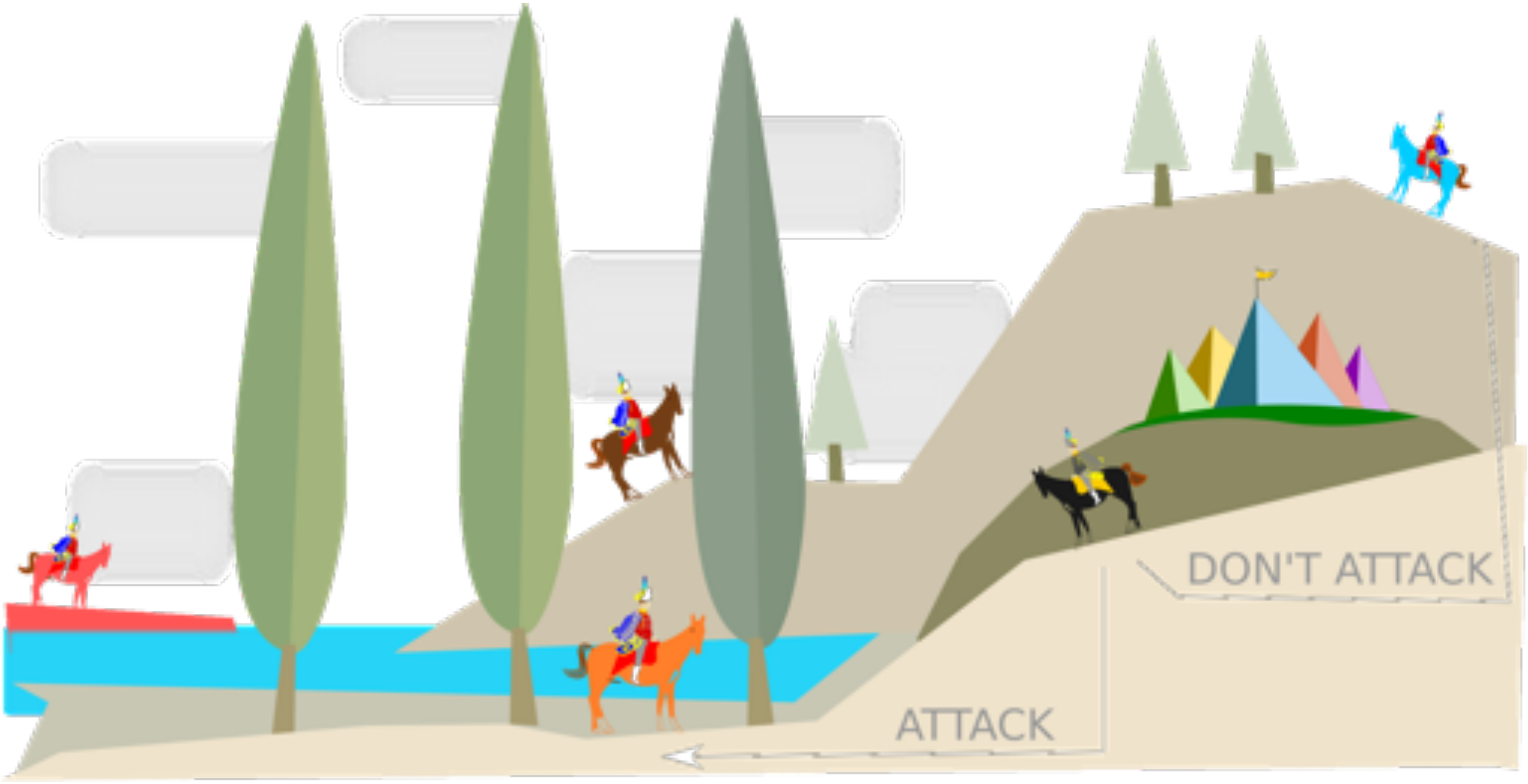
Advanced Network Security

5. Agreement and consensus I: concepts and protocols for crash failures

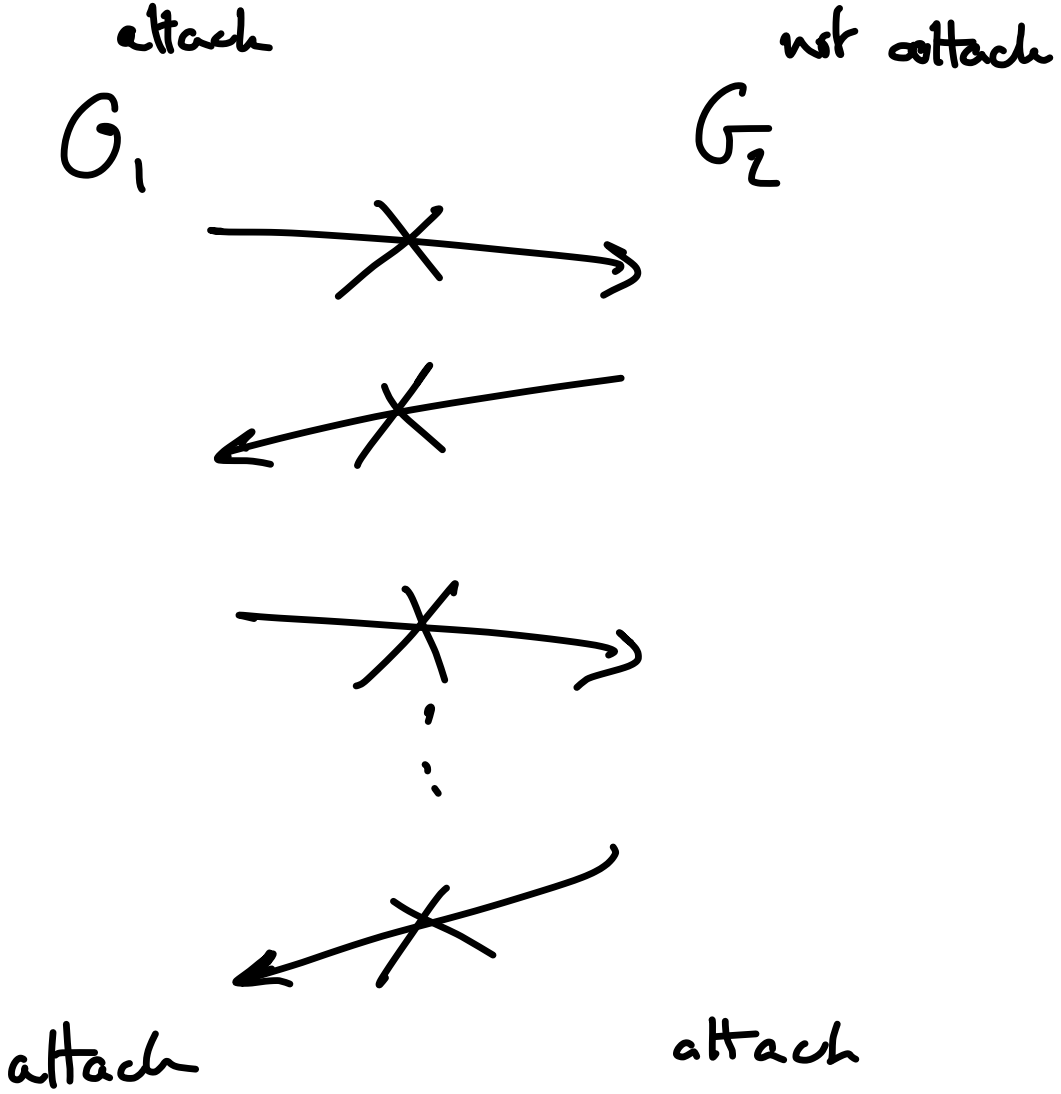
Jaap-Henk Hoepman

Digital Security (DS)
Radboud University Nijmegen, the Netherlands
@xotoxot // ✉ *jhh@cs.ru.nl* // 🖱 *www.cs.ru.nl/~jhh*

Byzantine generals


DON'T ATTACK
ATTACK

What if messages get lost?



