A Logical Ontology

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Abstract. We make an attempt to develop a Peircean ontology that is presupposed by propositional logic. The result of this can be seen as a first step towards linking Peirce's semiotic to his logic of relatives. Familiarity with Peirce's semiotic can be useful, but the thesis of this paper is intelligible without such a background.

1 Introduction

The aim of this paper is to develop an ontology presupposed by propositional logic on the basis of Peirce's semiotic ([9]). It will be argued that his definitions of signs involve the meaning of a proposition in the formal logical sense. It will be shown that if, as Peirce maintains, logic is equivalent to semiotic, propositional logic can be derived from a Peircean semiotic.

Peirce's classification of signs involves his triads, qualisign-sinsign-legisign, icon-index-symbol, and rheme-dicent-argument. The importance of these triads has been emphasised by most Peirce scholars ([2], [7], [8], [10]). Although Peirce developed also larger systems of signs, it will be advocated in this paper that his 'simple' classification is most practical.

By introducing a Peircean framework for propositional logic we make a first step towards linking Peirce's theory of signs to his logic of relatives. The latter theory serves as a basis for a model of logic known as 'Peirce algebras' ([1], [3]). Our analysis will point out that there is indeed a close relationship between the two theories.

The paper consists of three parts. In the first part, a recapitulation of Peirce's classification of signs will be followed by an attempt to derive a basic assumption from the properties of signs. Moreover, the validity of that assumption will be discussed in the context of human sign perception. In the second part, Peirce's signs will be analysed from the logical point of view. In the third part, the results of the analysis will be exemplified.

2 Towards the logical meaning of signs

Peirce's semiotic features an ingenious classification of signs as shown in fig. 1 (the meaning of the horizontal lines will be explained later on). Though Peirce distinguished between different kinds of signs[1], he did not, at least not to our knowledge, pursue an analysis of *how* those signs emerge. The barely hidden agenda of this paper is precisely to attempt to provide an answer to this question. It will be argued that (a) signs can arise via a binary *relation*, and (b) between *neigboring* signs. Based on this hypothesis, it will be shown that such relations have a formal logical meaning.

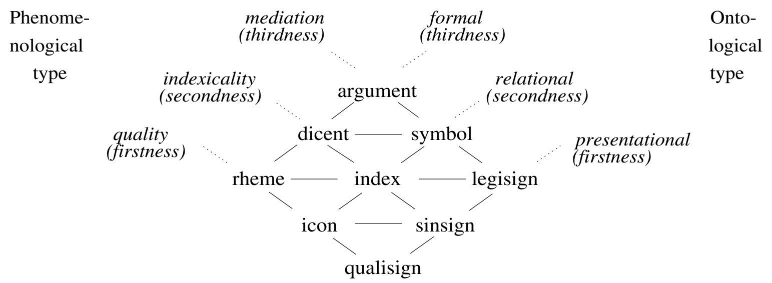

Fig. 1. Peirce's classification of signs

2.1 The relational nature of signs

Part (a) of our assumption above complies with Peirce's semiotic. In as much as signs are signifying events, they involve the aspect of secondness. Therefore, signs must involve a binary relation.

Part (b) can be justified on the basis of the category aspect of signs, as follows. Peirce's categories, firstness(1), secondness(2) and thirdness(3), however irreducible to one another, have a different degree of dependency in respect of one another. Though secondness cannot be reduced to firstness, it presupposes firstness, and, similarly, though thirdness cannot be reduced to either firstness or secondness, it presupposes both firstness

[1] Peirce's *typology* is concerned with the types of signs that arise in the representation of *a phenomenon*, i.e. how the sign behaves. But Peirce also introduced a classification of signs (consisting of ten classes) which considers the different meanings *a sign* can tell us about phenomena, i.e. the sign's affinity.

(through secondness) and secondness. This dependency of the categories can be formalised by the ordering 1<2<3.

From the dependency of categories it follows that a more elaborate sign must involve a less developed one. For example, a dicent must involve a rheme and, for the same reason, also an index (cf. fig. 1).

We define our neighboring relation on signs as follows. Let the category of a sign A from the phenomenological and ontological points of view be denoted as a pair (p_A, o_A) where $1 \leq p_A, o_A \leq 3$. Then, two different signs, A and B, are said to be *neighbors* if $|p_A - p_B| = |o_A - o_B| = 1$ and $p_A + o_A = p_B + o_B$. Neighboring signs are connected in fig. 1 by horizontal edges. These edges define a partitioning of signs into *levels*: level 0 (qualisign), level 1 (icon, sinsign), level 2 (rheme, index, legisign), level 3 (dicent, symbol) and level 4 (argument).

