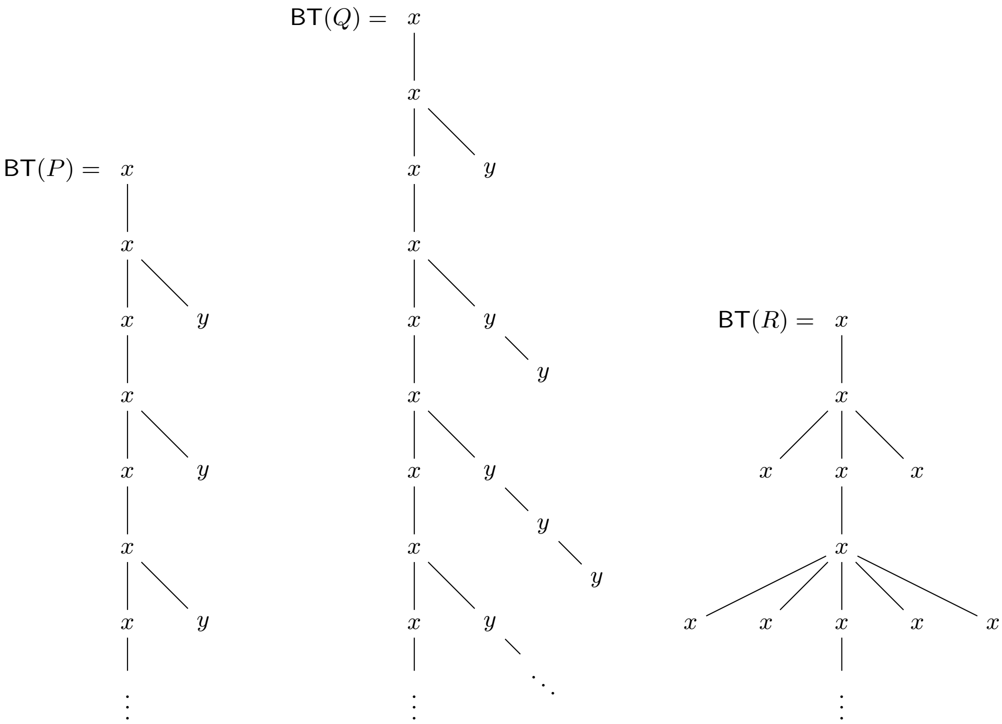
Semantics and Domain theory

Exercises 13

1. Define $T := \lambda x.x\,y\,(x\,x)$ and $M := T\,T$.
   (a) Draw the Böhm tree of M.
   (b) Describe the set of approximations of M, $\mathcal{A}(M)$. (You may describe this using a grammar or an inductive definition.)
2. Remember that the $\mathbf{S}$ combinator is defined as $\lambda x\,y\,z.x\,z\,(y\,z)$.
   (a) Draw the Böhm tree of $\mathbf{S}\,\mathbf{S}\,\mathbf{S}$.
   (b) Give the approximations of $\mathbf{S}\,\mathbf{S}\,\mathbf{S}$, that is, describe $\mathcal{A}(\mathbf{S}\,\mathbf{S}\,\mathbf{S})$.
3. Suppose that the terms B_1 and B_2 satisfy, respectively, $B_1 = x\,(x\,B_1\,B_1)\,B_1$ and $B_2 = x\,B_2\,(x\,B_2\,B_2)$.
   (a) Draw the Böhm trees of B_1 and B_2.
   (b) Do we have $\mathcal{E} \models B_1 = B_2$?
4. (a) Give a term P that has the Böhm tree given below.
   (b) (Hard) Give a term Q that has the Böhm tree given below.
   (c) (Challenge) Give a term R that has the Böhm tree given below.



5. Prove that all fixed point combinators have the same interpretation in $\mathcal{E}$
6. Let M and N be λ-terms that satisfy the following equations

$$M = \lambda xy.x\,(M\,x\,y)\,(M\,x\,y)$$
$$N = \lambda xy.x\,(N\,x\,x)\,(N\,x\,x)$$

   Prove that $M = N$ in $\mathcal{E}$.