A Theory of Diagnosis as Hypothesis Refinement*

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Abstract

In this paper, diagnosis is viewed as a two-stage process: domain knowledge is first interpreted in a diagnostic sense; next, observed findings are interpreted with respect to this interpreted knowledge and a given hypothesis, yielding a diagnosis. A set-theoretical framework is briefly discussed that captures this view on diagnosis; it is used to formalise various notions of diagnosis, those proposed in the literature included. Next, a theory of flexible diagnosis, called refinement diagnosis, is proposed and defined in terms of this framework. Relationships with notions of diagnosis known from the literature are investigated.

1 Introduction

In recent years, several theories of diagnosis have been proposed, providing different foundations for diagnostic problem solving in intelligent systems. Diagnostic problem solving is variously described in terms of *abductive reasoning* (cf. [CTT89, Poo94]), as a specific form of *consistency-based reasoning* (cf. [KMR92, Poo94]), or as *deductive reasoning* (cf. [BS84]). In the context of diagnosis, one usually speaks of *abductive diagnosis*, *consistency-based diagnosis* and *heuristic classification*, respectively. Abductive diagnosis is primarily used in systems incorporating causal models of normal or faulty behaviour. Consistency-based diagnosis appears especially suitable for models of normal structure and functional behaviour. Abductive and consistency-based diagnosis are often classified as *model-based* approaches to diagnosis. Heuristic classification is typically used in systems based on empirical knowledge.

Although the diagnostic frameworks mentioned above differ in several respects, in all of them diagnostic problem solving can be viewed as a special instance of *hypothetical reasoning*. In solving a diagnostic problem, a hypothesis is first generated and next tested with respect to diagnostic knowledge and observed findings. If it passes the tests, it is accepted and called a diagnosis; when it fails to pass the tests, it is rejected. This view of diagnosis is quite general, but it is still unnecessarily restrictive. Instead of simply rejecting a hypothesis that does not comply with all requirements, it seems more natural to adjust or refine it, when possible. Since knowledge bases are almost without exception incomplete, rigorous notions of diagnosis may be too strong. For example, in medicine it is usually better to arrive at a diagnosis that does not account for all observed findings, or that suggests findings that have not been observed, than to establish no diagnosis at all. The formalisation of this as *refinement diagnosis* is the subject of this paper.

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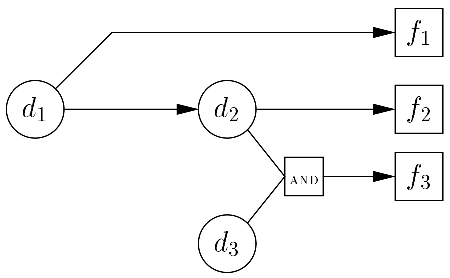


(a)

Meaning
d_1: influenza
d_2: tracheobronchitis
d_3: asthma

f_1: fever
f_2: sore throat
f_3: dyspnoea (shortness of breath)

(b)

Figure 1: Causal net.

2 A framework of diagnosis

In this section, we provide a brief overview of a set-theoretical framework of diagnosis that is used in the remainder of the paper for defining and analysing various notions of diagnosis, those mentioned in the literature included (cf. [Luc96]).

2.1 The representation of interactions

Consider the following piece of medical knowledge: "Influenza causes fever and infection of the trachea and bronchial tree, which causes a sore throat, but if the patient suffers from asthma, dyspnoea will occur as well." In Figure 1(a), the directed graph representation of the causal knowledge embodied in this medical description is depicted, where an arc denotes a cause-effect relationship. The medical meaning ascribed to the elements in the causal graph is indicated in Figure 1(b). Elements d_i represent disorders; elements f_i are observable findings. Note that in the graph the disorders d_1 and d_2 are causally related to each other.

Interactions among disorders can be captured more precisely by means of a mapping of sets of disorders to sets of observable findings, yielding a diagnostic interpretation of this knowledge. Such a mapping will be called an evidence function. Since the term 'disorder' is not used in technical domains, where instead the term 'fault' is commonly employed to indicate device problems, the term '*defect*' will be used in the following to denote both disorders in medicine and faults in technical devices.

