Semantics and Domain theory

Exercises 13

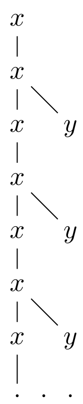
1. Prove that, for M a closed λ-term, if M has a head-normal-form, then there is a sequence of terms $P_1, \ldots, P_n$ such that $M\, P_1 \ldots P_n =_\beta \mathbf{I}$.
   (For closed terms, the reverse implication also holds, so this criterion is equivalent to *having a hnf*. This is where the terminology *solvable* comes from.)

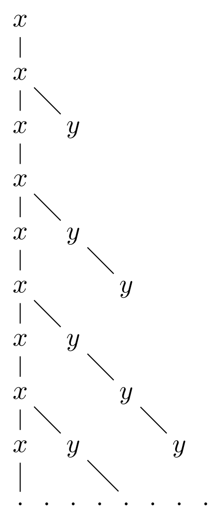
2. Define $T := \lambda x.x\, y\, (x\, x)$ and $M := T\, T$.
   (a) Draw the Böhm tree of M.
   (b) Describe the set of approximations of M, $\mathcal{A}(M)$.

3. Remember that the $\mathbf{S}$ combinator is defined as $\lambda x\, y\, z.x\, z\, (y\, z)$.
   (a) Draw the Böhm tree of $\mathbf{S\,S\,S}$.
   (b) Give the approximations of $\mathbf{S\,S\,S}$, that is, describe $\mathcal{A}(\mathbf{S\,S\,S})$.

4. Suppose that the term B satisfies $B = x\, B\, B$. Draw the Böhm tree of B.

5. (a) Give a term P that has the Böhm tree given below.
   (b) (Hard) Give a term Q that has the Böhm tree given below.

$\mathrm{BT}(P) =$ 

$\mathrm{BT}(Q) =$ 

6. Let M and N be λ-terms that satisfy the following equations

$$\begin{aligned} M &= \lambda xy.x\,(M\,x\,y)\,(M\,x\,y) \\ N &= \lambda xy.x\,(N\,x\,x)\,(N\,x\,x) \end{aligned}$$

   Prove that $M = N$ in D_A.