On a uniform ontology for logic and reasoning

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Abstract

We argue that Peircean category theory facilitates the definition of a uniform ontology and representation for logic and reasoning. Such a representation enables information in the two domains to be merged via structural coordination, which can be more efficient than via translation between different representations characterizing traditional models of knowledge representation. Efficient merging of different types of information can be necessary for a full scale analysis of documents.

Keywords

Peirce, sign aspect, process model, logic, reasoning, uniform representation.

Traditional ontology

Founded in Aristotle's theory of categories, traditional models of knowledge representation enable a specification of phenomena by means of subject-attribute relations. A weakness of this type of a definition lies in its inability to distinguish between such relations into ad-hoc and meaningful. For instance, in the phenomenon "Mary is wearing a red hat", the pair of relations, between "hat" and "red", and "Mary" and "red hat", are identically defined as subject-attribute relations, although the import of the two relations in the meaning of the sample phenomenon can be different.

Mathematics too is witnessed by the need for a distinction between those two types of relations above. An example are Peano's axioms. Although (1) zero and (2) successor already define all natural numbers, the need for a classification of numbers that are interesting from a certain perspective asks for (3) the axiom of induction.

Peircean ontology

We suggest that as solution to the above problem can be found in Peircean category theory. Peirce's categories are modes of organization or relation of matter and concepts (Peirce, 1931) (5.66-68). His three categories, monadic firstness, dyadic secondness, and triadic thirdness are related to one another by a relation of involvement and subservience: thirdness involves secondness, and transitively so, firstness; firstness may only appear through secondness, and in turn, through thirdness. We argue that the above relation between the categories, together with the potential of the categories to be applied recursively, enable the Peircean framework to be used for a process interpretation of phenomena.

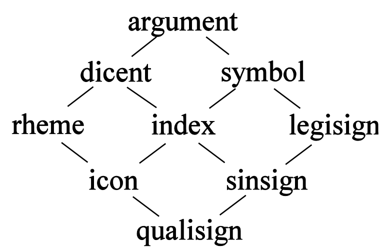

Fig.1: Hierarchy of Peircean sign aspects

Peircean sign theory

We may know about phenomena by means of signs. According to Peirce, the (potential) sign offers itself for interpretation and mediates its object to an interpreting agent. From a categorical stance, Peirce's sign concept above involves three types of a relation: (1) monadic sign in itself, (2) dyadic sign-object, (3) triadic sign-object-interpretant. By analyzing the relations involved in his sign concept, Peirce introduced nine sign aspects, or sub-categories, which can be arranged in a hierarchy (Peirce, 1931) (1.345); see Fig.1. By an ingenious analysis of Peirce's theory of interpretants, van Breemen (Sarbo et al., 2011) has shown that the Peircean hierarchy of sign aspects can be assigned the meaning of a process. As all phenomena can be subject to interpretation, all signs (3rdness) may become a potential sign (1stness), recursively, in a subsequent sign (interpretation) process.

Cognitive model

Obviously, interpretation can be looked at from the perspective of cognition as well. According to (Yarbus, 1967), the condition for interpretation is an interaction between an interpreting system, occurring in some state, and an independent stimulus, appearing as an effect. The interaction between state and effect takes place in a complementary context. The goal of interpretation is to find out why this effect (q_1) is occurring to this state (q_2), in this context (C). To this end, the input qualities are sorted (*sorting*; [q_1], [q_2], [C]), represented independently from one another (*abstraction*; q_1, q_2), completed with complementary information by the context (*complementation*; (q_1,C), (q_2,C)), and merged in a final relation (*predication*; (q_1,C)-(q_2,C)). See Fig.2.

