Consistency-based diagnosis (cont.)

Difference between predicted behaviour and observed behaviour $\Rightarrow$ defect!

Originators:

Plan for today

- Quick revision of basic concepts
- There are some optimisations possible for the hitting-set algorithm
- Consistency-based diagnosis is an example of non-monotonic reasoning (quite common in AI). We show why this is the case
- Finally, diagnoses can be seen as hypotheses that are revised when new observations are made. We extend the theory in this way

→ First, revision!
System specification

**SYStem specification** $SYS = (SD, COMPS)$:

- **SD (System Description):**
  
  $\forall x ((\text{MUL}(x) \land \neg \text{Ab}(x)) \rightarrow \text{in}_1(x) \times \text{in}_2(x) = \text{out}(x))$

  $\forall x ((\text{ADD}(x) \land \neg \text{Ab}(x)) \rightarrow \text{in}_1(x) + \text{in}_2(x) = \text{out}(x))$

  $\text{MUL}(M_1), \text{MUL}(M_2), \text{MUL}(M_3), \text{ADD}(A_1), \text{ADD}(A_2)$

  $\text{in}_1(A_1) = \text{out}(M_1), \text{in}_2(A_1) = \text{out}(M_2)$

  $\text{in}_1(A_2) = \text{out}(M_2), \text{in}_2(A_2) = \text{out}(M_3)$

- **COMPS** $= \{M_1, M_2, M_3, A_1, A_2\}$
Diagnostic problem

- System specification $SYS = (SD, COMPS)$
- Diagnostic problem $DP = (SYS, OBS)$, with $OBS$ a set of observations

Example:

$OBS = \{ \text{in}_1(M_1) = 3, \text{in}_2(M_1) = 2, \text{in}_1(M_2) = 3, \text{in}_2(M_2) = 2, \\
\text{in}_1(M_3) = 2, \text{in}_2(M_3) = 3, \text{out}(A_1) = 10, \text{out}(A_2) = 12 \}$
Diagnosis

- Diagnostic problem $DP = (SYS, OBS)$, with $OBS$ a set of observations

- A diagnosis $D$: smallest (subset minimal) set of components, such that

$$SD \cup OBS \cup \{Ab(c) \mid c \in D\} \cup \{\neg Ab(c) \mid c \in COMPS - D\}$$

- Faulty components
- Nonfaulty components

is consistent

- Remark: $\{Ab(c) \mid c \in D\}$ can be omitted (why?)

For the multiplier-adder:

$D = \{A_1\}, \{M_1\}, \{M_2, M_3\}, \{A_2, M_2\}$
Let \( CS \subseteq COMPS \) be a set of components, then \( CS \) is called a **conflict set** iff

\[
SD \cup OBS \cup \{\neg Ab(c) \mid c \in CS\}
\]

is inconsistent

**Proposition:** For each \( D \subseteq COMPS \) that is a **diagnosis** and each conflict set \( CS \) it holds that: \( D \cap CS \neq \emptyset \)

**Theorem:** \( D \) is a **diagnosis** for diagnostic problem \( DP = (SYS, OBS) \) iff \( D \) is a **minimal hitting set** for all conflict sets of \( DP \)
Hitting-set tree

Let $F$ be a set of sets
- Let $T = (V, E, l_V, l_E)$ be a labelled tree, with
- node labels

\[
l_V(v) = \begin{cases} 
S & \text{if } S \in F, S \neq \emptyset \\
\checkmark & \text{otherwise}
\end{cases}
\]

and

- edge labels if $l_V(v) = S$ then $\forall s \in S: l_E(v, v_s) = s$
Hitting sets

The hitting set $H(v)$ for node $v$ is defined as:

$$H(v) = \{l_E(u, w) \mid (u, w) \text{ is on the path from the root to } v\}$$

Example (incorrect why?):

- $l_V(a) = \{1, 3, 5\}$,
- $l_V(b) = \{2\}$, $l_V(e) = \checkmark$, etc.
- $l_E(a, b) = 1$, $l_E(a, c) = 3$,
- $l_E(b, e) = 2$, etc.
- $H(a) = \emptyset$
- $H(b) = \{1\}$
- $H(e) = \{1, 2\}$
- $H(f) = \{3, 4\}$
The hitting set $H(v)$ for node $v$ is defined as:

$$H(v) = \{ l_E(u, w) \mid (u, w) \text{ is on the path from the root to } v \}$$

Example (correct):

