1. Prove Proposition 3.3.1, that is prove that, given the partial function \( f : X \to Y \), the function \( f_\bot : X_\bot \to Y_\bot \) is continuous.

2. Prove Proposition 3.3.2, that is, prove that for each domain \( D \) the function \( \text{if} : B_\bot \times (D \times D) \to D \) defined by \( \text{if}(tt, (d, e)) = d \), \( \text{if}(ff, (d, e)) = e \) and \( \text{if}(\bot, (d, e)) = \bot \) is continuous.

3. (Exercise 3.4.2 of Winskel): Let \( X \) and \( Y \) be sets and \( X_\bot \) and \( Y_\bot \) be the corresponding flat domains. Show that a function \( f : X_\bot \to Y_\bot \) is continuous if and only if one of (a) or (b) holds:

   (a) \( f \) is strict, i.e. \( f(\bot) = \bot \).

   (b) \( f \) is constant, i.e. \( \forall x \in X (f(x) = f(\bot)) \).

4. Show that the following two definitions of the ordering between functions \( f, g : D \to E \) (see Slide 17) are equivalent.

   (a) \( f \sqsubseteq g := \forall d \in D (f(d) \sqsubseteq_E g(d)) \).

   (b) \( f \sqsubseteq' g := \forall d_1, d_2 \in D (d_1 \sqsubseteq_D d_2 \Rightarrow f(d_1) \sqsubseteq_E g(d_2)) \).

5. Prove that for \( D \) a domain and \( F : (D \to D) \to (D \to D) \) and \( g : D \to D \) continuous,

\[
\text{ev}(\text{fix}(F), \text{fix}(g)) = \sqcup_{k \geq 0} F^k(\bot')(f^k(\bot)),
\]

where \( \bot \) is in \( D \) and \( \bot' \) is in \( D \to D \) and \( \text{ev} \) is the evaluation function of Proposition 3.2.1.