Example logic programs

$$Parent(julia, augustus) \leftarrow$$
$$Female(julia) \leftarrow$$
$$Mother(x, y) \leftarrow \; Parent(x, y), Female(x)$$

$$Plus(0, x, x) \leftarrow$$
$$Plus(Sx, y, Sz) \leftarrow Plus(x, y, z)$$
$$Times(0, x, 0) \leftarrow$$
$$Times(Sx, y, u)) \leftarrow Times(x, y, z), Plus(z, y, u)$$

$$Member(x, \bullet(x, y)) \leftarrow$$
$$Member(x, \bullet(z, y)) \leftarrow Member(x, y)$$

Goal

A goal is a Horn clause with no head:
$\leftarrow B_1, \ldots, B_l$

The literals $B_1, \ldots, B_l$ are the subgoals. The empty clause is a goal without subgoals. For this we write $\leftarrow$.

Examples

$\leftarrow Mother(julia, augustus)$
$\leftarrow Parent(julia, x)$
$\leftarrow Parent(x, julia)$

$\leftarrow Plus(x, y, SSSS0), Times(x, y, SSSS0)$

Propositional resolution

A *resolution derivation* or *resolution proof* is a sequence of zero, one, or more applications of resolution. If φ is the last line in the derivation with assumptions from Π, then we write $\Pi \vdash \varphi$.
A *refutation* is a resolution proof of the empty clause $\leftarrow$.
The refutation $\Pi \vdash$ '$\leftarrow$' corresponds to the inconsistency $\Pi \models \bot$.

Example

1. $Parent(julia, augustus) \leftarrow$
2. $Female(julia) \leftarrow$
3. $Mother(julia, augustus) \leftarrow Parent(julia, augustus), Female(julia)$
4. $\leftarrow Mother(julia, augustus)$ (Goal)
5. $\leftarrow Parent(julia, augustus), Female(julia)$ (3,4)
6. $\leftarrow Parent(julia, augustus)$ (2,5)
7. $\leftarrow$ (1,6)

Completeness

Lvl Fact 16.3

Fact: Propositional resolution is (sound and) complete for deducing propositional consequences.

We need to be able to remove more than one occurrence of an atom at the time.

Try getting the empty clause from
$p, p \leftarrow$ and $\leftarrow p, p$
Now, we can only get
$p \leftarrow p$

Completeness

Lvl Fact 16.3

Fact: Propositional resolution is (sound and) complete for deducing propositional consequences.

We need to be able to remove more than one occurrence of an atom at the time.

Try getting the empty clause from
p, $p \leftarrow$ and $\leftarrow p$, p
Now, we can only get
$p \leftarrow p$

Be careful: do not remove two *different* atoms at the same time:
From $p, q \leftarrow$ and $\leftarrow p, q$, we do not get $\leftarrow$. Why?

Towards general resolution: Simultaneous substitution

Simultaneous substitution of terms t_1, ..., t_n for variables x_1, ..., x_n : $[x_1 = t_1, ..., x_n = t_n]$.

If θ is a substition, we write θA for the result of applying θ to A.

Unifiable
Two expressions A and B are *unifiable* if there is a substitution θ such that $\theta A = \theta B$. The substitution θ is called a *unifier*.
A substitution θ is a *most general unifier* (*mgu*) of A and B if all unifiers τ of A and B are more specific than θ.

Examples

- $P(x, f(y))$ and $P(a, f(g(z)))$ unify via $[x = a, y = g(z)]$
- $P(x, f(y))$ and $P(a, f(g(z)))$ *also* unify via

$[x{=}a, y{=}g(b), z{=}b]$

The first is an *mgu* (most general unifier)

- $Q(a, g(x, a), f(y))$ and $Q(a, g(f(b), a), x)$ unify via

$[x = f(b), y = b]$

- $R(h(x), c)$ en $R(f(a), y)$ do not unify

- $Mother(julia, augustus)$ en $Mother(x, y)$ unify via

$[x = julia, y = augustus]$

Unification algorithm

Lvl 16.5

Start with $Eq := [u = t]$. The variable Eq is called 'the equations'. Choose an equation from Eq having one of the following (mutually exclusive) forms. Apply the corresponding action. Repeat the procedure until no action can be performed.

- If $f(u_1, ..., u_n) = f(t_1, ..., t_n)$, replace by $u_1 = t_1, ..., u_n = t_n$;
- If $g(u_1, ..., u_m) = f(t_1, ..., t_n)$ and $g \neq f$ or $m \neq n$, failure;
- If $x = x$, remove;
- If $f(t_1, ..., t_n) = x$, replace by $x = f(t_1, ..., t_n)$;
- If $x = f(t_1, ..., t_n)$ and x occurs in $f(t_1, ..., t_n)$, failure.
- If $x = f(t_1, ..., t_n)$, x not in $f(t_1, ..., t_n)$, but x occurs in other equations: replace all other x by $f(t_1, ..., t_n)$;

After successful termination of the algorithm, Eq is an *mgu* of t and u.

Example

Do $Q(a, g(f(b), a), x)$ and $Q(a, g(x, a), f(y))$ unify?
As usual, a and b are constants, x and y are variables.

$Eq := [Q(a, g(f(b), a), x) = Q(a, g(x, a), f(y))]$
becomes $[a = a, g(f(b), a) = g(x, a), x = f(y)]$
becomes $[g(f(b), a) = g(x, a), x = f(y)]$
becomes $[f(b) = x, a = a, x = f(y)]$
becomes $[x = f(b), a = a, x = f(y)]$
becomes $[x = f(b), x = f(y)]$
becomes $[x = f(b), f(b) = f(y)]$
becomes $[x = f(b), b = y]$
becomes $[x = f(b), y = b]$

So, $[x = f(b), y = b]$ is a most general unifier of the two terms.