Types of faults: process failures

- **Stopping / Crash**
  - Process stops unexpectedly and does nothing after that, forever
- **Omission**
  - Process skips a step it is supposed to perform
    - ★ *e.g. sending a messages; this models message dropping on an edge (except that there is a limit on the number of affected edges…)*
- **Byzantine**
  - Process performs arbitrary actions, not specified by the protocol
    - ★ *e.g. sending different messages to different recipients*

Decision problems

- **Private inputs $C[p].in$, private decision outputs $C[q].decision$**
- **Termination condition**
  - Deterministic termination
    - ★ *Every correct process decides irrevocably, and stops/knows it decided*
  - Probabilistic termination (convergence)
    - ★ *Every correct process decides irrevocably with probability 1, and stops/knows it decided*
  - Implicit termination (stabilisation)
    - ★ *Every correct process decides, but never knows it decided (and may change decisions in the process); no such changes occur after a finite number of steps*
- **Consistency condition**
  - A global predicate over inputs and decision outputs
  - Problem specific

Solving decision problems

- **We assume a certain topology $G = (V, E)$, $n = |V|$**
  - Typically a clique
- **We assume certain faulty behaviour**
  - E.g. crash failures only
- **We assume at most $f < n$ processes are faulty**
  - Link failures are modelled as process failures
  - f expresses **robustness**; typically $f < n/3$ or $f < n/2$
  - Sometimes we specify certain processes can/cannot fail
- **We assume recipient knows sender of messages (authenticity)**
  - Not signatures, but because of point-to-point direct connections

f is ***assumption*** on number of faults. Real number of faults in an execution may be lower or equal (in which case algorithm is succesful) or not (in which case it fails)

Decision problem: replicated server

- **Suppose two (replicated) servers p, q hold the same data (input)**
- **Consistency condition:**
  - All correct processes decide on this input
- **Termination condition:**
  - deterministic
- **Assumptions**
  - Crash failures
  - At most one of the replicated servers fail

Decision problem: replicated server

- **Suppose two (replicated) servers p, q hold the same data (input)**
- **Consistency condition:**
  - All correct processes decide on this input
- **Termination condition:**
  - deterministic
- **Assumptions**
  - Crash failures
  - At most one of the replicated servers fail

- **Protocol for p, q**

forall $r \neq q, p$
do send $C[p].in$ **to** r
$C[p].decision = C[p].in$

Also sometimes written as $decide(C[p].in)$

- **Protocol for other processes r**

receive v
$C[r].decision = v$

Decision problem: replicated server

- **What if (replicated) servers p, q hold different data?**

- **What if both replicated servers fail?**

- **Protocol for p, q**

```
forall r ≠ q, p
do send C[p].in to r
C[p].decision = C[p].in
```

- **Protocol for other processes r**

```
receive v
C[r].decision = v
```

Decision problem: weak broadcast

- **One server p holds a bit**
  - Either 0 or 1
- **Consistency condition:**
  - All correct processes decide on the same value
  - If p does not crash, this should be p's input
- **Termination condition:**
  - stabilising
- **Assumptions**
  - Crash failures

Decision problem: weak broadcast

- **One server p holds a bit**
  - Either 0 or 1
- **Consistency condition:**
  - All correct processes decide on the same value
  - If p does not crash, this should be p's input
- **Termination condition:**
  - stabilising
- **Assumptions**
  - Crash failures

- **Protocol for p**

$C[p].decision = C[p].in$
if $C[p].in == 1$
then forall $r \neq p$
do send 1 **to** r

- **Protocol for other processes r**

$C[r].decision = 0$
receive 1
$C[r].decision = 1$
forall $q \neq r$
do send 1 **to** q

Decision problem: weak broadcast

- **What if p crashes?**

- **Why is this not deterministically terminating?**

- **Protocol for p**

```
C[p].decision = C[p].in
if C[p].in == 1
then forall r ≠ p
     do send 1 to r
```

- **Protocol for other processes r**

```
C[r].decision = 0
receive 1
C[r].decision = 1
forall q ≠ r
do send 1 to q
```

