Exercises Coalgebra for Lecture 8

The exercises labeled with **(*)** are optional and more advanced.

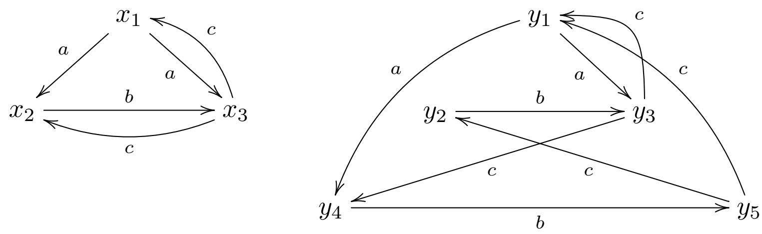
1. Use bisimulations up to congruence to prove the following identities between languages $L, K, M \in 2^{A^*}$.

   (a) $L(K + M) = LK + LM$

   (b) $(L^*)^* = L^*$

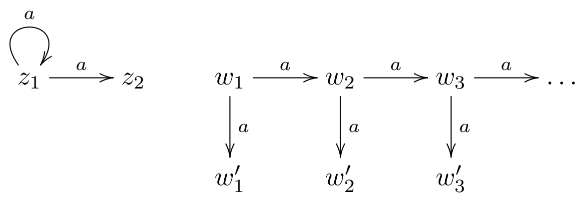
   (c) $L^* L^* = L^*$

2. (a) Show that x_1 and y_1 below are bisimilar.[1]



(1)

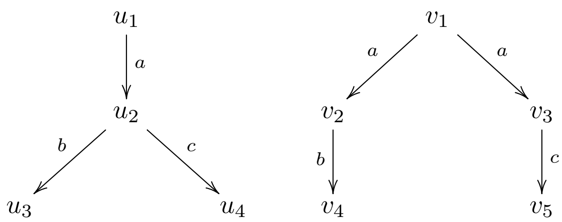
   (b) Show that z_1 and w_1 below are bisimilar.



(2)

   (c) Show that u_1 and v_1 below are *not* bisimilar.

(3)



[1] Exercise taken from: D. Sangiorgi, Introduction to Bisimulation and Coinduction, Cambridge Universty Press.

3. Let $f\colon X \to (\mathcal{P}(X))^A$ be a labelled transition system.

   (a) The *finite traces* of a state $x \in X$ are, informally, all those words w for which there exists a path starting from x, labelled by w. For instance, the set of traces of the state z_1 in (2) in the previous exercise is $\{\langle\rangle, a, aa, aaa, \ldots\}$ and, for state u_1 in (3), it is $\{\langle\rangle, a, ab, ac\}$. Define, by induction on words $w \in A^*$, a function $l\colon X \to 2^{A^*}$ that maps a state to the corresponding set of finite traces.

   (b) Define a deterministic automaton $\langle \epsilon, \delta \rangle\colon \mathcal{P}(X) \to 2 \times (\mathcal{P}(X))^A$ such that the accepted by a state $\{x\}$ is given by $l(x)$.

   (c) **(*)** Show that the above construction defines a functor from the category of transition systems (over A) to the category of deterministic automata (over A). What does this say about the relationship between bisimilarity and trace semantics?

4. **(*)** There are unique languages L and K over the alphabet $A = \{a, b\}$ such that

$$L = aLa + bLb + a + b + 1 \qquad K = aKa + bKb + aA^*b + bA^*a$$

   L is the language of *palindromes*: words which are equal to their own reverse. Prove that K is the language of all *non*-palindromes, by showing that the relation $R = \{(\overline{L}, K)\}$ is a bisimulation up to congruence, where $\overline{L}$ is the complement of L.

   Hint: to relate the derivatives, use that $\overline{La} = (\overline{L}a + A^*b + 1)$ (which holds for every language L).