More formally, let Δ denote a set of defects, and let Φ denote a set of findings. Positive defects d (findings f) and negative defects $\neg d$ (findings $\neg f$) denote *present* defects (findings) and *absent* defects (findings), respectively. It is assumed that $\neg \circ \neg = \iota$, where ι is the identity function. If a defect d or a finding f is not included in a set, it is assumed to be unknown. Let a set X_P denote a set of positive elements, and let X_N denote a set of negative elements, such that X_P and X_N are disjoint. It is assumed that $\Delta = \Delta_P \cup \Delta_N$ and $\Phi = \Phi_P \cup \Phi_N$. The power set of a set S is denoted by $\wp(S)$. Now, an *evidence function* e is a mapping

$$e : \wp(\Delta) \to \wp(\Phi) \cup \{\perp\}$$

such that: (1) for each $f \in \Phi$ there exists a set $D \subseteq \Delta$ with $f \in e(D)$ or $\neg f \in e(D)$ (and possibly both), (2) if $d, \neg d \in D$, for some $D \subseteq \Delta$, then $e(D) = \perp$, and (3) if $e(D) \neq \perp$ and $D' \subseteq D$, with $D, D' \subseteq \Delta$, then $e(D') \neq \perp$. If $e(D) \neq \perp$, it is said that $e(D)$ is the set of *observable findings* for D, and D is called *consistent*; otherwise, it is said that D is *inconsistent*. Finally, a triple $\Sigma = (\Delta, \Phi, e)$ is called a *diagnostic specification*.

For the medical knowledge depicted in Figure 1, it holds, among others, that:

$$\begin{array}{llll} e(\{d_1\}) & = \{f_1, f_2\} & e(\{d_3\}) & = \varnothing \\ e(\{d_2\}) & = \{f_2\} & e(\{d_2, d_3\}) & = \{f_2, f_3\} \\ e(\{d_1, d_2\}) & = e(\{d_1\}) & e(\{d_1, d_2, d_3\}) & = \{f_1, f_2, f_3\} \end{array}$$

The property $e(\{d_i\}) \subseteq e(\{d_1, d_2\})$, $i = 1, 2$, formally expresses that the interaction between d_1 and d_2 is monotonic; the evidence function e is monotonically increasing. An evidence function may also be monotonically decreasing, or nonmonotonic (examples are given below).

Various semantic properties of a domain for which a diagnostic system must be built can be expressed precisely in terms of evidence functions. Local as well as global interactions between defects can be expressed readily. A typical global property of evidence functions encountered in the literature is interaction freeness (cf. [PR90]). A set of defects Δ is called *interaction free* iff $e(D) = \bigcup_{d \in D} e(\{d\})$, for each consistent $D \subseteq \Delta$. This shows that an evidence function can be partially specified.

2.2 Notions of diagnosis

A specific evidence function provides a semantic interpretation of a knowledge base in terms of expected evidence for the combined occurrence of defects; yet, it does not yield a diagnosis. To employ an evidence function for the purpose of diagnosis, it must be interpreted with respect to actually observed findings. Such an interpretation of an evidence function and of observed findings can be viewed as a notion of diagnosis applied to solve a diagnostic problem at hand.

More formally, let $\mathcal{P} = (\Sigma, E)$ be a *diagnostic problem*, where $E \subseteq \Phi$ is a set of *observed findings*; it is assumed that if $f \in E$ then $\neg f \notin E$, i.e. contradictory observed findings are not allowed. Let R_Σ denote a notion of diagnosis R applied to Σ, then a mapping