The consensus problem

The consensus problem

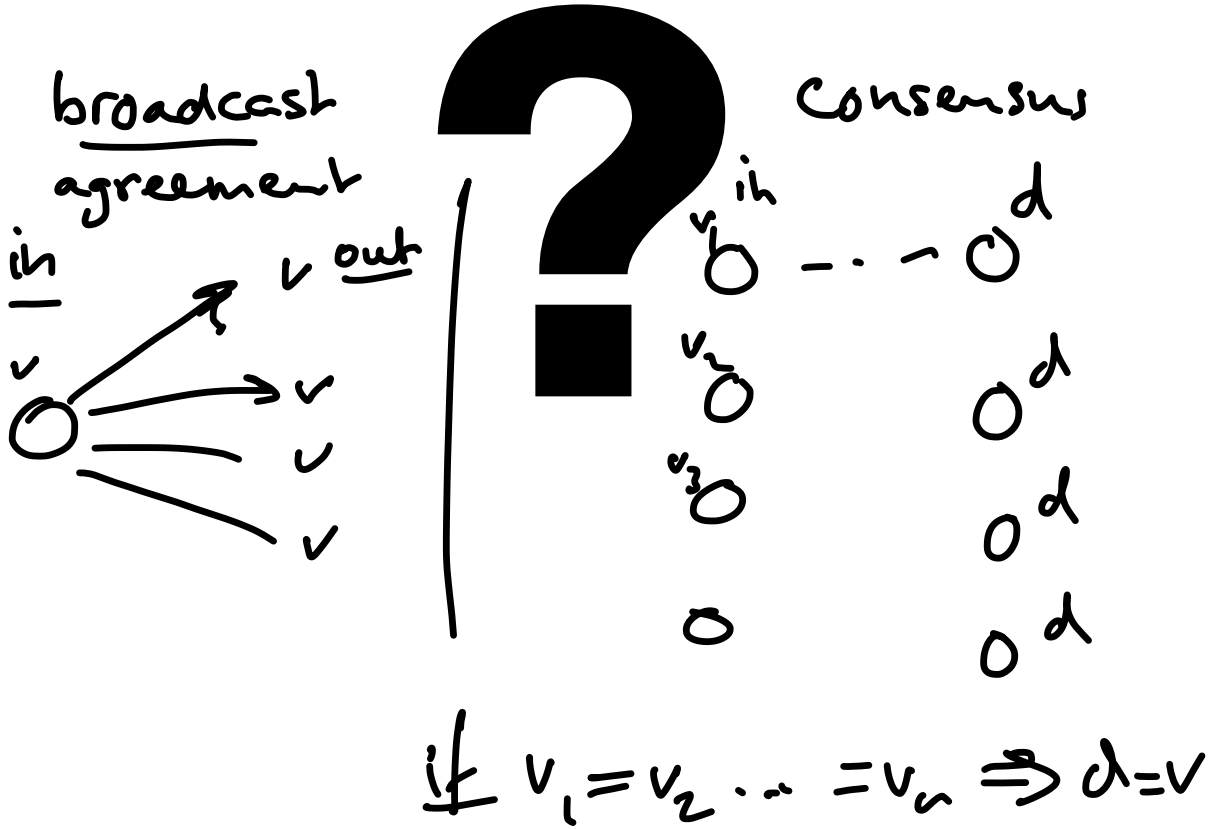
- ***All* processes have a *binary* input value (0 or 1)**
  - So it is different from a broadcast
- **Consistency condition**
  - All correct processes decide on the same value (*Agreement*)
  - If all processors have the same input value b, then all correct processors must decide b (*Validity*)
- **Termination condition**
  - Deterministic

Aside: solving consensus with broadcast

- **Atomic broadcast**
  - Sender p holds a bit
    - ★ *Either* 0 *or* 1
  - Consistency condition:
    - ★ *All correct processes decide on the same value (even when sender p fails)*
    - ★ *If p does not fail, all correct processes decide on sender p's input*
  - Termination condition: deterministic
- **Remember: no link failures**

a.k.a agreement

- **Consensus protocol for p?**



Aside: solving consensus with broadcast

- **Atomic broadcast**
  - Sender p holds a bit
    - ★ *Either* 0 *or* 1
  - Consistency condition:
    - ★ *All correct processes decide on the same value (even when sender p fails)*
    - ★ *If p does not fail, all correct processes decide on sender p's input*
  - Termination condition: deterministic

a.k.a agreement

- **Remember: no link failures**

- **Consensus protocol for p**

Initialise vector $V[]$
$V[p] = C[p].in$
broascast $C[p].in$
forall $r \neq p$
do receive $V[r]$
$C[p].decision = Majority\{V[r]\}$

