Talen en Automaten

Test 1, Tue 13$^{\text{th}}$ Dec, 2016
13h30 – 15h30

This test consists of **8** exercises over **3 pages**. Explain your approach, and **write your answers to the exercises on a separate folio (double pages) as indicated.**. You can score a maximum of 100 points, and each question indicates how many points it is worth. The test is closed book. You are NOT allowed to use a calculator, a computer or a mobile phone. You may answer in Dutch or in English. Please write clearly, and do not forget to put on each page: your name and your student number.

Notation Throughout the test, we denote for any alphabet A, $w \in A^*$ and $a \in A$ by $|w|_a$ the number of a's in w, as it was introduced in the lecture. Moreover, recall that v is a *subword* of w if $w = xvy$ for some words x, y.

Write your answers to Problems 1 and 2 on a separate folio (double page)

Problem 1.

Let $A = \{a, b, c\}$ be our finite alphabet and define $f : A^* \to A^*$ inductively as follows

$$\begin{array}{rcl} f(\lambda) & = & c \\ f(a\,v) & = & a\,f(v) \\ f(b\,v) & = & f(v)\,b \\ f(c\,v) & = & b\,f(v)\,b \end{array}$$

a) Give two different words w and v for which $f(w) = f(v) = bacbb$. **(5pt)**

b) Prove by induction the following property (for all $w \in A^*$) **(10pt)**

$$|f(w)|_b = |w|_b + 2|w|_c.$$

c) Give a regular language L such that $f(L) := \{f(w) \mid w \in L\}$ is not regular. (Describe $f(L)$ and argue why $f(L)$ is not regular; a full proof is not required.) **(5pt)**

Problem 2.

Consider the following language L over the alphabet $A = \{a, b, c\}$.

$$L = \{w \mid w \text{ does not contain } abc \text{ as subword}\}$$

a) Give a DFA that accepts L. Explain your answer. **(10pt)**

b) Show that your DFA accepts $abab$ and rejects $ababc$. **(5pt)**

Write your answers to Problems 3,4 and 5 on a separate folio (double page)

Problem 3.

Give a regular expression for the following language over the alphabet $A = \{a, b\}$. **(10pt)**

$$L = \{w \mid |w|_a \text{ is not a multiple of 3 and } w \text{ does not contain } bb \text{ as a substring}\}$$

Explain your answer.

Problem 4.

Consider the following language over the alphabet $A = \{a, b\}$.

$$L = \{(ab)^n w \mid w \text{ contains } n \text{ copies of } ab \text{ as subword}\}$$

a) Show that L is not regular. **(10pt)**

b) Now consider the language K over the alphabet $B = \{a, b, c\}$ given by **(5pt)**

$$K = \{w \in B^* \mid \text{ if } w \in A^* \text{ then } w \notin L\}$$

Is K regular? Explain your answer.

Problem 5.

Use the product construction to give a DFA that accepts the following language over the alphabet $A = \{a, b\}$. **(10pt)**

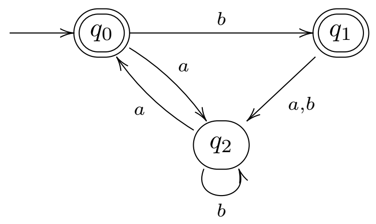
$$L = \{w \mid |w|_a \text{ is a multiple of 3 and } |w|_b \text{ is odd}\}$$

Show how your automaton arises as the product of two automata.

Write your answers to Problems 6,7 and 8 on a separate folio (double page)

Problem 6.

Let $A = \{a, b\}$ and the DFA M over A given by **(10pt)**



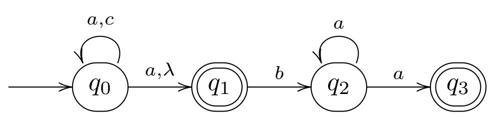
Use the procedure from the lecture to construct a regular expression e such that $\mathcal{L}(M) = \mathcal{L}(e)$. Show each intermediate step.

Problem 7.

Consider the regular expression $e = b^*(1 + a + aa + aaa)b^*$. Give an NFA-$\lambda$ M with at most four states such that $\mathcal{L}(M) = \mathcal{L}(e)$. Explain your answer. **(10pt)**

Problem 8.

Let $A = \{a, b, c\}$ and consider the following NFA-λ M over A. **(10pt)**



Use the powerset construction from the lecture to construct a DFA D with $\mathcal{L}(D) = \mathcal{L}(M)$. Indicate clearly from which subset of states in M a state in D originates.