Intuitively, the meaning of a sign interaction must be more than the meaning of any of its participants. This follows from the fact that such a sign involves the meaning of the interaction as well.

Formally, we can represent the meaning of a sign, e.g. $A = (p_A, o_A)$, by the expression $p_A + o_A$. Now, let A and B denote neighboring signs. Then, the interaction between A and B must be the sign $S = (p_S, o_S)$ such that $(p_S + o_S) = (p_A + o_A) + 1 = (p_B + o_B) + 1$. Informally, this means that neighboring signs of some level i $(1 \leq i \leq 3)$ 'generate' the signs of level $i+1$. Later it will be argued that the qualisign provides us, without interaction, with the icon and sinsign, which completes the justification of our hypothesis.

In the rest of the paper it will be advocated that the above model also applies to *logical* signs, that is, to signs from the logical point of view.

2.2 The diversity of qualities

From the dependency of categories it follows that for the cognition of signs (which have the aspect of thirdness) we must be able to perceive differences (which have the aspect of secondness) and those differences must be rooted in qualities (which have the aspect of firstness). If not, as Peirce points out (cf. 3.464)[2], anything could be in relation to anything else, positively or negatively, and 'to be in relation' would mean nothing. Therefore, we may assume that there exist in the 'real' world different sorts of qualities. In the next section we will argue that human sign processing is capable of distinguishing between such qualities.

[2] A reference to [9] will be given in the form: volume and paragraph number.

2.3 Continuants and occurrents

In as much as signs are for the purpose of communication between speaker and hearer, a classification of logical signs can refer to the speaker, or the listener. Although the resulting ontologies must be isomorphic, sign production and sign recognition can be different. In this paper we will focus on sign *recognition*.

Clearly, the recognition of any sign must begin with the sensation of the physical input. In the remaining we will assume a *human* receiver (however, this does not restrict the validity of our analysis as, according to Peirce, non–human 'systems' must be sign–using as well).

Physical stimuli which are subject to selective attention, enter the human receiver via the senses which transform the raw data into internal sensation continuously. The output of the senses, a bio-electric signal, is processed by the brain in percepts. The generation of such a percept is triggered by a change in the input, typically, or by the duration of some sampling time, e.g. in the case of visual perception. The time necessary for an observation is 35..50ms, or more, much larger than the time necessary for the perception of an input change[3] which is 1..10ms ([6]).

The brain compares the current percept with the previous one, and this enables it to distinguish between two sorts of input qualities: one, which was there and remained there, something stable, which we will call a *continuant*; and another, which, though it was not there, is there now (or the other way round), something changing, which we will call an *occurrent*. However we perceive different sorts of qualities, neither their difference, nor the qualities of the same sort can be observed as individual signs at this level of perception.

In sum, we can safely assume that human sign processing is based on *coherent* sensations of collections of *continuants* and *occurrents*. In the rest of the paper the term *input* will denote such collections of signs which, by virtue of their coherency, are inherently related to each other.

Our capability of perceiving coherent qualities is inevitably necessary for any sign recognition, but it is equally important that signs be relational. The former provides us with a basis for characterising the input in two different ways: as a collection of *parts*, and, as a *whole*. The latter allows us the 'generation' of all other signs by means of interaction, recursively, from the signs recognised in the input percept.

We argue that the differences between continuants and occurrents lie in the differences of the sensation of *space* and *time*. A continuant has the

[3] L. Schomaker, personal communication, 1999.

aspect of space as it refers to some 'thing', something stable or enduring, whereas an occurrent has the aspect of time as it refers to some 'event', something being in a state of flux. We emphasise that we only assume the *ability* of the sensation of such *differences*. Space and time are signs which arise as a result of learning, whereas the sensibility of the two sorts of qualities, space and time, is something we have from birth.

Our assumption of a human receiver allows us to talk about the existence of its memory, or, formally, a knowledge base ($\mathrm{K_b}$). In the remaining we will assume that memory is used as a 'lexicon' assisting in the recognition of the collections of continuants and occurrents as *signs*.

