1. (Part of Exercise 6.5.1.) Prove the Proposition on Slide 38 for the cases
   (a) $M' = x$
   (b) $M' = \text{fn} y : \sigma. P$
   (c) $M' = PQ$.

2. Prove Theorem 6.4.1 for the inductive cases $\Downarrow_{\text{pred}}$ and $\Downarrow_{\text{if1}}$.
   Remember that Theorem 6.4.1 states that for all closed expressions $M$ and $V$ and type $\tau$, if
   $M \Downarrow_\tau V$, then $[M] = [V]$. It is proved by induction on the derivation of $M \Downarrow_\tau V$.

3. Prove the following properties (by induction on $\tau$). Here, $M, M_1, M_2$ range over closed terms,
   $d_1, d_2$ are domain elements.
   (a) If $d_2 \sqsubseteq d_1$ and $d_1 \triangleleft_\tau M_1$, then $d_2 \triangleleft_\tau M_1$.
   (b) If $d_1 \triangleleft_\tau M_1$ and $\forall V (M_1 \Downarrow_\tau V \Rightarrow M_2 \Downarrow_\tau V)$, then
       \[ d_1 \triangleleft_\tau M_2 \]
   (c) The set $\{d \in [[\tau]] \mid d \triangleleft_\tau M\}$ is chain-closed.

   These properties constitute Lemma 7.2.1 (iii) and (ii)

4. Remember that $\triangleleft_\tau$ denotes the approximation relation (slide 39).
   Show that, if $d \triangleleft_{\text{nat}} M$, $e \triangleleft_{\text{nat}} N$ and $b \triangleleft_{\text{bool}} P$, then
   \[ \text{if}(b, d, e) \triangleleft_{\text{nat}} \text{if } P \text{ then } M \text{ else } N \]
   (This is the "if" inductive case in the proof of the Fundamental Property, Slide 40)