Semantics and Domain theory

Exercises 8

1. (a) Give a type τ, a term M, values V, V' and a context $C[-]$ such that $M \Downarrow_\tau V$ but $C[M] \Downarrow_\tau V' \neq C[V]$.

   (b) Give a type τ, a term M, a value V and a context $C[-]$ such that $M \Downarrow_\tau V$ but $C[M] \not\Downarrow_\tau$ ($C[M]$ has no value.)

   (c) Give a type τ, a term M, a value V and a context $C[-]$ such that $M \not\Downarrow_\tau$ but $C[M] \Downarrow_\tau V$

2. Prove that the following terms M and N are not contextually equivalent.

   (a) $M = \textbf{if } x \textbf{ then } 0 \textbf{ else } 1$ and $N = \textbf{if } y \textbf{ then } 0 \textbf{ else } 1$.

   (b) $M = \textbf{fn}\, x : \textbf{nat}.\textbf{succ}(\textbf{pred}\, x)$ and $N = \textbf{fn}\, x : \textbf{nat}.x$.

3. (Exercise 6.5.2.) Define $\Omega_\tau = \textbf{fix}(\textbf{fn}\, x : \tau.x)$

   (a) Show that $[\![\Omega_\tau]\!]$ is the least element of the domain $[\![\tau]\!]$.

   (b) Deduce that $[\![\textbf{fn}\, x : \tau.\Omega_\tau]\!] = [\![\Omega_{\tau\to\tau}]\!]$.

4. (a) Compute the denotational semantics of $M = \textbf{fn}\, x : \textbf{bool}.\, \textbf{fn}\, y : \textbf{nat}.\textbf{if}\, x\, \textbf{then}\, y\, \textbf{else}\, y$

   (b) Define a term N such that $[\![M]\!] = [\![N]\!]$ but $N \not\Downarrow M$.

5. Define terms $M, N : \textbf{nat} \to \textbf{nat}$ with $[\![M]\!] \sqsubseteq [\![N]\!]$ and $[\![M]\!] \neq [\![N]\!]$.

6. Verify that $[\![(\textbf{fn}\, x : \sigma.M)N]\!] = [\![M[N/x]]\!]$ for M, N with $\vdash N : \sigma$ and $x : \sigma \vdash M : \tau$. (Use the result on Slide 38, the Substitution Lemma.)