Summarised, the basic assumptions of our approach are the following: (1) Peircean logic can be based on a *coherent* set of qualities, contrary to the traditional ontology which refers to a set *formally* defined as the universe. (2) There are different sorts of qualities, and we are capable of distinguishing between them, physically. (3) We are able to characterise those qualities in two different ways, and, by virtue of the relational nature of signs, to recognise all other signs via interactions, recursively.

3 The ontology of logical signs

In this section we pursue an analysis of Peirce's definitions of signs and derive their formal logical meaning. In our ontology we will refer to the Boolean functions[4] depicted in fig. 2. We will visit Peirce's signs of growing complexity, stepwise.

A	B	f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}
f	f	f	f	f	f	f	f	f	f	t	t	t	t	t	t	t	t
f	t	f	f	f	f	t	t	t	t	f	f	f	f	t	t	t	t
t	f	f	f	t	t	f	f	t	t	f	f	t	t	f	f	t	t
t	t	f	t	f	t	f	t	f	t	f	t	f	t	f	t	f	t

Fig. 2. Boolean functions

3.1 Primary signs

Although the input itself appears by virtue of interaction between the stimulus and the human sensor(s), we do not experience that interaction

[4] f_0=0, f_1=$A*B$, f_2=$A*\neg B$, f_3=A, f_4=$\neg A*B$, f_5=B, f_6=$\neg A*B+A*\neg B$, f_7=$A+B$, f_8=$\neg A*\neg B$, f_9=$A*B+\neg A*\neg B$, f_{10}=$\neg B$, f_{11}=$A+\neg B$, f_{12}=$\neg A$, f_{13}=$\neg A+B$, f_{14}=$\neg A+\neg B$, f_{15}=1.

except as a perception, as we pointed out earlier, as "a quality which is a sign" (2.224), a qualisign. We will denote the collection of continuants and occurrents recognised as signs, respectively, as A and B. We will use the same letters for denoting the *type* of a sign, continuant or occurrent, as well.

Formally, a collection will be represented as a set. $\mathrm{U}=\mathrm{A}\cup\mathrm{B}\cup\mathrm{K_b}$ will be referred to as the *universe* ($\mathrm{K_b}$ is considered a constant). The variables X and Y will be used to refer to any type of sign; the term *logic* will denote Peircean logic.

The presence or absence of the constituent and occurrent qualities allow us to distinguish between four cases. These can be characterised by two variables A and B, and the Boolean values *true*(t) and *false*(f). For example, A=*false* and B=*true* denotes the case that the perceived input is recognised as the qualisigns B and 'not' A.

In logic, the value *true* refers to the perception of a sign, *as well as*, to the fact that any perceived sign is *true* by definition (and only a sign not present in the input can be *false*). From this it follows that a quality as a sign can refer to both X or ¬X. Although later it will become apparent that ¬X is the complement of X, this 'knowledge' is not available on the level of qualisigns, and negation (¬) is *not* an operation (likewise its opposite, 'assertion', is not an operation either). Negation will appear as a function on the second level of signs (cf. section 3.3).

The type of sign, qualisign, is represented by the Boolean functions A and B (f_3 and f_5), and $\neg A$ and $\neg B$ (f_{10} and f_{12}) which, respectively, correspond to the variables A and B defined as functions. Besides, we define two more functions as representations for the particular qualisigns: 0, denoting the sign that no input qualities are available (f_0); and 1, that all of them are present (f_{15}).

In sum, in our logical analysis we assume the potential existence of two different qualisigns which are related. No one of these signs can "actually act as a sign until it is embodied; but the embodiment has nothing to do with its character as a sign" (2.244). This aspect of the qualisign will be elucidated in the next section.

3.2 The signs of the first level

Qualisigns are signs which, though they can interact, do not actually do so. In that sense, they are undeveloped signs. Therefore, when a qualisign is embodied and becomes an icon or a sinsign, it will have the aspect of secondness degenerately.

> "Since a quality is whatever it is positively in itself, a quality can only denote an object by virtue of some common ingredient or similarity; so that a qualisign is necessarily an icon" (2.248).

Clearly, an icon must be a type A sign, in as much as it refers to the individual qualisigns, in their capacity of likeness to the input. The icon is a collection of such signs. In the remaining we will assume that each of Peirce's signs has a 'main' characteristics as a sign which is one of type A or B, or both A and B. For example, 'quality' signs (p=1) are of type A, 'presentational' ones (o=1) of type B.