- $l_V(a) = \{1, 3, 5\}$
- $l_V(b) = \{2\}$, $l_V(e) = \checkmark$, etc.
- $l_E(a, b) = 1$, $l_E(a, c) = 3$
- $l_E(b, e) = 2$, etc.
- $H(a) = \emptyset$
- $H(b) = \{1\}$
- $H(e) = \{1, 2\}$
- $H(f) = \{3, 4\}$
Example: full-adder

\[ \forall x ((\text{ANDG}(x) \land \neg \text{Ab}(x)) \rightarrow \text{out}(x) = \text{in}_1(x) \land \text{in}_2(x)) \]
\[ \forall x ((\text{ORG}(x) \land \neg \text{Ab}(x)) \rightarrow \text{out}(x) = \text{in}_1(x) \lor \text{in}_2(x)) \]
\[ \vdots \]
\[ \text{ORG}(O_1), \text{ANDG}(A_1), \text{XORG}(X_1), \ldots \]

\[ \text{COMPS} = \{ A_1, A_2, X_1, X_2, O_1 \} \]
Example HS tree

1. \( CS_1 \leftarrow \text{TP}(SD, \text{COMPS}, OBS); \)
   \( CS_1 \leftarrow \{X_1, X_2\} \)

2. \( CS_2 \leftarrow \text{TP}(SD, \text{COMPS} - \{X_1\}, OBS); \)
   \( CS_2 \leftarrow \checkmark \ (\text{diagnosis found}) \)

3. \( CS_3 \leftarrow \text{TP}(SD, \text{COMPS} - \{X_2\}, OBS); \)
   \( CS_3 \leftarrow \{X_1, A_2, O_1\} \)

Diagnoses \( D: \{X_1\}, \{X_2, A_2\}, \{X_2, O_1\} \)
(note that \( \{X_2, X_1\} \) not subset minimal)
Pruning of the hitting-set tree

Note that there is no need to extend the hitting set $H(v_4)$ (as $\{X_2, X_1\}$ is not subset minimal)

$\Rightarrow$ pruning of the hitting-set tree
Hitting-set tree

\[ F = \{\{1, 2\}, \{4, 5\}, \{1, 2, 3\}, \{3, 4\}\} \]

\[ v_1 \{1, 2, 3\} \]

\[ v_2 \{3, 4\} \]

\[ v_3 \{4, 5\} \]

\[ v_4 \{1, 2\} \]

\[ v_5 \{4, 5\} \]

\[ v_6 \]

\[ v_7 \]

\[ v_8 \{3, 4\} \]

\[ v_9 \{4, 5\} \]

\[ v_{10} \{4, 5\} \]

\[ v_{11} \]

\[ v_{12} \]

\[ v_{13} \]

\[ v_{14} \]

\[ v_{15} \]

\[ v_{16} \]

\[ v_{17} \]

\[ v_{18} \]

Minimal hitting set: \( H(v) \) is subset-minimal and \( l_V(v) = \checkmark \)

Examples: \( H(v_6) = \{1, 4\} \subset H(v_{11}) \); \( H(v_7) = \{2, 4\} \subset H(v_{14}) \)
Optimisation

- **Pruning** the HS tree:
  1. $H(v) = H(v')$: prune the subtree with root $v'$ and $l_V(v') = \times$
  2. $H(v) \subset H(v')$: ignore $v'$, $l_V(v') = \times$
  3. for $l_V(v) = S, l_V(v') = S' \in F$ with $S' \subset S$:
     $l_V(v) \leftarrow l_V(v')$ ($= S'$) and prune

![Diagram](image)

- **Reuse of labels** when $F$ is dynamic (as in diagnosis): if $S' \in F$ and $H(v) \cap S' = \emptyset$, then $l_V(v) = S'$
Example

\[ F = \{\{1, 2\}, \{4, 5\}, \{1, 2, 3\}, \{3, 4\}\} \]
Example

\[ F = \{\{1, 2\}, \{4, 5\}, \{1, 2, 3\}, \{3, 4\}\} \]

Rule (1) and (2)
Example

\[ F = \{\{1, 2\}, \{4, 5\}, \{1, 2, 3\}, \{3, 4\}\} \]
Example

\[ F = \{ \{1, 2\}, \{4, 5\}, \{1, 2, 3\}, \{3, 4\}\} \]

\[ v_1 \{1, 2, 3\} \rightarrow \{1, 2\} \]

\[ v_2 \{3, 4\} \]

\[ v_3 \{4, 5\} \]

\[ v_4 \{3, 4\} \]

\[ v_5 \{4, 5\} \]

\[ v_6 \]

\[ v_7 \]

\[ v_8 \{3, 4\} \]

\[ v_9 \]

\[ v_{10} \]

