pi \beta : * . \beta$.

Give a term of type

$$(\exists x : A.\varphi) \rightarrow (\Pi x : A.\varphi \rightarrow \bot) \rightarrow \bot$$  

You don’t have to give a typing derivation.