1. Show that the untyped \( \lambda \)-term \( \omega \) (= \( \lambda x.x \)) is not typable in PCF. That is: show that there are no \( \tau_1 \) and \( \tau_2 \) such that \( \vdash \text{fn } x : \tau_1, x x : \tau_2 \).

2. Suppose that the term \( \text{mult} : \text{nat} \to \text{nat} \to \text{nat} \) defines multiplication in PCF. Give a PCF term that defines the faculty function \( \text{fac} : \text{nat} \to \text{nat} \).

3. Show that, if \( Q \Downarrow V \), then \( (\text{fn } x : \tau.\text{fn } y : \tau.y)PQ \Downarrow V \). (NB. \( V \) denotes an arbitrary value.)

4. To prove that PCF evaluation is deterministic, we prove (in Proposition 5.4.1) that the following set is closed under the rules of Fig.3
   \[ \{(M, \tau, V)|M \Downarrow \tau V \land \forall V'(M \Downarrow \tau V' \Rightarrow V = V')\} \]
   Show this for the cases of the rules (\( \Downarrow_{\text{if1}} \)) and (\( \Downarrow_{\text{cbn}} \)).
   (An alternative way of looking at this is to prove the following:
   \( M \Downarrow \tau V \Rightarrow \forall V'(M \Downarrow \tau V' \Rightarrow V = V') \)
   by induction on the derivation of \( M \Downarrow \tau V \). Do only the cases when the last applied rule is (\( \Downarrow_{\text{if1}} \)) or (\( \Downarrow_{\text{cbn}} \)).)

5. (a) Give a type \( \tau \), a term \( M \), values \( V, V' \) and a context \( C[-] \) such that \( M \Downarrow \tau V \) but \( C[M] \Downarrow \tau V' \neq C[V] \).
   (b) Give a type \( \tau \), a term \( M \), a value \( V \) and a context \( C[-] \) such that \( M \Downarrow \tau V \) but \( C[M] \Downarrow \tau \) (\( C[M] \) has no value.)
   (c) Give a type \( \tau \), a term \( M \), a value \( V \) and a context \( C[-] \) such that \( M \Downarrow \tau \) but \( C[M] \Downarrow \tau V \)

6. Given the definition of plus (Exercise 5.6.3.)
   \[
   \text{plus} = \text{fix}(\text{fn } p : \text{nat} \to \text{nat}.\text{fn } x : \text{nat}.\text{fn } y : \text{nat}.
   \begin{cases}
   \text{if zero}(y) \text{ then } x \text{ else succ}(px \text{ pred}(y))
   \end{cases})
   \]
   Prove (by induction) that
   \[
   \forall m, n(\text{plus succ}^m(0) \text{ succ}^n(0) \Downarrow_{\text{nat}} \text{ succ}^{m+n}(0))
   \]

7. (If we come to this topic at the lecture!) Prove that the following terms \( M \) and \( N \) are not contextually equivalent.
   \[
   M = \text{if } x \text{ then } 0 \text{ else } 1
   \]
   \[
   N = \text{if } y \text{ then } 0 \text{ else } 1
   \]