Languages and Automata

Assignment 7, Tuesday 18$^{\text{th}}$ December, 2018

This (last) homework exercise is not graded; it's just to practice. Model solutions will be provided on the webpage of the course, and you can work on it during the last exercise class (Dec 21).

Goals: After completing these exercises successfully you should be able to construct and recognise languages of context sensitive grammars.

1 Context Sensitive Grammars

a) **i)** Give a context sensitive grammar that generates the language

$$\{a^n b^n c^n d^n \mid n \in \mathbb{N}\}.$$

You may abbreviate a group of three productions of the form $CB \longrightarrow XB$, $XB \longrightarrow XC$ and $XC \longrightarrow BC$ to $CB \rightsquigarrow_X BC$.

Solution: . . . .

$$\begin{aligned}
S &\to T \mid \lambda \\
T &\to ABTCD \mid ABCD \\
BA &\rightsquigarrow_X AB \\
DC &\rightsquigarrow_X CD \\
BC &\to bC \\
bC &\to bc \\
Bb &\to bb \\
Ab &\to ab \\
Aa &\to aa \\
cC &\to cc \\
cD &\to cd \\
dD &\to dd
\end{aligned}$$

□

ii) Show that the word $aabbccdd$ is generated. You may use the above abbreviation.

Solution: . . . .

$$\begin{aligned}
S &\to ABSCD \to ABABSCDCD \to ABABCDCD \rightsquigarrow_X AABBCDCD \\
&\rightsquigarrow_X AABBCCDD \to AABbCCDD \to AABbcCDD \to AAbbcCDD \\
&\to AabbcCDD \to aabbcCDD \to^* aabbccdd
\end{aligned}$$

□

b) Consider the following context sensitive grammar G

$$\begin{aligned}
S &\to XSY \mid a \mid b \\
Xa &\to aa \\
Xb &\to bb \\
Y &\to a
\end{aligned}$$

i) Describe the language generated by G using set notation.
Solution: . . . .

$$\{a^n \mid n \in \mathbb{N} \text{ is odd}\} \cup \{b^{n+1}a^n \mid n \in \mathbb{N}\}$$

□

ii) Is $\mathcal{L}(G)$ context-free? If so, give a CFG. **(1pt)**

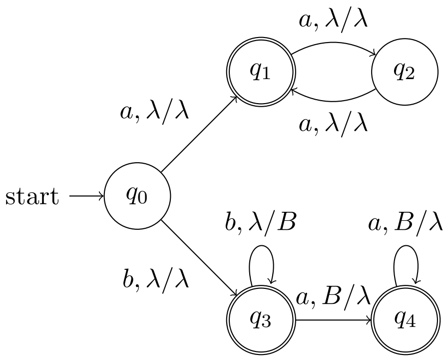
Solution: . . . .

$$\begin{aligned} S &\to T \mid U \\ T &\to aTa \mid a \\ U &\to bUa \mid b \end{aligned}$$

□

iii) Is $\mathcal{L}(G)$ deterministic context-free? If so, give a deterministic PDA.
Solution: . . . .

Here is a deterministic PDA



□

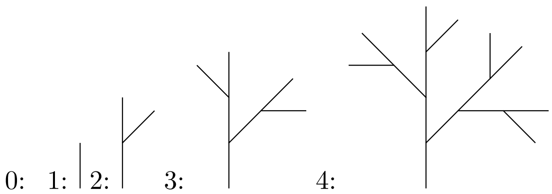
2 Fun Exercise – Lindenmayer Systems

Draw 4 iterations of the following Lindenmayer system with start symbol S.

$$S \to F[+S][X]$$
$$X \to F[-X][S]$$

Here + stands for a 45° rotation clockwise, and - stands for a 45° rotation counter-clockwise, F stands for a step forward.

Solution: ....



□

3 Fun Exercises – Turing Machines

In this exercise, a Turing machine (TM) accepts an input if there is a computation (remember that a TM is non-deterministic!) that ends in a final state, the tape can still contain arbitrary symbols. Moreover, one may use more symbols on the tape than used for the input. Finally, you are also allowed to use, besides $\rightarrow$ and $\leftarrow$, $\downarrow$, which keeps the current position of the head.

Let $P \subseteq \{a, ;, (,)\}^*$ encode the graph of the addition of natural numbers in unary encoding, that is,

$$P = \{(a^n; a^m; a^k) \mid n, m, k \in \mathbb{N} \text{ and } k = n + m\}.$$

a) Give a TM that accepts P, where you may assume that the input word on the tape is already of the form $(a^n; a^m; a^k)$ for some $n, m, k \in \mathbb{N}$ and that the head is on "(" in the beginning.

b) Show that the word $(; a; a)$ is accepted by your TM.

4 Fun Exercise – Turing Machines and PDAs

Recall the definition of a PDA_2 with two stacks from the last exercise sheet. Show that there is for each TM a PDA_2 that accepts the same language.