Languages and Automata

Final exam, 11 January 2019

8:30 – 10h30

This test consists of **5 exercises** over **9 pages**. Explain your approach, and **write your answers to the exercises on a separate folio (double pages) as indicated.** You can score a maximum of 100 points, and each question indicates how many points it is worth. The test is closed book. You are NOT allowed to use a calculator, a computer or a mobile phone. You should answer in English. Please write clearly, and do not forget to put on each page: your name and your student number.

Notation Throughout the test, we denote for any alphabet A, $w \in A^*$ and $a \in A$ by $|w|_a$ the number of a's in w, as it was introduced in the lecture. We denote the length of a word w by $|w|$.

Write your answers to Problems 1 and 2 on a separate folio (double page)

Problem 1.

For each of the following claims, state whether it is true or false. Give a brief explanation (no proofs needed) if it is true, and a counterexample if it is false. **(12pt)**

1. Suppose G is a context-free grammar. If the language $\mathcal{L}(G)$ is regular, then G must be a regular grammar.
2. The language $\{ww^R \mid w \in \{a,b\}^*\}$ is context-sensitive.
3. Let $L, K \subseteq A^*$ be languages over some alphabet A. If L is not regular, then also $L \cap K$ is not regular.

Problem 2.

Consider the following three languages over the alphabet $A = \{a, b, c\}$.

$$L_1 = \{a^i b^j c^k \mid i,j,k \geq 0 \text{ where } i = j \textbf{ or } j = k\}$$
$$L_2 = \{w \in \{a,b\}^* \mid |w|_a \text{ and } |w|_b \text{ are both even}\}$$
$$L_3 = \{a^i b^j c^k \mid i,j,k \geq 0 \text{ where } i = j = k\}$$

One of these languages is regular, one is context-free but not regular, and one is context-sensitive but not context-free.

a) Which language is regular? Show that it is, by giving a deterministic finite automaton which accepts it. Explain your answer. **(10pt)**

b) Give a regular expression for the language which is regular (from the answer to the previous question). Explain your answer. **(10pt)**

c) Pick one of the other languages from the above three and show that it is not regular. **(10pt)**

d) Which language is context-free but not regular? Give a pushdown automaton recognising this language. Explain your answer. **(10pt)**

Write your answers to Problems 3 and 4 on a separate folio (double page)

Problem 3.

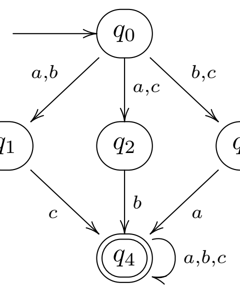
Let $A = \{a, b, c\}$.

a) Define, by induction, a function $f \colon A^* \to A^*$ that, given a word w, replaces every letter a in w by aca, every letter b by $babc$ and that removes every c. So, for instance, $f(abca) = acababcaca$ and $f(ccbba) = babcbabcaca$. **(5pt)**

b) Show, by induction, that for all words $w \in A^*$: $|f(w)|_c = |w|_a + |w|_b$. **(8pt)**

Problem 4.

Consider the following NFA M over the alphabet $A = \{a, b, c\}$:

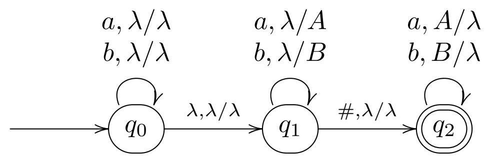


Use the construction from the lecture to give a deterministic finite automaton M' over A such that $\mathcal{L}(M) = \mathcal{L}(M')$. Clearly mark how the states of M' correspond to the states of M. **(10pt)**

Write your answers to Problem 5 on a separate folio (double page)

Problem 5.

Consider the following pushdown automaton M over the alphabet $A = \{a, b, \#\}$, with stack alphabet $\Gamma = \{A, B\}$.



a) Show that the word $abab\#ba$ is accepted, but the word $ab\#ab$ is not accepted. **(5pt)**

b) Give a precise description of the language $\mathcal{L}(M)$ accepted by M using set-notation. **(10pt)**

c) Give a context-free grammar G such that $\mathcal{L}(M) = \mathcal{L}(G)$. **(10pt)**