$$R_{\Sigma, e_{|H}} : \wp(\Phi) \to \wp(\Delta) \cup \{u\}$$

will either provide a diagnostic solution for a diagnostic problem $\mathcal{P}$, or indicate that no solution exists, denoted by u (undefined). Here, H denotes a *hypothesis*, which is taken to be a set of defects ($H \subseteq \Delta$), and $e_{|H}$, called the *restricted evidence function* of e, is a restriction of e with respect to the power set $\wp(H)$: $e_{|H} : \wp(H) \to \wp(\Phi) \cup \{\perp\}$, where for each $D \subseteq H$: $e_{|H}(D) = e(D)$. A restricted evidence function $e_{|H}$ can be thought of as the relevant part of a knowledge base with respect to a hypothesis H. An *R-diagnostic solution*, or *R-diagnosis* for short, with respect to a hypothesis $H \subseteq \Delta$, is now defined as the set $R_{\Sigma, e_{|H}}(E)$, where $R_{\Sigma, e_{|H}}(E) \subseteq H$ if a solution exists (cf. Figure 2).

A notion of diagnosis R provides the possibility to express interactions among defects and observed findings at the level of diagnosis, which we call dependencies. We may also have that a hypothesis can be split up into two subhypotheses, that can be examined independently: $R_{\Sigma, e_{|H \cup H'}}(E) = R_{\Sigma, e_{|H}}(E) \cup R_{\Sigma, e_{|H'}}(E)$, with $R_{\Sigma, e_{|H \cup H'}}(E) \neq u$. This means that the diagnostic solution with respect to the hypothesis $H \cup H'$ is obtained as the union of the solutions for the two separately examined hypotheses H and H'. This is called the *independence (or compositionality) assumption*. For many notions of diagnosis described in the literature, in particular for abductive diagnosis and consistency-based diagnosis, the independence assumption fails to hold.

To demonstrate how the definitions above can be employed, we consider a notion of diagnosis U, such that $U_{\Sigma, e_{|H}}(E) = H'$ if it holds that H' is the *only* subset of H such that

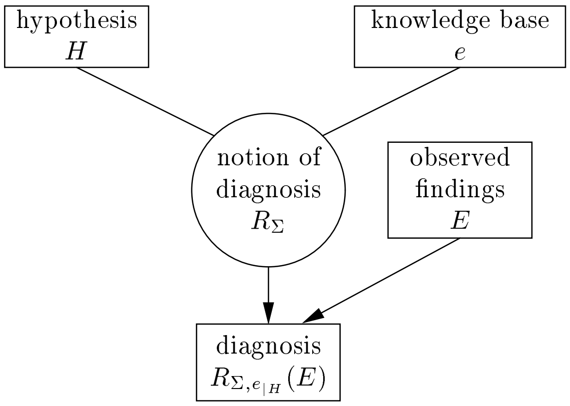


Figure 2: Schema of notion of diagnosis, diagnostic problem and solution.

$e_{|H}(H') \subseteq E$ and if $e_{|H}(H') = \varnothing$ then $E = \varnothing$; otherwise, $H' = u$. This notion of diagnosis expresses that a diagnosis consists of a unique subset of H which, on the one hand, can account for at least part of all observed findings, and, on the other hand, every finding associated with the set of defects that is taken as a diagnosis has been observed. Now, reconsider the medical example from Figure 1. Some interesting diagnostic conclusions are: $U_{\Sigma,e_{|\{d_2,d_3\}}}(\{f_2\}) = \{d_2\}$, i.e. a patient with only sore throat has tracheobronchitis, $U_{\Sigma,e_{|\{d_2,d_3\}}}(\{f_2, f_3\}) = u$, i.e. $\{d_2, d_3\}$ is not the only subset partly accounting for sore throat and dyspnoea as signs, and finally, $U_{\Sigma,e_{|\{d_2\}}}(\{f_2\}) = \{d_2\}$. In the first case, it is said that the hypothesis has been *adjusted*, in the second case, that the hypothesis is *rejected*, and in the last case, that the hypothesis has been *accepted*. This example demonstrates the flexibility of the approach.

It is also straightforward to define notions of diagnosis proposed in the literature. For example, consider the following typical instances of notions of diagnosis:

- *Abductive diagnosis using 'may' relations (weak-causality diagnosis)* [CTT89, PR90]:

$$\mathrm{WC}_{\Sigma,e_{|H}}(E) = \begin{cases} H & \text{if } e_{|H}(H) \supseteq E \\ u & \text{otherwise} \end{cases}$$

  i.e. all actually observed findings must have been defined as observable. A notion of *strong-causality diagnosis*, SC, is obtained by replacing $\supseteq$ by equality. This expresses that all observable findings must be observed, and vice versa.

