



Normalisation for general constructive propositional logic

Herman Geuvers
(joint work with Tonny Hurkens and Iris van der Giessen)

Institute for Computing and Information Science
Radboud University
Nijmegen, NL

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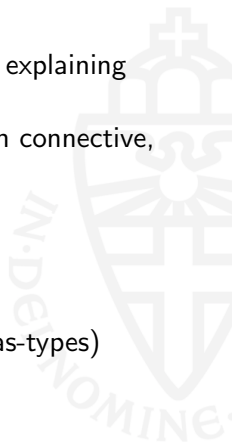
Constructive Logic

Constructively the meaning of a connective is fixed by explaining what a **proof** is that involves the connective.

Basically, this explains the **introduction rule(s)** for each connective, from which the elimination rules follow (Prawitz)

By analysing constructive proofs we then also get

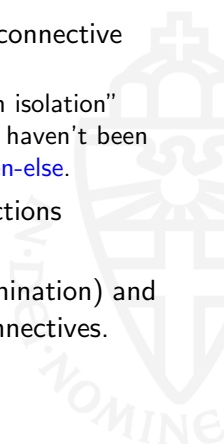
- consistency (from proof normalization),
- decidability (from the subformula property),
- Curry-Howard **proofs-as-terms** (and propositions-as-types) allowing to study normalization as term-reduction





Our work / Overview of the talk

- Derive **constructive** natural deduction rules for a connective from its **truth table** definition.
 - Gives natural deduction rules for a connective “in isolation”
 - Also gives constructive rules for connectives that haven't been studied constructively so far, like **nand** and **if-then-else**.
- Curry-Howard: give proof-terms for natural deductions
- Define proof-normalization as term-reduction
⇒ A general notion of **detour conversion** (cut-elimination) and **permutation conversion** for these constructive connectives.
- Weak normalization
- Strong normalization





Standard form for natural deduction rules

$$\frac{\Gamma \vdash A_1 \quad \dots \quad \Gamma \vdash A_n \quad \Gamma, B_1 \vdash D \quad \dots \quad \Gamma, B_m \vdash D}{\Gamma \vdash D}$$

If the conclusion of a rule is $\Gamma \vdash D$, then the hypotheses of the rule can be of one of two forms:

- ① $\Gamma, B \vdash D$. We are given extra data B to prove D from Γ . We call B a **Case**.
- ② $\Gamma \vdash A$. instead of proving D from Γ , we now need to prove A from Γ . We call A a **Lemma**.

One obvious advantage: we don't have to give Γ explicitly, as it can be retrieved:

$$\frac{\vdash A_1 \quad \dots \quad \vdash A_n \quad B_1 \vdash D \quad \dots \quad B_m \vdash D}{\vdash D}$$



Constructive natural deduction rules from truth tables

Let c be an n -ary connective c with truth table t_c .

Each row of t_c gives rise to an elimination rule or an introduction rule for c . (We write $\varphi = c(A_1, \dots, A_n)$.)

elimination

$$\frac{A_1 \quad \dots \quad A_n \quad | \quad \varphi}{p_1 \quad \dots \quad p_n \quad | \quad 0} \mapsto \frac{\vdash \varphi \dots \vdash A_j \text{ (if } p_j = 1) \dots A_i \vdash D \text{ (if } p_i = 0) \dots}{\vdash D} \text{el}$$

introduction

$$\frac{A_1 \quad \dots \quad A_n \quad | \quad \varphi}{q_1 \quad \dots \quad q_n \quad | \quad 1} \mapsto \frac{\dots \vdash A_j \text{ (if } q_j = 1) \dots A_i \vdash \varphi \text{ (if } q_i = 0) \dots}{\vdash \varphi} \text{in}^i$$

This is the **constructive** introduction rule; there is also a **classical** introduction rule, which we don't discuss now.



Definition of the logics

Given a set of connectives $\mathcal{C} := \{c_1, \dots, c_n\}$, the **constructive** natural deduction systems for \mathcal{C} , $\text{IPC}_{\mathcal{C}}$, has the following rules.

