Talen en Automaten

Retake Exam, Tue 1st May, 2018
8:30 – 11:30

This test consists of **7** problems over **2 pages. Explain your approach.** You can score a maximum of 100 points, and each question indicates how many points it is worth. The test is closed book. You are NOT allowed to use a calculator, a computer or a mobile phone. You may answer in Dutch or in English. Please write clearly, and do not forget to put on each page: your name and your student number.

Notation Throughout the test, we denote for any alphabet A, $w \in A^*$ and $a \in A$ by $|w|_a$ the number of a's in w, and by $|w|$ the length of the word w.

Problem 1.

For each of the following claims, state whether it is true or false. Support your claim with either a counterexample or a (short) argument/proof. **(16pt)**

1. If a language L contains finitely many words, it is regular.
2. If a language is context-free, it is not regular.
3. If L is regular, then $K \cap L$ is also regular.
4. The language $L = \mathcal{L}\left(b(a+b)^*\right) \cup \{a^n b^m \mid n \neq m\}$ is regular.

Problem 2.

You are in charge of the security of a large web-based company, and you should formally describe what a good password looks like. Fortunately, you know regular expressions...

Th alphabet of our passwords will be $\Sigma = \{a, b, c, A, B, C, 0, 1, 2\}$. A *safe password* is a word $w \in \Sigma^*$ such that w begins and ends with a number, w contains at least one lower-case letter and w contains at least one upper-case letter.

Give a regular expression e such that $\mathcal{L}(e) = \{w \in \Sigma^* \mid w \text{ is a safe password}\}$. **(10pt)**

Problem 3.

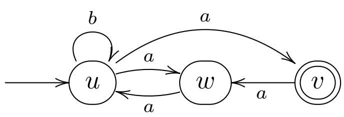
Let $A = \{a, b\}$ be our finite alphabet and define $f : A^* \to A^*$ inductively as follows:

$$f(\lambda) = \lambda \qquad f(av) = f(v)aba \qquad f(bv) = f(v)bb$$

Prove that $f(wa) = abaf(w)$ for all $w \in A^*$, by induction on w. **(10pt)**

Problem 4.

Consider the following NFA M over the alphabet $A = \{a, b\}$:



a) Show that aaa is accepted by M, and aab is not. **(5pt)**

b) Use the construction from the lecture to give a deterministic finite automaton N such that $\mathcal{L}(M) = \mathcal{L}(N)$. Clearly show how states in N relate to states in M. **(10pt)**

Problem 5.

Consider the following three languages over the alphabet $A = \{a, b, \#\}$.

$$L_1 = \{w\#v \mid w, v \in \{a,b\}^* \text{ and the first letter of } w \text{ equals the last letter of } v\}$$
$$L_2 = \{w\#v\#z \mid w, v \in \{a,b\}^* \text{ and } |w|_a = |v|_a = |z|_a\}$$
$$L_3 = \{w\#v\#z\#u \mid w, v, z, u \in \{a,b\}^* \text{ and } |w| = |v| \text{ and } |z| = |u|\}$$

Two of these languages are context-free, and the third is not context-free. One of the three languages is regular.

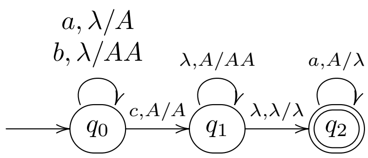
a) Which of the languages L_1, L_2, L_3 is regular? Give an DFA which accepts that language. **(10pt)**

b) Which of the languages L_1, L_2, L_3 is context-free but not regular? Give a context-free grammar which generates that language. **(10pt)**

c) Choose a non-regular language from L_1, L_2, L_3, and use the pumping lemma to prove that it is indeed not regular. **(10pt)**

Problem 6.

Consider the following push-down automaton M over the alphabet $A = \{a, b, c\}$.



a) Is M deterministic? Explain your answer. **(3pt)**

b) Describe the language $\mathcal{L}(M)$ using set notation. **(8pt)**

Problem 7.

Consider the language over the alphabet $A = \{(,), [,]\}$ given by the following grammar G:

$$S \to (S) \mid SS \mid [S] \mid \lambda$$

Give a PDA M such that $\mathcal{L}(G) = \mathcal{L}(M)$. **(8pt)**