

Exam S&DT 2014 (June)

Ex 6.

M has no head normal form, so $\llbracket M \rrbracket = \emptyset$

N has a head normal form, so $\llbracket N \rrbracket \neq \emptyset$ [we don't even need this, we just need $\llbracket M \rrbracket \subseteq \llbracket N \rrbracket$]

$$\begin{aligned}\llbracket \lambda x. x M \rrbracket &= G(\lambda d \in D_A. F(d) \llbracket M \rrbracket) \\ &= \{(\beta, b) \mid b \in F(\beta) \llbracket M \rrbracket\} \\ &= \{(\beta, b) \mid b \in \{b' \mid \exists \beta' \subseteq \llbracket M \rrbracket. (\beta', b') \in \beta\}\} \\ &= \{(\beta, b) \mid \exists \beta' \subseteq \llbracket M \rrbracket. (\beta', b) \in \beta\} \\ &\stackrel{\text{by } \otimes}{=} \{(\beta, b) \mid \exists \beta' \subseteq \llbracket N \rrbracket. (\beta', b) \in \beta\} \\ &= \llbracket \lambda x. x N \rrbracket\end{aligned}$$