Rule (3)
Non-monotonic reasoning

- Knowledge base $\text{KB}$
- **Add** knowledge to $\text{KB}$ and obtain new knowledge base $\text{KB'}$
- If $\text{KB} \vdash \text{Results} \text{ and } \text{KB'} \vdash \text{Results'}$ then $\text{Results} \subseteq \text{Results'}$ does not hold in general
  $\Rightarrow$ more knowledge does not always yield more results
- Consistency-based reasoning is an example of non-monotonic reasoning. Why?

$$\text{SD} \cup \text{OBS} \cup \{\neg \text{Ab}(c) \mid c \in \text{COMPS} - D\} \not\equiv \bot$$

e.g., larger $\text{OBS}$ or $\text{SD}$ may make $D$ smaller or different
Default logic

\[ DT = (W, R) \] is a default theory, where
\[ W = \{ \text{Elephant(john)} \} \], i.e., John is an elephant, and the following default \( R \):

\[
\begin{align*}
\text{Elephant}(x) : \text{Grey}(x) \\
\text{Grey}(x)
\end{align*}
\]

If being grey is consistent with our knowledge, conclude ‘grey’, so conclude \( \text{Grey}(\text{john}) \)

- general form default

\[
\text{prerequisite} : \text{justifications} \\
\text{consequent}
\]
Reasoning in default logic

Let $DT = (W, R)$ be a default theory ($W$ a set of logical formulas and a set of defaults $R$):

- $E = \text{Th}(E)$ (so-called fixed point)
- $W \subseteq E$
- $E$ includes the maximal set of conclusions obtained by applying defaults in $R$
- If $\frac{A; B_1, \ldots, B_n}{C} \in R$, $A \in E$ and $\neg B_1, \ldots, \neg B_n \notin W$, then $C \in E$

$E$ is called an extension and $\text{Th}$ is the derivation operator (deduction + default rule application)
Example

$DT = (W, R)$, where

$$W = \{ \text{Elephant(clyde)}, \neg\text{Grey(john)} \}$$

i.e., Clyde is an elephant and John is not grey, and the following default $R$:

\[
\begin{align*}
\text{Elephant}(x) : \text{Grey}(x) \\
\text{Grey}(x)
\end{align*}
\]

‘elephants are normally grey’

Extension: $E = \{ \text{Elephant(clyde), Grey(clyde), } \neg\text{Grey(john)} \}$
Diagnosis as non-monotonic reasoning

- Map a diagnostic problem to a default theory DT
- A diagnosis $D$ predicts a formula $\varphi$ iff

$$SD \cup OBS \cup \{\neg Ab(c) \mid c \in \text{COMPS} - D\} \cup \{Ab(c) \mid c \in D\} \models \varphi$$

- **Lemma:** $DT = (W, R)$ is a default theory with extension $E$ iff

$$E = \text{Th} (W \cup \{L \mid :L/L \in \Delta\})$$

with subset-maximal set of defaults $\Delta \subseteq R$ such that

$$W \cup \{L \mid :L/L \in \Delta\} \not\models \bot$$

- **Remark:** $:\psi/\psi$ is a so-called normal default (default without prerequisite and justification that is the same as the conclusion)
Theorem. Let $DP = (SYS, OBS)$ be a diagnostic problem. Let

$$DT = \left( SD \cup OBS, \left\{ \frac{\neg Ab(c)}{\neg Ab(c)} \mid c \in COMPS \right\} \right)$$

be a default theory with extension $E$, then $D$ is a diagnosis for $DP$ iff $E = \{ \varphi \mid D \text{ predicts } \varphi \}$

Proof: $E$ is an extension of $DT$, thus (Lemma):

$$SD \cup OBS \cup \left\{ \neg Ab(c) \mid \frac{\neg Ab(c)}{\neg Ab(c)} \in \Delta \right\} \not\models \bot$$

with $\Delta \subseteq R$ such that $\Delta$ subset-maximal. Suppose that $D = \{ c \mid c \in COMPS, \frac{\neg Ab(c)}{\neg Ab(c)} \not\in \Delta \}$, then ...
Theorem. Let $DP = (SYS, OBS)$ be a diagnostic problem. Let

$$DT = \left( SD \cup OBS, \left\{ \frac{\neg Ab(c)}{\neg Ab(c)} \mid c \in COMPS \right\} \right)$$

be a default theory with extension $E$, then $D$ is a diagnosis for $DP$ iff $E = \{ \varphi \mid D \text{ predicts } \varphi \}$

Proof (continued):

$$\left\{ \neg Ab(c) \mid \frac{\neg Ab(c)}{\neg Ab(c)} \in \Delta \right\} = \left\{ \neg Ab(c) \mid c \in COMPS - D \right\}$$

