Talen en Automaten

Additional assignments for exercise class on Fri 30th Nov, 2018

1 Non-regular languages via closure properties

Show that $L = \{wv \in \{a,b\}^* \mid |w| = |v| \text{ and } w \neq v\}$ is not regular.

Solution: ....

Suppose L were regular, then by the closure properties also

$$K = L \cup \{w \in \{a,b\}^* \mid |w| \text{ is odd }\}$$

would be regular. Hence, the complement $\overline{K}$ would be regular. But it is easy to check that

$$\overline{K} = \{ww \mid w \in \{a,b\}^*\}$$

This language has been shown to be non-regular in the lecture. Thus we arrive at a contradiction and L cannot be regular. □

2 Non-regular languages via distinguishable words

Show that the language

$$L = \{vca^n \mid v \in \{a,b,c\}^* \text{ with } |v| < n, \text{ for some } n \in \mathbb{N}\}$$

is not regular.

Solution: ....

Consider the infinite collection of words

$$W = \mathcal{L}(a^*c) = \{c, ac, aac, aaac, \ldots\}$$

We show that any two words in W are distinguishable. Let a^ic, $a^jc \in W$ with $i < j$. These are distinguished by the word a^j: we have

$$a^ica^j \in L$$

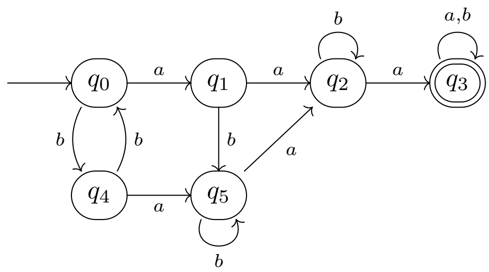
since $i < j$, but

$$a^jca^j \notin L\,.$$

Thus, W is an infinite collection of distinguishable words, hence L is not regular. □

3 Minimisation

Minimise the following automaton, using the algorithm from the lecture:



Solution: ....

The first partition separates accepting and non-accepting states:

$$P_0 = \{\{q_0, q_1, q_2, q_4, q_5\}, \{q_3\}\}$$

Then q_2 gets separated from its group, as it makes an a-transition to q_3 (all the other transitions are within the group:

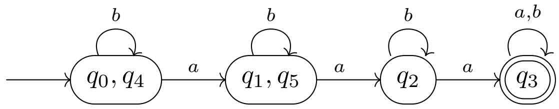
$$P_1 = \{\{q_0, q_1, q_4, q_5\}, \{q_2\}, \{q_3\}\}$$

Next step:

$$P_2 = \{\{q_0, q_4\}, \{q_1, q_5\}, \{q_2\}, \{q_3\}\}$$

And at this point we stabilise, that is, $P_3 = P_2$.

We can now draw the minimal automaton:



□