- *Consistency-based diagnosis* [KMR92]:

$$\mathrm{CB}_{\Sigma,e_{|H}}(E) = \begin{cases} H & \text{if } \forall f \in E : f \in e_{|H}(H) \ \vee \neg f \notin e_{|H}(H) \\ u & \text{otherwise} \end{cases}$$

  i.e. observed findings may not contradict with observable findings.

These notions of diagnoses are the set-theoretic analogues of the corresponding notions of diagnosis mentioned above, which will be used below as reference points.

It is informative to relate these notions of diagnosis to each other in terms of a restriction relation $\sqsubseteq$; it holds that $R \sqsubseteq R'$ iff the diagnoses resulting from the notion of diagnosis R are a subset of those resulting from R' (for any legal diagnostic specification Σ). It is easily seen that: SC $\sqsubseteq$ WC $\sqsubseteq$ CB.

3 Refinement diagnosis

There exists a wide range of possibilities that can be chosen as a basis for notions of refinement diagnosis. Which of the possible choices yields the most natural result depends on the relationship between the properties of a notion of diagnosis R, on the one hand, i.e. the interpretation of the set of observed findings given a specific knowledge base, and the properties of a given evidence function e, on the other hand.

Two classes of refinement diagnosis will be studied here. Firstly, the class of notions of refinement diagnosis, called *most general diagnosis*, is examined, where the least upper bound of accepted hypotheses (with respect to set inclusion) is taken as a diagnostic solution. Secondly, the class of notions of refinement diagnosis, called *most specific diagnosis*, based on taking the greatest lower bound of accepted hypotheses is studied. In most general diagnosis, the smallest set of defects that includes every accepted subhypothesis is considered most plausible; in contrast, in most specific diagnosis, the largest set of defects that is included in every accepted subhypothesis is considered most plausible.

3.1 Most general diagnosis

Notions of most general diagnosis capture the idea that if a specific diagnostic hypothesis is not accepted, then the 'nearest' subhypothesis should be taken instead. The least upper bound with respect to set inclusion of the set of accepted subhypotheses is an example of such a 'nearest' subhypothesis. The notions of most general diagnosis enforce independence of diagnostic components in the sense of the previous section.

The notion of *most general subset diagnosis*, denoted by GS, is defined as follows:

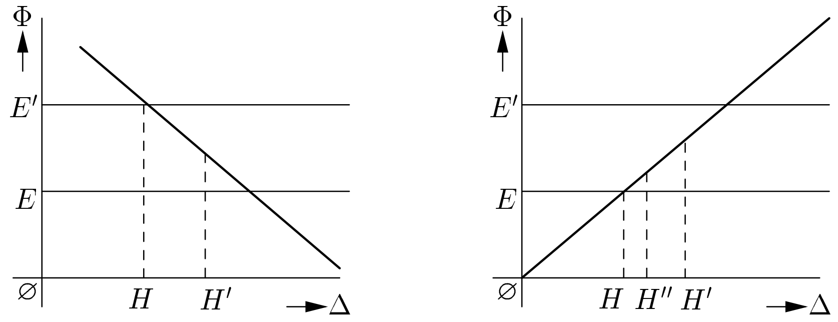
$$\mathrm{GS}_{\Sigma,e_{|H}}(E) = \begin{cases} \bigcup_{\substack{H' \subseteq H \\ e_{|H}(H') \subseteq E}} H' & \text{if } H \text{ is consistent, and} \\ & \quad \exists H' \subseteq H : e_{|H}(H') \subseteq E \\ u & \text{otherwise} \end{cases}$$

Intuitively, a most general subset diagnosis is the smallest set of defects that includes all accepted subhypotheses of a given hypothesis, where an accepted subhypothesis concerns observable findings that all have been observed.

