NB. We write \( \text{State} \to \text{State} \) for the set of partial functions from \( \text{State} \) to \( \text{State} \).

1. Do exercise 1.2.1. of Winskel at the end of Chapter 1.

2. (a) Define the denotational semantics of \( \text{repeat } P \text{ until } b \) as a fixed point of a function \( g : (\text{State} \to \text{State}) \to (\text{State} \to \text{State}) \).
   (NB this program executes statement \( P \) and then checks the boolean \( b \); if \( b \) holds, execution stops, if \( b \) doesn’t hold, it iterates.)
   (b) Define a denotational semantics for the statement \( \text{for } x := e_1 \text{ to } e_2 \text{ do } P \):
   i. First with \( e_1, e_2 \) fixed numbers in \( \mathbb{Z} \), say \( n \) and \( m \).
   ii. Discuss some of the choices and problems with giving the general semantics, where \( e_1 \) and \( e_2 \) are arbitrary expressions.
   What semantics would you give to \( \text{for } x := 1 \text{ to } x + 1 \text{ do } \text{skip} \)? And to \( \text{for } x := 1 \text{ to } 3 \text{ do } x := x - 1 \)?

3. Consider the function \( f : (\text{State} \to \text{State}) \to (\text{State} \to \text{State}) \) as defined by Winskel on slide 6, but now with \( \text{State} = V \to \mathbb{Z} \):

\[
\begin{align*}
  f(w)(s) & := s & \text{if } s(x) \leq 0 \\
  f(w)(s) & := w(s[x \mapsto s(x) - 1, y \mapsto s(x) \ast s(y)]) & \text{if } s(x) > 0
\end{align*}
\]

(a) Do exercise 1.2.2. of Winskel at the end of Chapter 1.

(b) Prove \( f(w_\infty) = w_\infty \) for \( w_\infty : \text{State} \to \text{State} \) as defined a la Winskel’s notes. (First redefine \( w_\infty \) for our notion of State.)

(c) Prove that \( \forall s \in \text{State} : \exists n [f^n(\bot)(s) = f^{n+1}(\bot)(s)] \).