By virtue of their coherency, any of the perceived qualities must be a perfect icon of the input. Therefore, the icon can be represented by the Boolean function $A+B$ (the 'or' function; f_7). This coincides with Peirce's conception of the icon as a relational quality, as "a first [i.e. the qualities] with accretive secondness [i.e. the 'or' relation]" (2.276).

The other sign of the first level is the sinsign.

> "A sinsign (where the syllable *sin* is taken as meaning 'being only once', as in *single* ...) is an actually existing thing or event which is a sign. It can only be through its qualities; so that it involves a qualisign, or rather, several qualisigns. But these qualisigns are of a peculiar kind and only form a sign through being actually embodied" (2.245).

The sinsign refers to an event defined by the collection of qualisigns, appearing *simultaneously*, and embodied as a 'whole'. Clearly, the sinsign is a type B sign. From the Boolean point of view, the sinsign can be represented by the function $A*B$ (the 'and' function; f_1).

The sinsign is a "second whose secondness [i.e. the (degenerate) 'and' relation] is a second to a firstness [i.e. the qualities], namely that of being a particular. The secondness of a sinsign is an accretion" (1.528).

In the Boolean representation, icon and sinsign have different 'structure'. In what follows, an icon will be called a "+" sign, whereas a sinsign a "$*$" sign.

The signs of the first level refer to the input percept as a particularity: as a particular event, a *whole* (sinsign), and, as a particular object, a collection of *parts* (icon).

3.3 The signs of the second level

On the second level of signs we find the rheme, the index and the legisign. In this section we will argue that each of these signs can be yielded by interaction, and between icon and sinsign.

The most unusual sign of the second level is the index. Peirce exemplified such a sign by the functioning wind vane which links two different signs, the icon of the wind vane and the sinsign of the event. But linking two different 'entities' requires that these are brought to a common denominator by conversion. As a result we get the index sign of the wind vane representing it, roughly, as the sign of the iconic wind vane pointing in the direction of the wind and being blown by that, or equivalently, the sign of the wind pointing in the direction of the wind vane and blowing it.

The index has two 'faces', or forms, which can be converted to one another, and so, the type of the index must be A and B. In propositional logic such a conversion is provided by the equalities: $\neg(A+B)=\neg A*\neg B$ (Shäffer function; f_{14}); $\neg(A*B)=\neg A+\neg B$ (Peirce function; f_8).

These equalities, known as the De Morgan rules, introduce the negated forms for the "+" and "*" signs and, at the same time, the conversion between the incomparable signs, icon and sinsign. These equations postulate the index as the common denominator of icon and sinsign, a sign yielded by 'contraction'.

In general, the conversion of the index is not for free. In order to be able to determine the converse of a sign we must have knowledge about the 'real' world (cf. $\mathrm{K_b}$). The possibility of the conversion of a sign can be seen as the validation of the input as a 'real' sign: if we can do the conversion, then we must have knowledge about it, and although we have not yet completely recognised it, we can distinguish it from other signs that we know already. Those signs will be called the *context* of the particular percept.

Because icon and sinsign refer to the same input qualities, the conversion 'learns' us how the input recognised as a collection of parts corresponds to something as a whole, and the other way round. But we should not forget that the index is only a "genuine secondness" as Tejera pointed out so clearly ([10]), a "pointing finger, directing attention, but asserting nothing".

In sum, the index can be characterised as the class of signs, the common (main) characteristics of which is the conversion of signs from one form to another. Formally, the index is represented by the conversion functions, or alternatively, by the *negation* operation.

The conversion of the index is used in the *elimination* of the aspect of particularity from the signs of the first level, and the recognition of the input as an 'abstract' sign. Accordingly, the two other cases of the icon–sinsign interaction will correspond to signs yielded by 'extraction'. Again, we will elucidate this by our example of the wind vane.

In the first case, the sinsign (event) is extracted from the icon of the wind vane yielding the collection of the abstract, unrelated concepts of the wind and the wind vane. The fact that these concepts appear together implies their (lawlike) compatibility. This provides us with the definition of the legisign which is the representation of the input of an abstract event (which the sinsign is a concrete instance of).

> "A law that is a sign. This law is usually established by men. It is not a single object, but a general type which, it has been agreed, shall be significant" (2.246).

In the second case, the icon is extracted from the sinsign, yielding the abstract sign of the wind, and of the wind vane, representing their object, not related to anything else, as a possible. Such a sign is a rheme.