For the example in Figure 1 with $E = \{f_1, f_2\}$, we have that $\mathrm{GS}_{\Sigma,e_{|\{d_1,d_2\}}}(E) = \{d_1, d_2\}$, which is also an abductive diagnosis, because $\mathrm{SC}_{\Sigma,e_{|\{d_1,d_2\}}}(E) = \{d_1, d_2\}$. However, it holds that $\mathrm{GS}_{\Sigma,e_{|\{d_1,d_2\}}}(\{f_2\}) = \{d_2\}$, whereas $\mathrm{SC}_{\Sigma,e_{|\{d_1,d_2\}}}(\{f_2\}) = u$. Hence, $e(\{d_1, d_2\})$ predicts a finding that cannot be accounted for, causing the defect d_1 to be ignored. This may be a suitable approach to domains in which neglecting a particular defect may be dangerous.

In Figure 3, the relationship between a diagnostic hypothesis H, the set of observed findings E and the resulting diagnosis $\mathrm{GS}_{\Sigma,e_{|H}}(E)$ is summarised by schematically depicting these sets as if they were real numbers and by taking set inclusion as the $\leq$ total order on the real numbers. If most general subset diagnosis is applied to a monotonically decreasing evidence function, the resulting diagnosis is either undefined or equal to the given hypothesis H. This contrasts with GS applied to a monotonically increasing evidence function, which may also yield subsets of the hypothesis as a diagnosis. $\mathrm{GS}_{\Sigma,e_{|H'}}(E) = H''$ in Figure 3(b) is intended to illustrate that $e(H'')$ may even be a superset of E. Similar properties can be investigated if the evidence function e is nonmonotonic.

Where most general subset diagnosis can be viewed as a more flexible version of strong-causality diagnosis SC, which for certain evidence functions is as little restrictive as consisten-



$\mathrm{GS}_{\Sigma,e_{|H}}(E') = H$
$\mathrm{GS}_{\Sigma,e_{|H'}}(E') = H'$
$\mathrm{GS}_{\Sigma,e_{|H'}}(E) = u$

(a)

$\mathrm{GS}_{\Sigma,e_{|H}}(E) = H$
$\mathrm{GS}_{\Sigma,e_{|H'}}(E) = H''$
$\mathrm{GS}_{\Sigma,e_{|H'}}(E') = H'$

(b)

Figure 3: Monotonically decreasing (a) and increasing (b) evidence functions.

cy-based diagnosis, a similar, flexible notion of diagnosis can be designed for weak-causality diagnosis. This suggests replacing the subset relation in most general subset diagnosis by the superset relation, yielding the notion of most general superset diagnosis GO (the letter 'O' stands for 'cOntains').

The notion of *most general superset diagnosis*, denoted by GO, is defined as follows:

$$\mathrm{GO}_{\Sigma,e_{|H}}(E) = \begin{cases} \bigcup_{\substack{H' \subseteq H \\ e_{|H}(H') \supseteq E}} H' & \text{if } H \text{ is consistent, and } \exists H' \subseteq H : e_{|H}(H') \supseteq E \\ u & \text{otherwise} \end{cases}$$

Most general superset diagnosis has much in common with weak-causality diagnosis WC defined in the previous section. As is true for weak-causality diagnosis WC, most general superset diagnosis restricted to monotonically increasing evidence functions is very unrestrictive, which is revealed by the fact that $\mathrm{GO}_{\Sigma,e_{|H}}(\varnothing) = H$ if $e(H) \neq \perp$, meaning that all defects constituting the hypothesis may have occurred, even if no findings have been observed. Note that the same diagnosis would have been produced by weak-causality diagnosis WC in this case. By adopting some criterion of parsimony, such as minimality according to set inclusion, the unrestrictiveness is alleviated; the empty diagnosis $\varnothing$ would then be produced.

