Talen en Automaten

Assignment 3, Tue 20th Nov, 2018

Exercise teachers. The student groups are supervised by the following teachers:

Teacher	E-Mail	Room	Time
Menno Bartels	m.m.bartels@student.ru.nl	HG00.065	8:30 – 10:15
Maris Galesloot	m.galesloot@student.ru.nl	HG00.086	8:30 – 10:15
Leon Gondelman	leon.gondelman@gmail.com	HG00.114	8:30 – 10:15
Ellen Gunnarsdóttir	E.Gunnarsdottir@student.ru.nl	HG00.308	8:30 – 10:15
Toine Hulshof	T.Hulshof@student.ru.nl	HG00.633	8:30 – 10:15
Alexis Linard	A.linard@cs.ru.nl	HG00.310	8:30 – 10:15
Jan Martens	j.martens@student.ru.nl	HG01.028	8:30 – 10:15
Serena Rietbergen	serena.rietbergen@student.ru.nl	HG01.029	8:30 – 10:15
Bas Steeg	bas.steeg@student.ru.nl	HG01.028	10:30 – 12:15
Nienke Wessel	N.Wessel@student.ru.nl	E1.09	10:30 – 12:15
Bas Hofmans	B.Hofmans@student.ru.nl	HG00.308	15:30 – 17:15
Amber Pater	A.Pater@student.ru.nl	HG00.310	15:30 – 17:15

Postboxes are located in the Mercator building on the ground floor. There will be boxes labelled with *LnA* and the corresponding group teacher's name. There will be 1 box, the *Uitleverbak*, for work that hasn't been picked up at the exercise hours.

Handing in your answers: There are two options:

1. E-mail: Send your solutions by e-mail to your exercise class teacher (see above) with subject "**L&A: assignment 3**". This e-mail should only contain a single PDF document as attachment (unless explicitly stated otherwise). Before sending an e-mail make sure:
   - the file is a PDF document
   - your name is part of the filename (for example MyName_assignment-3.pdf)
   - your name and student number are included in the document.
   - please do not submit photographs (scans of handwritten notes are fine).
2. Post box: Put your solutions in the appropriate post box (see above). Before putting your solutions in the post box make sure:
   - your name, student number, and IC, KI, Wiskunde or Science are written clearly on the document.

Deadline: Tue 27th Nov, 2018, 8:30 (in Nijmegen!)

Goals: After completing these exercises successfully you should be able to construct an NFA from a language description, to construct an NFA-λ from a regular expression, to turn an NFA-λ into an NFA and to determinise an NFA.

There are 4 mandatory exercises, worth **10 points** in total. There are 2 more, extra hard, exercises. Be aware that this exercise is just for fun, you cannot earn any points with it.

1 NFAs and Their Languages

Let $A = \{0, 1, 2\}$ and let L be the set of words in which the last digit occurs at least twice, with no larger digit in between the last two occurences:

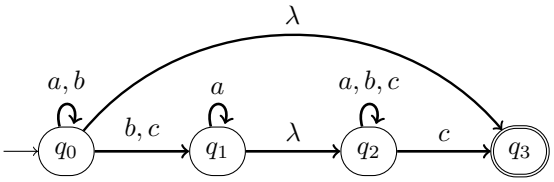
$$L = \{w \in A^* \mid \exists x \in A.\ \exists u, v \in A^*.\ w = uxvx, \text{ and there is no } y \text{ in } v \text{ s.t. } x < y\},$$

where $0 < 1 < 2$. For example, $00, 2111, 2102 \in L$, but $1, 121 \notin L$.

Construct an NFA that accepts L, and show that 2101 is accepted, but 121 not. **(3pt)**

2 NFA-λ

a) Let M be the NFA-λ over the alphabet $A = \{a, b, c\}$ given by the following graph.



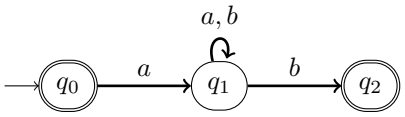
Give an NFA (without λ-transitions) with the same set of states, accepting the same language. **(1pt)**

b) An *NFA-λ with multiple initial states* over an alphabet A is a tuple $\mathcal{A} = (Q, I, \delta, F)$ where $I \subseteq Q$ is a set of initial states; Q, δ and F are the same as with NFA-λ. A word $w \in A^*$ is accepted if there exists a state $q_0 \in I$ such that $\delta^*(q_0, w) \cap F \neq \emptyset$.

Give, for any NFA-λ $\mathcal{A} = (Q, I, \delta, F)$ with multiple initial states, an NFA-λ $\mathcal{A}' = (Q', q_0, \delta', F)$ (with one initial state) such that $\mathcal{A}$ and $\mathcal{A}'$ accept the same words. Explain your answer. **(2pt)**

3 From NFA to DFA

Let M be the NFA over the alphabet $A = \{a, b\}$ given by the following graph.



Use the powerset construction from the lecture to turn M into a DFA M' that accepts the same language. Leave out unreachable states, and clearly indicate how a state of M' corresponds to a subset of states of M. **(2pt)**

4 From Regular Expression to NFA-λ

Let e be the regular expression $(b + a^*)a$.

Use the "toolkit" from the lecture to construct an NFA-λ that accepts $\mathcal{L}(e)$. The (non-trivial) intermediate steps must be given as part of the solution. **(2pt)**

5 Fun Exercises – Properties of Regular Languages

a) Using that regular languages are closed under complement and intersection, show that for regular languages L_1, L_2 their difference $L_1 \setminus L_2$ is regular as well.

b) Give an algorithm that decides whether for a regular expression e its language $\mathcal{L}(e)$ is empty.

c) Give an algorithm that checks for given regular expressions e_1, e_2 whether their languages are equal: $\mathcal{L}(e_1) = \mathcal{L}(e_2)$.

6 Fun Exercise – Constructing an NFA-λ

Give an NFA-λ over $A = \{a\}$ such that it rejects some string and the length of the shortest rejected string is strictly greater than the number of states.