SADT Exam July 2 2014

Ex. 2

$$g(\mathrm{fix}\, f) = g\Big(\bigsqcup_{n\in\mathbb{N}} f^n(\bot)\Big) = \bigsqcup_{n\in\mathbb{N}} g(f^n(\bot))$$

$$= \bigsqcup_n f^n(g(\bot)) = \bigsqcup_{n\in\mathbb{N}} f^{n+1}(\bot) = \mathrm{fix}\, f$$

So $\mathrm{fix}\, f$ is a fixed point of g

so $\mathrm{fix}\, g \sqsubseteq \mathrm{fix}\, f$.

Similarly: $\mathrm{fix}\, f \sqsubseteq \mathrm{fix}\, g$

So $\mathrm{fix}\, f = \mathrm{fix}\, g$.

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$\boxed{\text{Ex. 3}}$

(a) pimp is monotone in each of its arguments, so it is monotone

<u>Proof</u>: NB: $pimp(ff, x) = tt$ for all x

$pimp(x, tt) = tt$ for all x

The only interesting cases to consider are

(1) $a \neq \bot$ $\quad pimp(\bot, x) \overset{?}{\sqsubseteq} pimp(a, x)$

(2) $a \neq \bot$ $\quad pimp(x, \bot) \overset{?}{\sqsubseteq} pimp(x, a)$

<u>$a = tt$</u> then $x = tt$ & both are tt

$x = \bot$ then $pimp(\bot, \bot) = \bot = pimp(tt, \bot)$

$x = ff$ then $pimp(\bot, ff) = \bot \sqsubseteq pimp(tt, ff) = ff$

<u>$a = ff$</u> then $pimp(ff, x) = tt \sqsupseteq pimp(\bot, x)$ for all x

<u>$a = tt$</u> then $pimp(x, tt) = tt \sqsupseteq pimp(x, \bot)$ for all x

<u>$a = ff$</u> — $x = tt$ $\quad pimp(tt, \bot) = \bot$ &

$x = \bot$ $\quad pimp(\bot, \bot) = \bot$ &

$x = ff$ $\quad pimp(ff, \bot) = tt \sqsubseteq pimp(ff, ff)$ &

(b) pimp is continuous because monotone $\Rightarrow$ continuous for functions on domains where all chains are eventually finite.

(c) We can define por using pimp. Suppose that M defines pimp.

Then $P := \text{fn } x, y : bool.\ M\ (neg\ x)\ y$ defines por. because

$[\![P]\!]\ (ff, ff) = ff$

$[\![P]\!]\ (tt, x) = tt$

$[\![P]\!]\ (x, tt) = tt$

(d) Change $pimp(\bot, tt)$ to $\bot$. Take $N = \text{fn } x, y : bool.$ if x then y else true

then $[\![N]\!]$ is the adapted function