An alternative to the definition of subset diagnosis is to consider all sets of defects D that have at least one finding f in common with the findings E observed. This leads to the following definition of the notion of *most general intersection diagnosis*, denoted by GI:

$$\mathrm{GI}_{\Sigma,e_{|H}}(E) = \begin{cases} \bigcup_{\substack{H' \subseteq H \\ (E = \varnothing \vee e_{|H}(H') = \varnothing \vee \\ e_{|H}(H') \cap E \neq \varnothing)}} H' & \text{if } H \text{ is consistent, and } (E = \varnothing \text{ or } \exists H' \subseteq H : e_{|H}(H') = \varnothing \text{ or } e_{|H}(H') \cap E \neq \varnothing) \\ u & \text{otherwise} \end{cases}$$

If the sets of observed and observable findings are nonempty, intersection diagnosis with respect to H stands for the least upper bound of subsets of defects of $H \subseteq \Delta$, where for each subset of defects H' admitted to the most general intersection diagnosis $\mathrm{GI}_{\Sigma,e_{|H}}(E)$, the associated set of observable findings $e_{|H}(H')$ is empty or has at least one finding in common with the set of observed findings E.

The advantage of most general intersection diagnosis over most general subset and superset diagnosis is that when both $e_{|H}(H')$ and E are nonempty, not all findings associated with a set of defects need be observed and not all observed findings need be accounted for.

$$\begin{array}{ccccc}
 & & \mathrm{GS} & & \\
 & \sqsubseteq & & & \\
\mathrm{SC} & & & & \mathrm{GO} \\
 & & & \sqsubseteq & \\
 & \sqsubseteq & \mathrm{WC} & & \\
 & & & \sqsubseteq & \mathrm{GI}
\end{array}$$

Figure 4: Restriction taxonomy of notions of diagnosis.

3.2 Comparison

The restriction relationships between the notions of diagnosis defined above are shown in Figure 4. Notions of diagnosis can also be classified in terms of elements included in individual diagnoses using set inclusion; the *subdiagnostic relation* $\trianglelefteq$ does exactly this. The three notions of diagnosis discussed above stand in a subdiagnostic relation to each other: GS $\trianglelefteq$ GI and GO $\trianglelefteq$ GI, i.e., a most general intersection diagnosis will always contain at least as many elements as the corresponding most general subset and superset diagnoses.

3.3 Most specific diagnosis

Rather than taking the least upper bound of a set of accepted subhypotheses of a given hypothesis, taking the greatest lower bound provides another approach to refinement diagnosis. We shall refer to notions of diagnosis based on taking the greatest lower bound as notions of *most specific diagnosis*. Where the concept of most general diagnosis formalises notions of diagnosis that yield diagnoses that include *every* accepted subhypothesis, most specific diagnosis formalises notions of diagnosis that yield diagnoses that are *common* to every accepted subhypothesis. In general it holds for a notion of most specific diagnosis S that if $S_{\Sigma,e_{|H}}(E) = \varnothing$ and $S_{\Sigma,e_{|H'}}(E) = H''$, then, by definition, $S_{\Sigma,e_{|H \cup H'}}(E) = \varnothing$. Hence, notions of most specific diagnosis are very restrictive.

As with the notion of most general subset diagnosis, in the notion of most specific subset diagnosis, subhypotheses are admitted to a diagnosis if their associated sets of findings are included in the set of observed findings of a diagnostic problem. However, of these accepted subhypotheses, only the defects the subhypotheses have in common constitute a diagnosis. Hence, the notion of *most specific subset diagnosis*, denoted by SS, is defined as follows:

$$\mathrm{SS}_{\Sigma,e_{|H}}(E) = \begin{cases} \bigcap_{\substack{H' \subseteq H \\ e_{|H}(H') \subseteq E}} H' & \text{if } H \text{ is consistent, and} \\ & \quad \exists H' \subseteq H : e_{|H}(H') \subseteq E \\ u & \text{otherwise} \end{cases}$$

This notion of diagnosis is extremely restrictive. For example, if an evidence function is interaction free, then the most specific subset diagnosis will be always equal to the empty set.