Recipient recognizes sender

- **In other words: atomic broadcast and consensus are very similar**

Consensus for crash failures

- **Assume at most $f < n$ crash failures**
- **Synchronous protocol**
  - Computation proceeds in rounds
  - At start of round r, all processors send all messages for round r
  - Before proceeding to round $r + 1$ all processors receive all round r messages
    - ★ *If they arrive, they arrive in this round; otherwise they are lost forever*

Gossip style protocol: intuition

- **“Gossip”**
  - Tell everyone you know as soon as you hear something new
  - “Spread information like a virus”

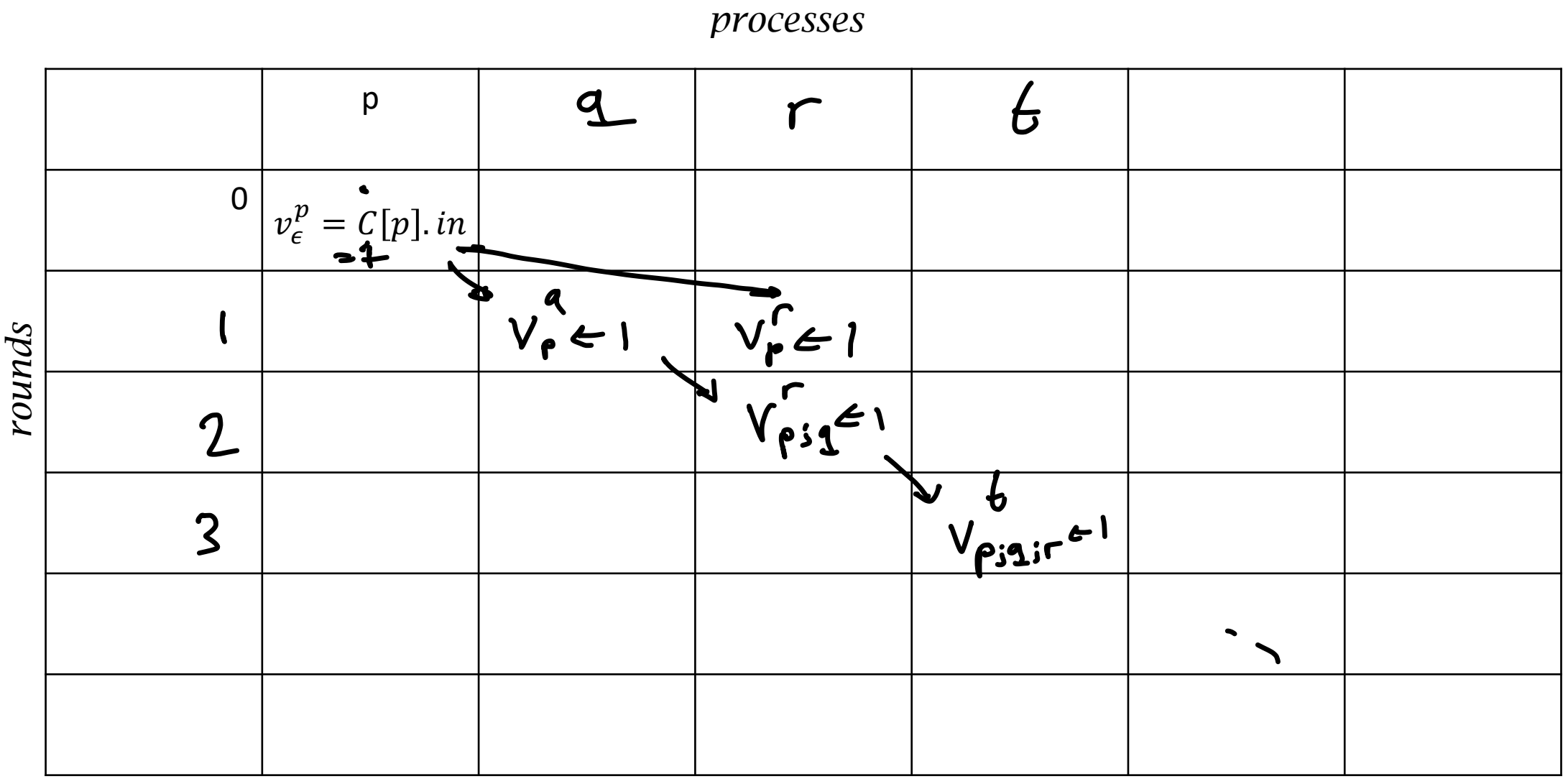
- **Why is gossip a good strategy to solve consensus when facing crash failures?**
  - If process crashes after telling *some but not all* processes about its value, other processes may have different views
  - Gossip ensures that as soon as *at least one correct process* learns about a value, it will tell all other (correct) processes about this value