- The **axiom rule**

$$\frac{}{\Gamma \vdash A} \text{ axiom} \quad (\text{if } A \in \Gamma)$$

- The **constructive introduction rules** for the connectives in \mathcal{C} , as derived from the truth table.
- The **elimination rules** for the connectives in \mathcal{C} , as derived from the truth table.



Examples

Derivation rules for \wedge (3 elim rules and one intro rule):

A	B	$A \wedge B$
0	0	0
0	1	0
1	0	0
1	1	1

$$\frac{\vdash A \wedge B \quad A \vdash D \quad B \vdash D}{\vdash D} \wedge\text{-el}_a$$

$$\frac{\vdash A \wedge B \quad A \vdash D \quad \vdash B}{\vdash D} \wedge\text{-el}_b$$

$$\frac{\vdash A \wedge B \quad \vdash A \quad B \vdash D}{\vdash D} \wedge\text{-el}_c$$

$$\frac{\vdash A \quad \vdash B}{\vdash A \wedge B} \wedge\text{-in}$$

- These rules can be shown to be equivalent to the well-known derivation rules.
- These rules can be optimized to 3 rules.



Examples

Rules for \neg : 1 elimination rule and 1 introduction rule.

A	$\neg A$
0	1
1	0

Derivation rules:

$$\frac{\vdash \neg A \quad \vdash A}{\vdash D} \neg\text{-el} \qquad \frac{A \vdash \neg A}{\vdash \neg A} \neg\text{-in}$$





The well-known constructive connectives

The optimised rules for \wedge , \neg , \vee , \rightarrow , \top and \perp we obtain are:

$$\begin{array}{c}
 \frac{\vdash A \quad \vdash B}{\vdash A \wedge B} \wedge\text{-in} \qquad \frac{\vdash A \wedge B}{\vdash A} \wedge\text{-el}_1 \qquad \frac{\vdash A \wedge B}{\vdash B} \wedge\text{-el}_2 \\
 \\
 \frac{\vdash A \vee B \quad A \vdash D \quad B \vdash D}{\vdash D} \vee\text{-el} \qquad \frac{\vdash A}{\vdash A \vee B} \vee\text{-in}_1 \qquad \frac{\vdash B}{\vdash A \vee B} \vee\text{-in}_2 \\
 \\
 \frac{\vdash A \rightarrow B \quad \vdash A}{\vdash B} \rightarrow\text{-el} \qquad \frac{\vdash B}{\vdash A \rightarrow B} \rightarrow\text{-in}_1 \qquad \frac{A \vdash A \rightarrow B}{\vdash A \rightarrow B} \rightarrow\text{-in}_2 \\
 \\
 \frac{\vdash \neg A \quad \vdash A}{\vdash D} \neg\text{-el} \qquad \frac{A \vdash \neg A}{\vdash \neg A} \neg\text{-in} \qquad \frac{}{\vdash \top} \top\text{-in} \qquad \frac{\vdash \perp}{\vdash D} \perp\text{-el}
 \end{array}$$



Sheffer stroke or NAND connective [I]

The truth table for $\text{nand}(A, B)$, which we write as $A \uparrow B$ is as follows.

A	B	$A \uparrow B$
0	0	1
0	1	1
1	0	1
1	1	0

From this we derive the following optimized rules.

$\frac{A \vdash A \uparrow B}{\vdash A \uparrow B} \uparrow\text{-inl}$	$\frac{B \vdash A \uparrow B}{\vdash A \uparrow B} \uparrow\text{-inr}$	$\frac{\vdash A \uparrow B \quad \vdash A \quad \vdash B}{\vdash D} \uparrow\text{-el}$
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Sheffer stroke or NAND connective [II]

The usual connectives can be defined in terms of nand.

$$\begin{aligned} \neg A &:= A \uparrow A \\ A \vee B &:= (A \uparrow A) \uparrow (B \uparrow B) \\ A \wedge B &:= (A \uparrow B) \uparrow (A \uparrow B) \\ A \rightarrow B &:= A \uparrow (B \uparrow B) \end{aligned}$$

This gives rise to an embedding $(-)^{\uparrow}$ of intuitionistic propositional logic \vdash_i into the nand-logic \vdash_{\uparrow} .