If the evidence function is monotonically decreasing, then most specific subset diagnosis tries to construct the smallest diagnosis possible. It may be view as a flexible form of kernel,

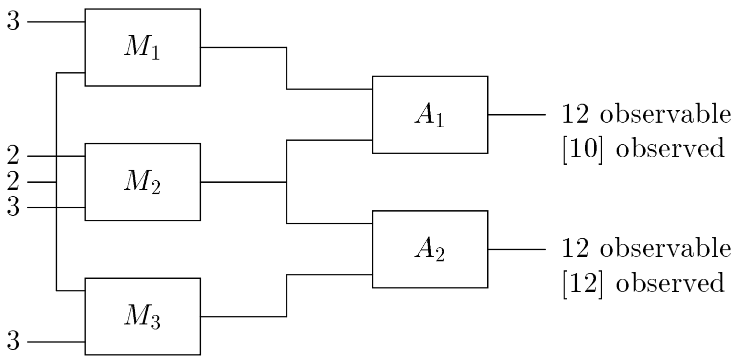


Figure 5: Multiplier-adder circuit.

consistency-based diagnosis in the sense of [KMR92] (a *kernel diagnosis* is a set of defects common to certain diagnoses). The reason for the similarity between kernel diagnosis in consistency-based diagnosis and most specific subset diagnosis is that any hypothesis H' for which $e_{|H}(H') \subseteq E$ is also consistent with E.

The correspondence between kernel diagnosis and most specific subset diagnosis is illustrated by an example taken from [KMR92]. Consider Figure 5, which depicts an electronic circuit with three multipliers, referred to as M_1, M_2 and M_3, and two adders, denoted by A_1 and A_2. Let $\Sigma = (\Delta, \Phi, e)$ be a diagnostic specification representing the circuit. The fact that some multiplier M_i (adder A_j) is defective, is denoted by m_i (a_j); if it is nondefective, this is indicated by $\neg m_i$ ($\neg a_j$). The normal output of the circuit for fixed input, $O_1 = 12$ and $O_2 = 12$, is denoted by o_1 and o_2; abnormal output is denoted by $\neg o_j$, $j = 1, 2$. Only normal behaviour is modelled, although abnormal output of the real system can be observed.

The following values of the evidence function e are among those that correspond to the circuit's normal behaviour:

$$
\begin{array}{llll}
e(\{\neg m_1, \neg m_2, \neg m_3, \neg a_1, \neg a_2\}) = \{o_1, o_2\} & & e(\{a_1\}) = \{o_2\} & \\
e(\{\neg m_1, \neg m_2, \neg m_3, a_1, \neg a_2\}) & = \{o_2\} & \vdots & \\
e(\{\neg m_1, \neg m_2, \neg m_3, a_1, a_2\}) & = \varnothing & e(\varnothing) & = \{o_1, o_2\}
\end{array}
$$

($e(\varnothing)$ denotes that it is unknown whether defects are present or absent.) The most specific subset diagnosis with respect to the hypothesis $H = \{a_1\}$ is equal to

$$\mathrm{SS}_{\Sigma, e_{|\{a_1\}}}(\{\neg o_1, o_2\}) = \{a_1\}$$

which is indeed a kernel diagnosis for the diagnostic problem $\mathcal{P} = (\Sigma, E)$, with $E = \{\neg o_1, o_2\}$ (i.e. $O_1 = 10$ and $O_2 = 12$), using consistency-based diagnosis. Note that

$$\mathrm{SS}_{\Sigma, e_{|H}}(\{\neg o_1, o_2\}) = \{a_1\}$$

if $a_1 \in H$ and $\{m_1\}, \{m_2, m_3\}$ and $\{m_2, a_2\} \not\subseteq H$ (these are the alternative kernel diagnoses), for example, $H = \{\neg m_1, \neg m_2, \neg m_3, a_1, \neg a_2\}$.