> "A sign of qualitative possibility ... representing ... such a ... possible object. Any rheme, perhaps, will afford some information; but it is not interpreted as doing so" (2.250).

From the definition of a rheme, it follows that a rheme is a type A sign, whereas a legisign is a sign of type B. In Boolean logic, 'extraction' of Y from X corresponds to the relative difference operation (\) formally defined as: $X \backslash Y = X * \neg Y$.

With respect to the two cases above, we will distinguish between two definitions of the relative difference which can be justified by the differences between the icon and the sinsign: (1a) The icon consists of individual signs, (1b) but which form a whole; whereas (2a) the sinsign itself is a whole, (2b) but which consists of individual signs.

We can represent, respectively, the legisign as icon\sinsign, and the rheme as sinsign\icon: $(A+B) \backslash (A*B) =\{\text{2a}\}\ (A+B)*\ (\neg A + \neg B) =\{\text{1b}\}\ (A+B)*\neg A + (A+B)*\neg B = \neg A*B + A*\neg B\ (f_6)$; and $(A*B) \backslash (A+B) =\{\text{1a}\}\ (A*B) \backslash A,\ (A*B) \backslash B =\{\text{2b}\}\ A*\neg A,\ B*\neg A,\ A*\neg B,\ B*\neg B = \neg A*B,\ A*\neg B\ (f_4,\ f_2)$.

The case of the legisign above is simple, because its Peircean and Boolean logical definitions are isomorphic. However, in the case of the rheme, we do not immediately have the Boolean interpretation. Informally, the Peircean logical derivation reads as follows: extract A and B separately, respectively, from $A*B$, and from A and B. The pair of functions is explained as follows: $A+B$ can be interpreted as A, or B, or $A+B$ (as a whole), therefore we can extract them from $A*B$ individually (notice, that in the last case we get the empty set which is omitted). We emphasise that A and B could *only* be extracted from each other because of their inherent relatedness.

The signs of the second level (except for the index) are the abstract conceptions of the input: as abstract 'entities' having no reference to anything else (rheme), and as abstract event, 'prepared' from those entities according to some general rule (legisign).

3.4 The signs of the third level

On the third level of signs we find the dicent and the symbol sign. We argue that these are abstract signs also reflecting the particularities of the input, contrary to the signs of the second level (rheme and legisign) which are abstractions of the input in the general sense.

By interpreting the rule-like legisign with respect to the context defined by the index, we get its instantiations represented, equivalently, by its converse, as a *property*, a symbol sign. Because negation does not change the sign's type, the symbol must be a type B sign.

Formally, this amounts to the application of the operation of the index (negation) to the legisign: $\neg(A{*}\neg B{+}\neg A{*}B)$ $=A{*}B{+}\neg A{*}\neg B$ (f_9). This derivation exposes the symbol as an abstract property, represented as an equivalency relation: $A{*}B$ is expressive of the relation of the input qualities corresponding to each other, whereas $\neg A{*}\neg B$ is the sign of the 'background', speaking metaphorically.

Peirce defines the symbol sign as follows.

> "A symbol ... refers to the object that it denotes by virtue of a law, usually an association of general ideas, which ... cause[s] the symbol to be interpreted as referring to that object. It is thus itself a general type of law ... a legisign". "Not only is it general itself, but the object to which it refers is of a general nature. Now that which is general has its being in the instances which it will determine. There must, therefore, be existent instances of what the symbol denotes .... The symbol will indirectly ... be affected by those instances; and thus ... involve ... an index of a peculiar kind" (2.249).

The Boolean representation of the dicent sign can be given by similar arguments: $\neg(A{*}\neg B), \neg(\neg A{*}B){=}\ \neg A{+}B,\ \ A{+}\neg B$ (f_{11}, f_{13}).

The dicent sign is yielded from the rheme by actualising it with respect to the context defined by the index. The dicent is represented by the continuants in relation to their complementing occurrents (as $A{+}\neg B$, or, alternatively, as the causality relation $A{\leftarrow}B$), and the other way round.

> "[The dicent is] a sign, which, for its interpretant, is a sign of actual existence. A [dicent] necessarily involves, as part of it, a rheme, to describe the fact which

it is interpreted as indicating. But this is a peculiar kind of rheme; and while it is essential to the [dicent], it by no means constitutes it" (2.251). "[The] dicent ... is a proposition or a quasi-proposition" (2.250).