PROPOSITION For A a formula in propositional logic,

$$\vdash_i \neg\neg A \quad \iff \quad \vdash_{\uparrow} (A)^{\uparrow}.$$





Detours (cuts) in constructive logic

Remember that the rules for c arise from rows in the truth table t_c :

A_1	...	A_n	$c(A_1, \dots, A_n)$
p_1	...	p_n	0
q_1	...	q_n	1

DEFINITION A **detour convertibility** is a pattern of the following form, where $\varphi = c(A_1, \dots, A_n)$.

$$\frac{\dots \frac{\boxed{\Sigma_j}}{\Gamma \vdash A_j} \dots \frac{\boxed{\Sigma_i}}{\Gamma, A_i \vdash \varphi} \dots}{\Gamma \vdash \varphi} \text{ in} \quad \frac{\dots \frac{\boxed{\Pi_k}}{\Gamma \vdash A_k} \dots \frac{\boxed{\Pi_\ell}}{\Gamma, A_\ell \vdash D} \dots}{\Gamma \vdash D} \text{ el}$$

- $q_j = 1$ for A_j and $q_i = 0$ for A_i
- $p_k = 1$ for A_k and $p_\ell = 0$ for A_ℓ



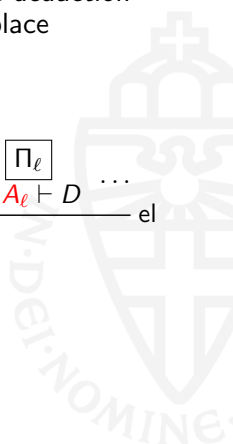
Eliminating a detour (detour conversion) (I)

The *elimination of a detour* is defined by replacing the deduction pattern by another one. If $A_j = A_\ell$ (for some j, ℓ), replace

$$\frac{\begin{array}{c} \dots \frac{\boxed{\Sigma_j}}{\Gamma \vdash A_j} \dots \frac{\boxed{\Sigma_i}}{\Gamma, A_i \vdash \varphi} \dots \\ \hline \Gamma \vdash \varphi \end{array} \text{ in} \quad \begin{array}{c} \dots \frac{\boxed{\Pi_k}}{\Gamma \vdash A_k} \dots \frac{\boxed{\Pi_\ell}}{\Gamma, A_\ell \vdash D} \dots \\ \hline \Gamma \vdash D \end{array} \text{ el}}{\Gamma \vdash D}$$

by

$$\begin{array}{c} \frac{\boxed{\Sigma_j}}{\Gamma \vdash A_j} \dots \frac{\boxed{\Sigma_j}}{\Gamma \vdash A_j} \\ \frac{\boxed{\Pi_\ell}}{\Gamma \vdash D} \end{array}$$





Eliminating a detour (detour conversion) (II)

If $A_i = A_k$ (for some i, k), replace

$$\frac{\dots \frac{\boxed{\Sigma_j}}{\Gamma \vdash A_j} \dots \frac{\boxed{\Sigma_i}}{\Gamma, A_i \vdash \varphi} \dots}{\Gamma \vdash \varphi} \text{ in } \dots \frac{\boxed{\Pi_k}}{\Gamma \vdash A_k} \dots \frac{\boxed{\Pi_\ell}}{\Gamma, A_\ell \vdash D} \dots}{\Gamma \vdash D} \text{ el}$$

by

$$\frac{\frac{\boxed{\Pi_k}}{\Gamma \vdash A_k} \dots \frac{\boxed{\Pi_k}}{\Gamma \vdash A_k} \quad \frac{\boxed{\Sigma_i}}{\Gamma \vdash \varphi} \quad \dots \quad \frac{\boxed{\Pi_k}}{\Gamma \vdash A_k} \quad \dots \quad \frac{\boxed{\Pi_\ell}}{\Gamma, A_\ell \vdash D} \quad \dots}{\Gamma \vdash D} \text{ el}$$