Most specific superset diagnosis is a more restrictive, and possibly more suitable, notion of diagnosis than most general superset diagnosis. The notion of *most specific superset diagnosis*, denoted by SO, is defined as follows:

$$
\mathrm{SO}_{\Sigma, e_{|H}}(E) = \begin{cases} \bigcap_{\substack{H' \subseteq H \\ e_{|H}(H') \supseteq E}} H' & \text{if } H \text{ is consistent, and} \\ & \quad \exists H' \subseteq H : e_{|H}(H') \supseteq E \\ u & \text{otherwise} \end{cases}
$$

The basic idea of most specific superset diagnosis is that the observed findings that are common to the accepted subhypotheses are due to the common defects of accepted subhypotheses. If the evidence function to which most specific superset diagnosis is applied, is monotonically increasing, the result may be intuitively attractive.

Reconsider Figure 1. For $E = \{f_2, f_3\}$ (i.e. the patient has a sore throat and dyspnoea), the most specific superset diagnosis is equal to

$$\mathrm{SO}_{\Sigma, e_{|\{d_1,d_2,d_3\}}}(E) = \{d_3\}$$

because, it holds that $e_{|H}(\{d_1, d_3\}) \supseteq E$, $e_{|H}(\{d_2, d_3\}) \supseteq E$ and $e_{|H}(\{d_1, d_2, d_3\}) \supseteq E$, where $H = \{d_1, d_2, d_3\}$. All other subsets of H have associated sets of findings that are no supersets of E. The defect d_3 stands for asthma. While both d_1 and d_2 participate in subhypotheses that also account for E, only the defect d_3 occurs in all accepted subhypotheses, i.e. it turns out to be essential. It seems therefore intuitively right to accept d_3 as the most plausible diagnosis.

As discussed above, the notion of most general intersection diagnosis is very unrestrictive. All defects that, either individually or in combination with other defects, have findings in common with the set of observed findings, are included in a diagnosis. The notion of *most specific intersection diagnosis*, denoted by SI, is much more restrictive than most general intersection diagnosis; it is defined as follows:

$$\mathrm{SI}_{\Sigma, e_{|H}}(E) = \begin{cases} \bigcap_{\substack{H' \subseteq H \\ (E = \varnothing \vee e_{|H}(H') = \varnothing \vee \\ e_{|H}(H') \cap E \neq \varnothing)}} H' & \begin{array}{l}\text{if } H \text{ is consistent, and } (E = \varnothing \text{ or} \\ \quad \exists H' \subseteq H : e_{|H}(H') = \varnothing \text{ or} \\ \quad e_{|H}(H') \cap E \neq \varnothing)\end{array} \\ u & \text{otherwise} \end{cases}$$

If the evidence function e is monotonically increasing, the resulting diagnosis will be equal to the empty set if the function values $e(\{d\})$ have many observable findings in common.

3.4 Comparison

Although the notions of most specific diagnosis are very restrictive, they do not stand in a simple restriction relation to the other notions of diagnosis. However, it is easy to see that $\mathrm{SS}_{\Sigma, e_{|H}}(E) \subseteq \mathrm{GS}_{\Sigma, e_{|H}}(E)$ holds for each consistent $H \subseteq \Delta$. Similar set inclusion relations hold for the other notions of diagnosis. We state without proof that SS $\trianglelefteq$ GS, SO $\trianglelefteq$ GO, and SI $\trianglelefteq$ GI hold.

4 Discussion

Above, we have introduced a set-theoretical framework of diagnosis, which was used as a tool for the formalisation of various notions of refinement diagnosis. As was shown, the particular properties of evidence functions to which a notion of diagnosis is applied, are important for assessing the appropriateness of a notion of diagnosis. The notions of diagnosis discussed in this paper are certainly not the only notions of diagnosis possible. In particular, investigating these notions of diagnosis for their suitability in certain problem domains would yield valuable insight.

In [TH96], a framework of 'approximate diagnosis' is proposed, taking logical abductive diagnosis as a point of departure. The ideas in that paper are related to those presented in this paper, but in contrast to our approach, approximate diagnosis is studied in terms of an approximate logical deduction relation. A feature of using an approximate deduction relation is that the meaning of the domain knowledge, e.g. causal knowledge, used in diagnostic problem solving is affected by changes to the deduction relation. In our set-theoretical framework, however, domain knowledge and notions of diagnoses are kept separate, which eases design. Further research is required to elucidate the relative merits of the two approaches.

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