3.5 The input as a proposition

The recognition of the input is completed by the interaction of the dicent and the symbol, the yield of which will be an argument sign. From the logical point of view such an interaction, which is a syllogism (degenerately), is beyond the scope of Boolean logic.

The dicent–symbol interaction corresponds to, what we may also call as *predication*, the generation of a proposition by 'merging' the meaning of symbol ('predicate') and dicent ('subject'). Predication, which we denote by '&&' indicating its strong relationship with the 'and' function, is defined as follows: $X \&\& X*Y{=}X$, $Y \&\& X*Y{=}Y$, and the empty set, otherwise. The occurrence of $X(Y)$ on the right hand side denotes $X(Y)$ *extended* with the meaning of $Y(X)$.

Formally, the argument sign corresponds to implication: $(\neg A{+}B)\&\&(A{*}B{+}\neg A{*}\neg B){=}\neg A\&\&(A{*}B{+}\neg A{*}\neg B){+}B\&\&(A{*}B{+}\neg A{*}\neg B){=}\neg A{+}B$; and $(A+\neg B)\&\&(A{*}B+\neg A{*}\neg B){=}A\&\&(A{*}B{+}\neg A{*}\neg B){+}\neg B\&\&(A{*}B{+}\neg A{*}\neg B){=}A{+}\neg B$, but this implication[5] is a particular one between two signs, each involving both the continuants and the occurrents, in their particular as well as their abstract meaning. The two forms of the argument correspond to the two views of the input, as the 'event' referring to the 'entities' it is inherently related to, and the other way round.

"... an argument is a sign which is understood to represent its object in its character as a sign" (2.252).

Because the argument is embedded, typically, in the context of other signs, the type of the argument will depend on that context. For the time being we will assume that the argument is an A and B type sign.

Let us mention briefly that the above interpretation of the dicent–symbol interaction complies with Aristotle's conception of a proposition. In as much as the dicent refers to a particular (abstract) object, and the symbol to a universal (abstract) property, we can identify in the logical sign of the dicent two propositions (degenerately, in the logical sense), a particular affirmative and a particular negative,[6] respectively $A{+}\neg B$ and $\neg A{+}B$; and, similarly, a universal affirmative and a universal negative in the logical sign of the symbol, respectively $A{*}B$ and $\neg A{*}\neg B$.

[5] This coincides with Łukasiewicz' conception of Aristotle's syllogisms ([4]).
[6] Notice that $A{+}\neg B = A{\leftarrow}B$, and $\neg A{+}B = (\neg A){+}\neg(\neg B) = (\neg A){\leftarrow}(\neg B)$.

The final classification of the Boolean functions, as well as the types of Peirce's signs are displayed in fig. 3 (respectively, on the left and right hand side).

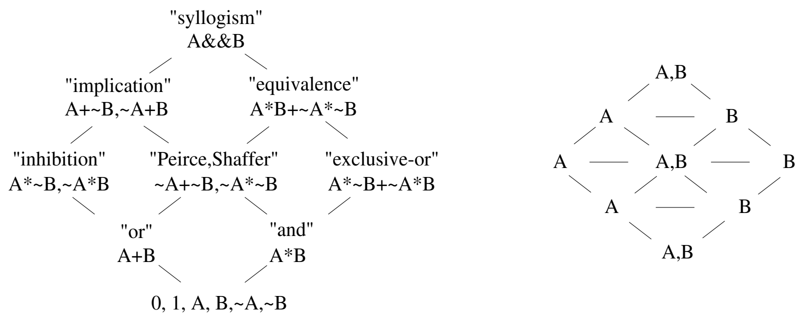

Fig. 3. The classification of Boolean functions and their types

4 An example

We exemplify our ontology by analysing the logical signs of the sample input "*John loves Mary*" considered as a visual perception (for example). Signs are represented algebraically.

The Peircean logic properly includes the Boolean one, and this inclusion is witnessed in the example by the mappings of the Boolean operators $+$, $*$, $\neg$ to functions that apply on the different levels of signs.