Gossip style protocol: details

- **Processor p keeps track of the values others told him about in variables v_σ^p**
  - $v^p_{q_1,q_2,..,q_k}$ means: q_k told p, that q_{k-1} told q_k, …. that q_1's value is v
  - Initially all $\perp$
  - $v_\epsilon^p = C[p].in$

- **And when processor p learns a value v_σ^p**
  - it will tell everyone that doesn't know yet (i.e. all $p \notin \sigma$) about this by sending a message $m^q_{\sigma;p} = \langle v_\sigma^p, \sigma \rangle$ to q
  - Which q stores in $v^q_{\sigma;p}$

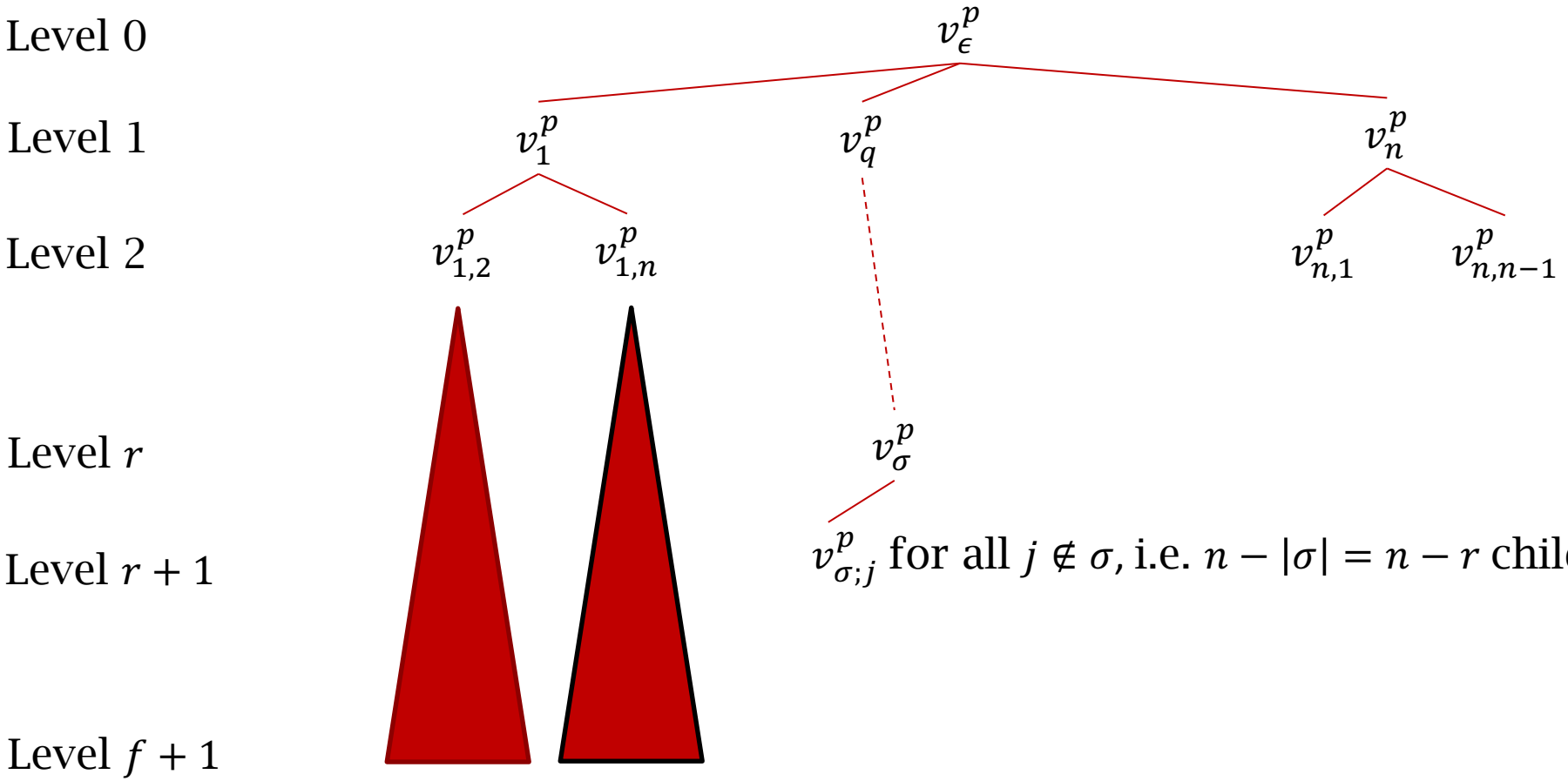
Gossip: propagation of initial value of node p