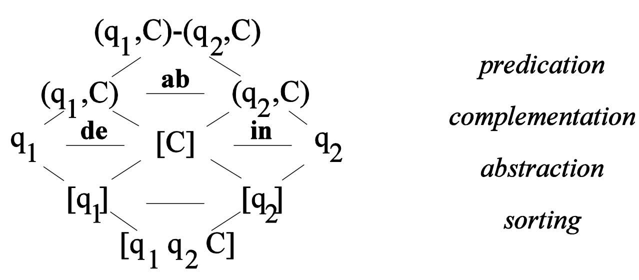

Fig.2: Process model. Events are displayed by a horizontal edge. Bold face labels designate an involved aspect of reasoning: **de**(ductive), **in**(ductive), **ab**(ductive). A comma symbol (",") stands for a synonymous interpretation.

Following (Sarbo et al., 2011), all interpretation moments (or positions) in Fig.2 involve a Peircean sign aspect, the ordering of the positions is isomorphic to the hierarchy of sign aspects, in Fig.1. With this relation between the process model and the Peircean sign aspects (and categories), the possibility of a uniform ontology for logic and reasoning comes into sight.

Logical ontology

A logical analysis of the process model (Fig.2), revealing its 'naive' logical meaning, was first introduced in (Sarbo et al., 2011); see Fig.3. To this end, input qualities are interpreted as variables: *A* (state) and *B* (effect). Qualities that are in focus, and complementary, are represented by a variable stated positively (e.g., *A*), and negatively (e.g., ~*A*), respectively. Logical expressions assigned to the input representations involve the aspect of a Boolean relation, e.g., the logical expression of [q_2], which is *A*B*, involves the aspect of a logical-and relation.

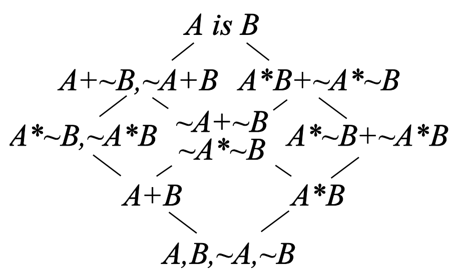

Fig.3: Logical expressions. The presence of all 16 Boolean relations on two variables (0 and 1 are omitted) is an expression of the completeness of the process model from a 'naive' logical stance.

Reasoning and syllogistic

Reasoning has three basic types: deduction, induction and abduction. According to Peirce (Piece, 1931) (2.623), deduction is the type of reasoning which examines the premises and concludes their *necessary* or probable truth. The major premise lays down a rule, the minor premise states a case under the rule, the conclusion applies the rule to the case and states the result. Induction is the type of reasoning which adopts a conclusion as approximate, because it results from a series of inferences which generally lead to the truth. Now a rule is concluded from a case and result. As a consequence, the conclusion may not be necessary, but only *plausible*. Abduction is the type of reasoning which provisionally adopts a hypothesis, as an explanation of a result, because every possible consequence of the hypothesis can be verified experimentally, so a disagreement with the facts can be revealed. In this type of an inference, a case is concluded from a rule and result. By making a hypothesis, the truth of the conclusion may not be plausible, but only *hypothetical*. Through refraining from the element of repetition in induction and the element of experimental verification in abduction, and by focusing on a single inference act, a relation between the three types of reasoning, as permutations of rule, case and result, can be revealed.

Syllogistic is a theory of deduction, in which the conclusion is derived from the major and minor premises. Aristotle introduced three figures for this type of inferencing of which, figure-1 is perfect, or obviously true, the other two figures, which can be transformed to figure-1, are less transparent; see Fig.1.

figure-1	figure-2	figure-3
$A_1\ B$	$B\ A_1$	$A_1\ B$
$\underline{A\ B_1}$	$\underline{A\ B_1}$	$\underline{B_1\ A}$
$A\ B$	$A\ B$	$A\ B$

Fig.4: Aristotle's three syllogistic figures. The common term is defined by $A_1=B_1$. Syllogistic expressions are given in italics.

A promise of this paper is the existence of a relation between the three types of reasoning and Aristotle's three syllogistic figures. To this end we offer an analysis to our model, in Fig.2, this time from a reasoning perspective. According to this view, in *complementation* of q_1, context information about a property is added to the qualitative possibility represented by the abstract input state. This event involves the aspect of a deduction. In *complementation* of q_2, the habitual input effect is tested for a measure provided by the context. This event involves the aspect of an induction. The final relation, merging an expression of the input state, as actual existent, and the input effect, as characteristic property, into a single representation, involves the sign aspect *proposition*. As, in a single 'run', the input can be represented from a single perspective only, *predication* involves the aspect of an abduction (cf. a hypothesis).