Observation

$$\frac{\dots \quad \boxed{\Sigma_j} \quad \Gamma \vdash A_j \quad \dots \quad \boxed{\Sigma_i} \quad \Gamma, A_i \vdash \varphi \quad \dots}{\Gamma \vdash \varphi} \text{ in} \quad \dots \quad \boxed{\Pi_k} \quad \Gamma \vdash A_k \quad \dots \quad \boxed{\Pi_\ell} \quad \Gamma, A_\ell \vdash D \quad \dots}{\Gamma \vdash D} \text{ el}$$

- There can be several “matching” (i, k) or (j, ℓ) pairs.
- So: detour conversion (“ β -rule”) is non-deterministic in general.



Permutation convertibility: Definition

Let c and c' be connectives of arity n and n' , with elimination rules r and r' respectively. A **permutation convertibility** in a derivation is a pattern of the following form, where $\Phi = c(B_1, \dots, B_n)$, $\Psi = c'(A_1, \dots, A_{n'})$.

$$\frac{\frac{\vdots \boxed{\Sigma_j} \quad \vdots \boxed{\Sigma_i}}{\vdash \Psi \dots \vdash A_j \quad \dots \quad A_i \vdash \Phi \quad \dots} \text{el}_{r'}}{\vdash \Phi} \quad \frac{\vdots \boxed{\Pi_k} \quad \vdots \boxed{\Pi_\ell}}{\dots \vdash B_k \quad \dots \quad B_\ell \vdash D \quad \dots} \text{el}_r}{\vdash D}$$

- A_j ranges over all propositions that have a 1 in the truth table of c' ; A_i ranges over all propositions that have a 0,
- B_k ranges over all propositions that have a 1 in the truth table of c ; B_ℓ ranges over all propositions that have a 0.



Permutation conversion

The **permutation conversion** is defined by replacing the derivation pattern on the previous slide by

$$\frac{\vdash \Psi \dots \vdash A_j \quad \dots \quad \frac{\vdots \boxed{\Sigma_j} \quad \vdots \boxed{\Sigma_i} \quad A_i \vdash \Phi \quad \dots \quad \vdots \boxed{\Pi_k} \quad A_i \vdash B_k \quad \dots \quad \vdots \boxed{\Pi_\ell} \quad A_i, B_\ell \vdash D \quad \dots}{A_i \vdash D} \text{el}_r}{\vdash D} \text{el}_{r'}$$

This gives rise to copying of sub-derivations: for every A_i we copy the sub-derivations Π_1, \dots, Π_n .



Curry-Howard proofs-as-terms

We define rules for the judgment $\Gamma \vdash t : A$, where

- A is a formula,
- Γ is a set of declarations $\{x_1 : A_1, \dots, x_m : A_m\}$, where the A_i are formulas and the x_i are term-variables,
- t is a **proof-term**:

$$t ::= x \mid \{\bar{t}; \overline{\lambda x : A.t}\}_r \mid t \cdot_r [\bar{t}; \overline{\lambda x : A.t}]$$

where x ranges over variables and r ranges over the rules.

For a connective $c \in \mathcal{C}$, r an introduction rule for c and r' an elimination rule for c , we have

- an **introduction term** $\{\bar{t}; \overline{\lambda x : A.t}\}_r$
- an **elimination term** $t \cdot_{r'} [\bar{t}; \overline{\lambda x : A.t}]$



Curry-Howard typing rules

Let $\Phi = c(A_1, \dots, A_n)$ and r a rule for c .

$$\begin{array}{c}
 \frac{}{\Gamma \vdash x_i : A_i} \text{ if } x_i : A_i \in \Gamma \\
 \\
 \frac{\dots \Gamma \vdash p_j : A_j \dots \quad \dots \Gamma, y_i : A_i \vdash q_i : \Phi \dots}{\Gamma \vdash \{\bar{p} ; \overline{\lambda y : A.q}\}_r : \Phi} \text{ in} \\
 \\
 \frac{\Gamma \vdash t : \Phi \quad \dots \Gamma \vdash p_k : A_k \dots \quad \dots \Gamma, y_\ell : A_\ell \vdash q_\ell : D}{\Gamma \vdash t \cdot_r [\bar{p} ; \overline{\lambda y : A.q}] : D} \text{ el}
 \end{array}$$

Here, \bar{p} is the sequence of terms $p_1, \dots, p_{m'}$ for all the 1-entries in rule r of the truth table, and $\overline{\lambda y : A.q}$ is the sequence of terms $\lambda y_1 : A_1.q_1, \dots, \lambda y_m : A_m.q_m$ for all the 0-entries in r .