Consensus: main approach

$v^p_{q_1,q_2,..,q_k}$ means: q_k told p,
that q_{k-1} told q_k,
....
that q_1's value is v
Initially all $\perp$
$v^p_\epsilon = C[p].in$

- **Each processor p builds the following tree T_p**



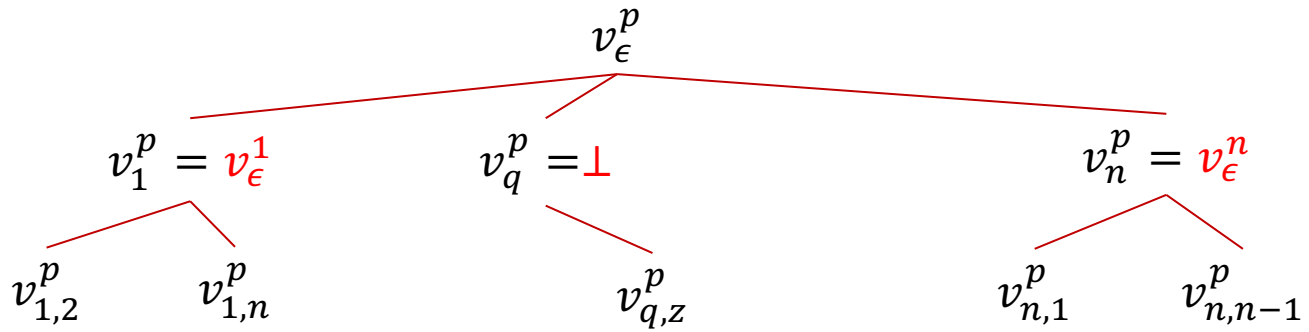
Building the tree: protocol for p

$v^p_{q_1,q_2,..,q_k}$ means: q_k told p,
that q_{k-1} told q_k,
....
that q_1's value is v
Initially all $\perp$
$v^p_\epsilon = C[p].in$

- **Before round 1**
  - Initialise tree. Set all $v^p_\sigma = \perp$ and $v^p_\epsilon = C[p].in$
- **Round** $r, 1 \leq r \leq f + 1$
  - For all σ with $|\sigma| = r - 1 \wedge p \notin \sigma$, send v^p_σ to all processors q (including p)
    - ★ *Call this message* $m^q_{\sigma;p}$
  - Receive all $m^p_{\sigma;x}$ addressed to p and store in $v^p_{\sigma;x}$ (in the tree at level r)
    - ★ *By the protocol* $x \notin \sigma$ *so* p *receives* $n - (r-1)$ *such messages (one from each* x*)*