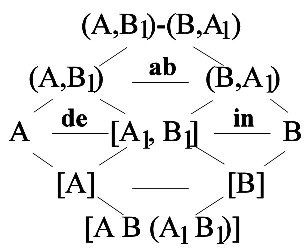

figure-1

$A_1\ B$
$\underline{A\ B_1}$
$A\ B$

Fig.5: Process model applied to figure-1. The major and minor premises, '$A_1\ B$' and '$A\ B_1$', are represented from the perspective of their import in the conclusion; the common term is defined by $A_1=B_1$.

Syllogistic processing

Following a semiotic analysis of the process model, in Fig.2, general information, involving the aspect *rule*, is represented by the abstract input effect (q_2); observed information, involving the aspect *qualitative possibility*, by the abstract input state (q_1). According to Peirce, the characteristic meaning of a premise can be 'general' or 'experienced'. He maintained that only minor premises can come from experience, major premises have their truth in the brain. From this we conclude that, in syllogistic as a process, the major and minor premise must be represented by the positions, q_2 (effect) and q_1 (state), respectively; *complementation* must stand for an expression of the input premises in context, *predication* for drawing the conclusion.

A process model of figure-1 can be defined by the assignment: q_1=A, q_2=B, C=(A_1 B_1), enabling the minor and major premises, '*A B_1*' and '*A_1 B*', to be represented as a quality by [A B (A_1 B_1)]; the conclusion, '*A is B*', as a relation, by (A,B_1)-(B,A_1). See also Fig.5. Following our theory, a model of figure-2, and figure-3, assumes that the major and minor premise, respectively, is converted, preceding syllogistic processing. Following experimental research by (Dickstein, 1981), conversion is a natural operation by the brain/mind.

Conclusion

The isomorphic relation between the hierarchy of Peircean sign aspects (Fig.1), the process model (Fig.2) and its logical interpretation (Fig.3, Fig.5) shows the possibility of a uniform ontology for logic and reasoning. The proposed model of syllogistic assumes the use of a conversion of a premise. Conversion of information involved by an observation (cf. minor premise) boils down to an exchange of a state and an effect, which can be simple. Conversion of general knowledge (cf. major premise) asks for a representation of a collection of converse observations, which can be more laborious. We argue that the computational effort required by a conversion of a premise can be used for an explanation of the differences, in perception, between the three Aristotelian figures, with figure-1 (no conversion) as obvious, figure-3 (conversion of the minor) and figure-2 (conversion of the major) as increasingly less transparent. Through the relation between syllogistic and reasoning, as explained in the paper, the above difference in computational effort can be used for an explanation of the truth involved by the three types of reasoning as well: deduction as necessarily true, induction as plausible, and abduction as only hypothetically plausible. Preliminary results of an experimental verification of the above relation between the types of reasoning and the three syllogistic figures can be found in (Sarbo and Yang, 2015).

References

Collected papers by C.S. Peirce (1931-58); Harward University Press; Cambridge, MA

Sarbo, J.J, Farkas, J.I, van Breemen, A.J.J. (2011); Knowledge in Formation: A Computational Theory of Interpretation. Springer, Berlin

Yarbus, A.L. (1967); Eye Movements and Vision; Plenum Press, New York

L.S. Dickstein, L.S. (1981); The meaning of conversion in syllogistic reasoning, Bulletin of the Psychonomic Society, Vol 18, no.3 (pp. 35-138)

Sarbo, J.J., Yang, J.H. (2015); A Semiotic Approach to Critical Reasoning, Proc.16th Int. Conf. On Informatics and Semiotics in Organizations (ed. Liu, K, Nakat, W., Galarreta, D); (pp. 10-20)