Reductions on terms for detours

Term reduction rules that correspond to **detour conversions**.

- For simplicity we write the “matching cases” as last term of the sequence.
- For the $A_j = A_\ell$ case, that is, $p_j : A_j$ and $y_\ell : A_\ell$ with $A_j = A_\ell$:

$$\{\overline{p}, \overline{p_j} ; \overline{\lambda x. q}\} \cdot [\overline{s} ; \overline{\lambda y. r}, \overline{\lambda y_\ell. r_\ell}] \longrightarrow_a r_\ell[y_\ell := p_j]$$

- For the $A_i = A_k$ case, that is, $x_i : A_i$ and $s_k : A_k$ with $A_i = A_k$:

$$\{\overline{p} ; \overline{\lambda x. q}, \overline{\lambda x_i. q_i}\} \cdot [\overline{s}, \overline{s_k} ; \overline{\lambda y. r}] \longrightarrow_a q_i[x_i := s_k] \cdot [\overline{s}, \overline{s_k} ; \overline{\lambda y. r}]$$

$\overline{p}, \overline{p_j}$ should be understood as a sequence $p_1, \dots, p_j, \dots, p_{m'}$, where the p_j that matches the r_ℓ in $\overline{\lambda y. r}, \overline{\lambda y_\ell. r_\ell}$ has been singled out.

NB There is always (at least one) **matching case**, because intro/elim rules comes from different lines in the truth table.



Reductions on terms for permutations

We add the following reduction rules for **permutation conversions**.

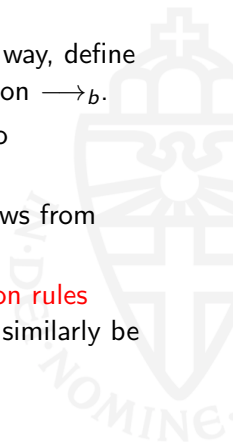
$$(t \cdot_r [\bar{p}; \overline{\lambda x. q}]) \cdot_{r'} [\bar{s}; \overline{\lambda y. r}] \longrightarrow_b t \cdot_r [\bar{p}; \overline{\lambda x. (q \cdot_{r'} [\bar{s}; \overline{\lambda y. r}])}]$$

Here, $\overline{\lambda x. (q \cdot [\bar{s}; \overline{\lambda y. r}])}$ should be understood as a sequence $\lambda x_1. q_1, \dots, \lambda x_m. q_m$ where each q_j is replaced by $q_j \cdot_{r'} [\bar{s}; \overline{\lambda y. r}]$.



Optimized reductions on optimized terms

- On optimized terms, one can also, in a canonical way, define detour conversion \longrightarrow_a and permutation conversion \longrightarrow_b .
- Detour reduction on optimised terms translates to (multi-step) detour reduction on the full terms.
- So, strong normalization on optimised terms follows from strong normalization on full terms.
- Other well-known rules, like the **general elimination rules** studied by Schroeder-Heister and Von Plato, can similarly be translated to our full rules.