The protocol in action: round 1

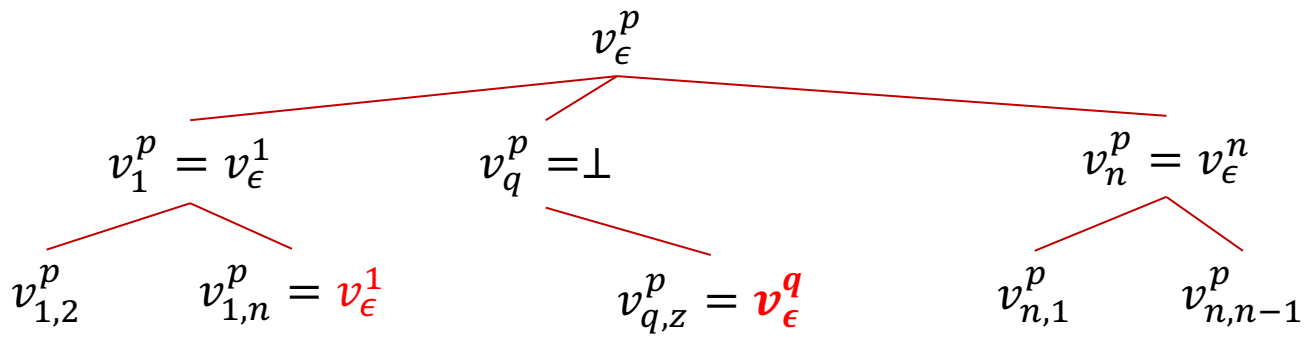
$v^p_{q_1,q_2,..,q_k}$ means: q_k told p,
that q_{k-1} told q_k,
....
that q_1's value is v
Initially all $\perp$
$v^p_\epsilon = C[p].in$



Processor q crashes

The protocol in action: round 2

$v^p_{q_1,q_2,..,q_k}$ means: q_k told p,
that q_{k-1} told q_k,
....
that q_1's value is v
Initially all $\perp$
$v^p_\epsilon = C[p].in$



For crash failures we have either $v^p_{q;\sigma} = \perp$ or $v^p_{q;\sigma} = v^q_\epsilon$

z tells p that q told z its value is v;
So q crashed after sending to z

If z is honest, z will tell this to all honest nodes

Deciding on a value

- **Let $V_p = \{v \mid v = v_\sigma^p \in T_p \wedge v \neq \perp\}$, i.e. the set of different values in T_p**
- **What are the possible values for V_p?**

Deciding on a value

- **Let $V_p = \{v \mid v = v_\sigma^p \in T_p \wedge v \neq \perp\}$, i.e. the set of different values in T_p**
- **What are the possible values for V_p?**
  - If inputs are binary, then {}, {0}, {1}, {0,1}

- **If $|V_p| = 1$, i.e. $V_p = \{v\}$**
  - p decides on v
- **Otherwise**
  - p decides on a default value v_{def}, say 0

Correctness

- **Lemma: suppose both processors p and q are correct (i.e don't fail). Then if $v \in V_p$ then $v \in V_q$**
- **Proof**
  - If $v \in V_p$ then $v = v_\sigma^p$ for some σ with $p \notin \sigma$
    - ★ *If $p \in \sigma$, i.e. $\sigma = \alpha; p; \beta$ then p sent $v = m_{\alpha;p}^p$ and hence $v = v_\alpha^p$ too, with $p \notin \alpha$*
  - If $|\sigma| < f + 1$ then p will send $m_{\sigma;p}^q = v_\sigma^p = v$ to q and then $v_{\sigma;p}^q = v$ and so $v \in V_q$
  - If $|\sigma| = f + 1$ then there is a non faulty processor z with $\sigma = \alpha; z; \beta$ such that $v_\alpha^z = v_\sigma^p$ Then at round $|\alpha| + 1$ processor z sent $v = v_\alpha^z$ to q as well (as message $v = m_{\alpha;z}^q$). Hence $v_{\alpha;z}^q = v$. Again $v \in V_q$

Correctness

- **By lemma previous slide, for any two correct processors we have agreement**
  - If $|V_p| > 1$ then $|V_q| > 1$ so both decide on the same value v_{def}
  - If $|V_p| = 1$ then $V_p = V_q = \{v\}$ for some v on which both decide
- **If all processors start with the same value v, then all nodes in any tree equals v or $\perp$. Therefore $V_p = \{v\}$ for all correct p who therefore decides on v**