Semantics and Domain theory

Exercises 2

NB. We write $\mathsf{State} \rightharpoonup \mathsf{State}$ for the set of partial functions from State to State.

1. Do exercise 1.2.1. of Winskel at the end of Chapter 1.

2. (a) Define the denotational semantics of **repeat** P **until** b as a fixed point of a function $g : (\mathsf{State} \rightharpoonup \mathsf{State}) \to (\mathsf{State} \rightharpoonup \mathsf{State})$.
   (NB this program executes statement P and then checks the boolean b; if b holds, execution stops, if b doesn't hold, it iterates.)

   (b) Define a denotational semantics for the statement **for** $x := e_1$ **to** e_2 **do** P:

   i. First with e_1, e_2 fixed numbers in $\mathbb{Z}$, say n and m.

   ii. Discuss some of the choices and problems with giving the general semantics, where e_1 and e_2 are arbitrary expressions.
   What semantics would you give to **for** $x := 1$ **to** $x + 1$ **do** **skip** ? And to **for** $x := 1$ **to** 3 **do** $x := x - 1$?

3. Consider the function $f : (\mathsf{State} \rightharpoonup \mathsf{State}) \to (\mathsf{State} \rightharpoonup \mathsf{State})$ as defined by Winskel on slide 6, but now with $\mathsf{State} = \mathsf{V} \to \mathbb{Z}$:

$$
\begin{array}{lcll}
f(w)(s) & := & s & \text{if } s(x) \leq 0 \\
f(w)(s) & := & w(s[x \mapsto s(x) - 1, y \mapsto s(x) * s(y)]) & \text{if } s(x) > 0
\end{array}
$$

   (a) Do exercise 1.2.2. of Winskel at the end of Chapter 1.

   (b) Prove $f(w_\infty) = w_\infty$ for $w_\infty : \mathsf{State} \rightharpoonup \mathsf{State}$ as defined a la Winskel's notes. (First redefine w_∞ for our notion of State.)

   (c) Prove that $\forall\, s \in \mathsf{State}\, \exists\, n[f^n(\perp)(s) = f^{n+1}(\perp)(s)]$.