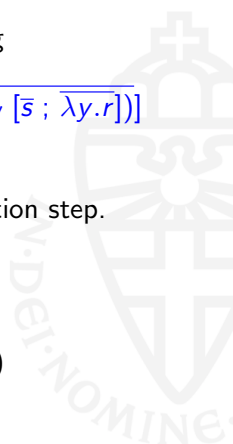
Normalization

THEOREM The reduction \longrightarrow_b is strongly normalizing

$$(t \cdot_r [\bar{p}; \overline{\lambda x. q}]) \cdot_{r'} [\bar{s}; \overline{\lambda y. r}] \longrightarrow_b t \cdot_r [\bar{p}; \overline{\lambda x. (q \cdot_{r'} [\bar{s}; \overline{\lambda y. r}])}]$$

PROOF The measure $| - |$ decreases with every reduction step.

$$\begin{aligned} |x| &:= 1 \\ |\{\bar{p}; \overline{\lambda y. q}\}| &:= \Sigma |p_i| + \Sigma |q_j| \\ |t \cdot [\bar{s}; \overline{\lambda y. u}]| &:= |t|(2 + \Sigma |s_k| + \Sigma |u_\ell|) \end{aligned}$$





Normalization

THEOREM The reduction \rightarrow_a is strongly normalizing.

$$\{\overline{p}, \overline{p_j}; \overline{\lambda x. q}\} \cdot [\overline{s}; \overline{\lambda y. r}, \overline{\lambda y_e. r_e}] \rightarrow_a r_e[y_e := p_j]$$

(for the $A_j = A_e$ case, $p_j : A_j$ and $y_e : A_e$ with $A_j = A_e$)

$$\{\overline{p}; \overline{\lambda x. q}, \overline{\lambda x_i. q_i}\} \cdot [\overline{s}, \overline{s_k}; \overline{\lambda y. r}] \rightarrow_a q_i[x_i := s_k] \cdot [\overline{s}, \overline{s_k}; \overline{\lambda y. r}]$$

(for the $A_i = A_k$ case, $x_i : A_i$ and $s_k : A_k$ with $A_i = A_k$)

PROOF We adapt the saturated sets method of Tait.

COROLLARY the combination \rightarrow_{ab} is weakly normalizing.

Basically: take the \rightarrow_b -normal-form and then contract the innermost \rightarrow_a -redex of **highest rank**. (This generalizes the Gandy-Turing SN proof for simple type theory, $\lambda \rightarrow$.)



Strong Normalization

New: we have obtained a proof of Strong Normalization for general IPC_C .

Rough outline of the proof (generalizing a proof of SN for IPC by Philippe De Groote):

- Define a “double negation” translation from IPC_C formulas to $\lambda \rightarrow$ -types.
- Define a reduction preserving “CPS” translation from IPC_C terms to $\lambda \rightarrow$ -parallel.
($\lambda \rightarrow$ extended with $[M_1, \dots, M_n] : A$ if $M_i : A$ for $1 \leq i \leq n$.)
- Prove SN for $\lambda \rightarrow$ -parallel.



$\lambda \rightarrow$ -parallel

- Types: $\sigma ::= o \mid (\sigma \rightarrow \sigma)$
- Terms: $M ::= x \mid (M M) \mid (\lambda x.M) \mid [M_1, \dots, M_n] \ (n > 1)$.
- Typing rules

$$\frac{\Gamma \vdash M : A \rightarrow B \quad \Gamma \vdash N : A}{\Gamma \vdash M N : B} \qquad \frac{\Gamma, x : A \vdash M : B}{\Gamma \vdash \lambda x.M : A \rightarrow B}$$

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A} \qquad \frac{\Gamma \vdash M_1 : A \quad \dots \quad \Gamma \vdash M_n : A}{\Gamma \vdash [M_1, \dots, M_n] : A}$$

- Reduction rules: $(\lambda x.M) N \longrightarrow_{\beta} M[x := N]$ plus

$$[M_1, \dots, M_n] N \longrightarrow_{\beta} [M_1 N, \dots, M_n N]$$

SN can be proved by adapting the well-known Tait proof.



Translating formulas to types (outline)

Abbreviate $\neg A := A \rightarrow o$.

- For a proposition letter, $\widehat{A} := \neg\neg A$.
- For $\Phi = c(A_1, \dots, A_n)$ with elimination rules r_1, \dots, r_t

$$\widehat{\Phi} := \neg(E_1 \rightarrow \dots \rightarrow E_t \rightarrow o),$$

where

$$E_s := \widehat{A}_{k_1} \rightarrow \dots \rightarrow \widehat{A}_{k_m} \rightarrow \neg\widehat{A}_{l_1} \rightarrow \dots \rightarrow \neg\widehat{A}_{l_{n-m}} \rightarrow o$$

with the A_k the 1-entries and the A_l are the 0-entries in the truth table.