Signs are relations, but which can also form sets (collections). Therefore we need a model which treats sets and relations as equals. Such a framework can be found in a Peirce algebra. We will use the following definitions (borrowed from Peirce algebra's): *converse* $\hat{R}=\{(y,x)|(x,y)\in R\}$, *composition* $R;S = \{(x,y) \mid \exists z[(x,z) \in R \wedge (z,y) \in S]\}$, and *direct product* $R\times S = \{(x_R \times x_S, y_R \times y_S) \mid (x_R, y_R) \in R \wedge (x_S, y_S) \in S\}$.

We will represent a relation as a set of pairs, and emphasise its type by underlining its "main" component. We define the A and B sets as: A$=\{(\underline{x},y)|x{=}C \wedge y{=}\ O\}$, B$=\{(x,\underline{y})|x{=}C \wedge y{=}\ O\}$ where, respectively, C and O, denote the continuants and occurrents perceived.

We will use the abbreviations, respectively, J, l and M, for *John*, *loves* and *Mary*, and JM for the set $\{J,M\}$. A singleton set will be represented by its element. Negation ($\neg$) is mapped to the function converse.

Level 0 We assume that we perceive in the input, l as an occurrent, and J and M as continuant signs (but other choices are possible as well):
Qualisign A=$(\underline{JM}, l)$, B=$(JM, \underline{l})$.

Level 1 We map $+$ and $*$ to identical functors (i.e. to structure preserving functions) which complies with the secondness of the signs of this level: *Icon* $(\underline{JM}, l)+(JM, \underline{l})$, *Sinsign* $(\underline{JM}, l)*(JM, \underline{l})$ which, respectively, represent the input as a union of parts, and as a whole.

Level 2 The operators $+$ and $*$ are mapped to union and composition, respectively. This complies with their secondness interpreted as 'modification' of one of the arguments by the other:
Legisign $(\underline{JM}, l) * (\underline{l}, JM) + (l, \underline{JM}) * (JM, \underline{l}) = (\underline{JM}, JM) + (l, \underline{l}) = JM + l$;
Rheme $(\underline{JM}, JM)$, $(l, \underline{l}) = JM,\ l$
where (JM, JM) is simplified as JM because the object (or event) of a sign relating to itself must be the sign itself; similarly, (l, l) is simplified as l. The representation of the index, which is a pair of functions, is omitted.

The rheme refers to JM and l as abstract concepts, e.g. as names. The legisign describes the perception as an abstract event defined by the union of the abstract notions l and JM which, therefore, must be compatible.

Level 3 We map $+$ and $*$ to disjoint union and direct product, which is in accordance with their thirdness interpreted as follows: the arguments contribute to the generation of a sign, together. The dicent is a proposition of the input in terms of the possible 'casts' of the continuants and occurrents in the 'play' of the event, as a causality relation; whereas the symbol is the expression of the input in terms of the possible 'acts' of the event in that 'play', as a property:
Symbol $(\underline{JM}, l)\times(JM, \underline{l})+(l, \underline{JM})\times(\underline{l}, JM)=(JM\times JM, l\times l)+(l\times l, JM\times JM)=JM_l+l_{JM}$;
Dicent $(\underline{JM}, l)\leftarrow(JM, \underline{l})$, $(\underline{JM}, l)\rightarrow(JM, \underline{l}) = JM \leftarrow l,\ JM \rightarrow l$
where, by virtue of the uniqueness of the input qualities, $JM \times JM$ must be either $J \times M$, or $M \times J$, or simply, JM; likewise, $l \times l$ can be simplified as l. JM_l and l_{JM}, respectively, denote (JM, l) and (l, JM) as a property. In the final forms of the signs, the 'main' components are only displayed.

Level 4 The argument merges the meaning of the dicent and the symbol:
Argument $(JM \leftarrow l), (JM \rightarrow l)\&\&(JM_l + l_{JM}) = JM_l \leftarrow l_{JM}, JM_l \rightarrow l_{JM}$
where the two signs can be paraphrased, respectively, as *John loves Mary* or *Mary loves John*, and *(There is) love (between) John (and) Mary* (i.e. from *John* to *Mary*, or the other way round).

Conclusion and further research

From Peirce's definitions of signs a propositional logical meaning is derived. It is argued that signs, both general and logical ones, can emerge via an interaction.

Logic is closely related to natural language. Recently we developed a Peircean model of syntax ([5]) which now can be combined with our model of logic. It turns out that, in the Peircean view, logic and language are isomorphic. Further research on this issue is our highest priority. Another interesting question that must be addressed in the future, is the relation between Peirce's signs and formal concepts ([11]).

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