Languages and Automata

Assignment 5, Tuesday 4$^{\text{th}}$ December, 2018

Exercise teachers. The student groups are supervised by the following teachers:

Teacher	**E-Mail**	**Room**	**Time**
Menno Bartels	`m.m.bartels@student.ru.nl`	HG00.065	8:30 – 10:15
Maris Galesloot	`m.galesloot@student.ru.nl`	HG00.086	8:30 – 10:15
Leon Gondelman	`leon.gondelman@gmail.com`	HG00.114	8:30 – 10:15
Ellen Gunnarsdóttir	`E.Gunnarsdottir@student.ru.nl`	HG00.308	8:30 – 10:15
Toine Hulshof	`T.Hulshof@student.ru.nl`	HG00.633	8:30 – 10:15
Alexis Linard	`A.linard@cs.ru.nl`	HG00.310	8:30 – 10:15
Jan Martens	`j.martens@student.ru.nl`	HG01.028	8:30 – 10:15
Serena Rietbergen	`serena.rietbergen@student.ru.nl`	HG01.029	8:30 – 10:15
Bas Steeg	`bas.steeg@student.ru.nl`	HG01.028	10:30 – 12:15
Nienke Wessel	`N.Wessel@student.ru.nl`	E1.09	10:30 – 12:15
Bas Hofmans	`B.Hofmans@student.ru.nl`	HG00.308	15:30 – 17:15
Amber Pater	`A.Pater@student.ru.nl`	HG00.310	15:30 – 17:15

Postboxes are located in the Mercator building on the ground floor. There will be boxes labelled with *LnA* and the corresponding group teacher's name. There will be 1 box, the *Uitleverbak*, for work that hasn't been picked up at the exercise hours.

Handing in your answers: There are two options:

1. E-mail: Send your solutions by e-mail to your exercise class teacher (see above) with subject "**L&A: assignment 5**". This e-mail should only contain a single PDF document as attachment (unless explicitly stated otherwise). Before sending an e-mail make sure:
   - the file is a PDF document
   - your name is part of the filename (for example MyName_assignment-5.pdf)
   - your name and student number are included in the document.
   - please do not submit photographs (scans of handwritten notes are fine).
2. Post box: Put your solutions in the appropriate post box (see above). Before putting your solutions in the post box make sure:
   - your name, student number, and IC, KI, Wiskunde or Science are written clearly on the document.

Deadline: Tuesday 11$^{\text{th}}$ December, 2018, 8:30 (in Nijmegen!)

Goals: After completing these exercises successfully you should be able to read context-free grammars, write down grammars for context-free languages and regular languages, and work with basic closure properties of context-free languages. The total number of points is 10.

There are 4 mandatory exercises, worth **10 points** in total. There are 3 more, extra hard, exercises. Be aware that these exercises are just for fun, you cannot earn any points with them.

1 Ambiguous grammars

Let $\Sigma = \{a, b\}$. Consider the following context-free grammar

$$G_1 = \boxed{S \quad \rightarrow \quad a\,S \mid S\,b \mid a\,b \mid S\,S}$$

a) Show that the grammar is ambiguous by giving two left-most derivations of $a\,a\,b\,b$. **(1pt)**

b) Give a regular expression for $L_1 := \mathcal{L}(G_1)$. **(1pt)**

c) Give a non-ambiguous regular grammar for L_1. **(1pt)**

2 Constructing context-free grammars

For each of the following languages construct a context-free grammar that generates the language, and **explain why your answer is correct**. **(4pt)**

$$\begin{aligned} L_1 &= \{a^n b^{n+m} a^m \mid n, m \geq 0\} \\ L_2 &= \{w \in \{a, b, r\}^* \mid \text{every } a \text{ (in } w\text{) is followed directly by } b\text{, and } w \text{ ends with } brrr\} \end{aligned}$$

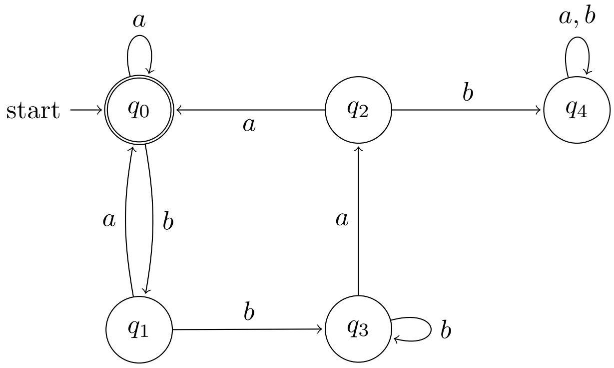
3 Closure properties

For each of the following statements, decide whether or not they are correct. Justify your answer with a proof (if the statement is correct) or a counterexample (if it is not correct). **(2pt)**

1. If L is context-free, then the complement $\overline{L}$ is *not* context-free.
2. If L is context-free and K is regular, then $L \cap K$ is regular.

4 From DFA to regular grammar

Consider the following DFA M:



Construct a context-free grammar that generates $\mathcal{L}(M)$. **(1pt)**

Fun Exercises

1. Construct a context-free grammar that generates the following language:

$$L_5 = \{w \in \{a, b\}^* \mid |w| = 2k + 1 \text{ and } w_1 = w_{k+1}\},$$

where w_i denotes the i-th symbol in a word w. That is, L_5 consists of all words of odd length that have the same symbol in the first and middle positions.

2. $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \times, +, (,)\}$. Construct a context-free grammar that generates the language

$$L_6 = \{w \in \Sigma^* \mid w \text{ is a well-formed arithmetical expressions}\}$$

NB. $2+3+4\times 5$ and $((2+3)+4)\times 5$ and $(((2+3)))+4\times 5$ are well-formed. $2+(3+4\times 5$ and $(2+3)+4)\times 5$ and $)($ are not.

3. Suppose that L and L' are context-free languages. Show that both L^* and $L\,L'$ are context-free languages.