For example

$$\widehat{A \wedge B} = \neg(\neg\neg\widehat{A} \rightarrow \neg\neg\widehat{B} \rightarrow o)$$

$$\widehat{A \vee B} = \neg((\neg\widehat{A} \rightarrow \neg\widehat{B} \rightarrow o) \rightarrow o)$$



Translating proof-terms to $\lambda \rightarrow$ -parallel terms (outline)

Let $\Phi = c(A_1, \dots, A_n)$ have elimination rules r_1, \dots, r_t .

- $\widehat{x} := \lambda h.x h$.
- Elimination term:

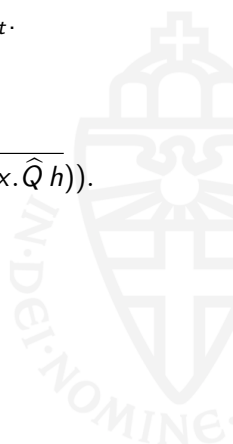
$$M \cdot_{r_s} \widehat{[N ; \lambda x.Q]} := \lambda h.\widehat{M} (\lambda g_1 \dots g_t.g_s \widehat{N}(\lambda x.\widehat{Q} h)).$$

- Introduction term

$$\{\widehat{N ; \lambda y.M}\}_r := \lambda h.h e_1^h \dots e_t^h,$$

where e_s^h is the possibly parallel term containing

- $\widehat{\lambda f.f_\ell \widehat{N}_j}$ for ℓ in rule r_s and j with $A_j = A_\ell$.
- $\widehat{\lambda f.(\lambda y_i.\widehat{M}_i h)}$ for k in rule r_s and i with $A_i = A_k$.





Translating proof-terms to $\lambda \rightarrow$ -parallel terms (outline)

Using the translation \widehat{M} we define a second translation $\widehat{\widehat{M}}$. (This is a generalization of the CPS translation $\overline{\overline{M}}$ of Plotkin, that De Groote also uses.)

We can prove

- If $M \rightarrow_b N$, then $\widehat{\widehat{M}} = \widehat{\widehat{N}}$
- If $\widehat{\widehat{M}} \subset K$ ($\widehat{\widehat{M}}$ is a **subterm** of K), then

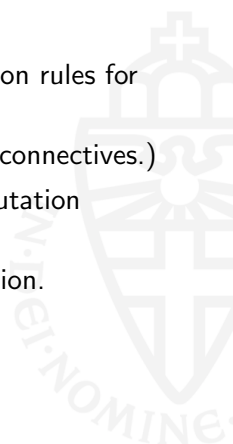
$$\begin{array}{ccccc}
 M & \mapsto & \widehat{\widehat{M}} & \subset & K \\
 \downarrow a & & & & \downarrow \beta + \\
 N & \mapsto & \widehat{\widehat{N}} & \subset & \exists K'
 \end{array}$$

From this we can derive Strong Normalization.



Conclusions

- Simple general way to derive constructive deduction rules for (new) connectives.
- Study connectives “in isolation”. (Without other connectives.)
- General definition of detour conversion and permutation conversion.
- General Curry-Howard proofs-as-terms interpretation.
- General Strong Normalization proof.





Future work and Related

- Meaning of the new connectives as inductive data types.
- Study conditions for the set of rules to be Church-Rosser.
- General definition of classical detour/permutation conversion
- Relation with other well-known term calculi for classical logic: subtraction logic (Crolard), $\lambda\mu$ (Parigot), $\bar{\lambda}\mu\tilde{\mu}$ (Curien, Herbelin).

Related work:

- Roy Dyckhoff, Peter Milne, Jan von Plato and Sara Negri, Peter Schroeder-Heister, . . .
- “Harmony” in logic (following Prawitz)



Questions?