Thus, $E = Th(SD \cup OBS \cup \left\{ \neg Ab(c) \mid c \in COMPS - D \right\})$, and $E = \{ \varphi \mid D \text{ predicts } \varphi \}$
Example

\[ \text{DP} = (\text{SYS}, \text{OBS}), \text{ with} \]
\[ \text{SD} = \{ \forall x((\text{ANDG}(x) \land \neg \text{Ab}(x)) \rightarrow (\text{out}(x) = \text{and}(\text{in}_1(x), \text{in}_2(x))))), \]
\[ \forall x((\text{XORG}(x) \land \neg \text{Ab}(x)) \rightarrow (\text{out}(x) = \text{xorg}(\text{in}_1(x), \text{in}_2(x))))), \]
\[ \text{ANDG}(A), \text{XORG}(X), \]
\[ \text{out}(A) = \text{in}_1(X) \} \]

\[ \text{COMPS} = \{ A, X \} \text{ and } \text{OBS} = \{ \text{in}_1(A) = 1, \text{in}_2(A) = 1, \text{in}_2(X) = 0, \text{out}(A) = 0, \text{out}(X) = 1 \} \]

Default theory \[ \text{DT} = (\text{SD} \cup \text{OBS}, R), \text{ with} \]
\[ R = \left\{ \frac{\neg \text{Ab}(A)}{\neg \text{Ab}(A)}, \frac{\neg \text{Ab}(X)}{\neg \text{Ab}(X)} \right\} \]

\[ E = \text{Th}(\text{SD} \cup \text{OBS} \cup \{ \neg \text{Ab}(X) \}), \text{ e.g., } \text{Ab}(A) \in E \text{ (we can predict that } A \text{ is abnormal)} \]
Extra measurements

Recall: a diagnosis $D$ predicts a formula $\varphi$ iff

$$SD \cup OBS \cup \{\neg Ab(c) \mid c \in COMPS - D\} \models \varphi$$

if $\varphi$ is a set of extra observations (measurements), then:

1. Every diagnosis $D$ for $DP = (SYS, OBS)$ that predicts $\varphi$ is also a diagnosis for $DP = (SYS, OBS \cup \{\varphi\})$, i.e., the measurement $\varphi$ confirms $D$

2. No diagnosis for $DP = (SYS, OBS)$ that predicts $\neg \varphi$ is also a diagnosis for $DP = (SYS, OBS \cup \{\varphi\})$, i.e., the measurement $\varphi$ disconfirms $D$

3. Any diagnosis $D$ for $DP = (SYS, OBS \cup \{\varphi\})$ which is not a diagnosis for $DP' = (SYS, OBS)$ is a strict superset of a diagnosis of $DP'$ which predicts $\neg \varphi$
Example

- Diagnostic problem $\text{DP} = (\text{SYS}, \text{OBS})$, with
- Set of observations
  \[
  \text{OBS} = \{ \text{in}_1(M_1) = 3, \text{in}_2(M_1) = 2, \text{in}_1(M_2) = 3, \text{in}_2(M_2) = 2, \\
  \text{in}_1(M_3) = 2, \text{in}_2(M_3) = 3, \text{out}(A_1) = 10, \text{out}(A_2) = 12 \}
  \]
- Predictions w.r.t. $\text{out}(M_2)$: diagnosis $D_1 = \{ M_1 \}$ predicts $\text{out}(M_2) = 6$, $D_2 = \{ A_1 \}$ predicts $\text{out}(M_2) = 6$, $D_3 = \{ M_2, M_3 \}$ predicts $\text{out}(M_2) = 4$, $D_4 = \{ M_2, A_2 \}$ predicts $\text{out}(M_2) = 4$
• Diagnostic problem $\text{DP} = (\text{SYS}, \text{OBS})$, with

• Set of observations (with new one on $M_2$):

$$\text{OBS} = \{in_1(M_1) = 3, in_2(M_1) = 2, in_1(M_2) = 3, in_2(M_2) = 2, in_1(M_3) = 2, in_2(M_3) = 3, out(A_1) = 10, out(A_2) = 12, out(M_2) = 5\}$$

• The new observation $out(M_2) = 5$ disconfirms all previous diagnoses
Example (continued)

- Diagnostic problem \( \text{DP} = (\text{SYS}, \text{OBS}) \), with

- Set of observations (with new one on \( M_2 \)):
  \[
  \text{OBS} = \{ \text{in}_1(M_1) = 3, \text{in}_2(M_1) = 2, \text{in}_1(M_2) = 3, \text{in}_2(M_2) = 2, \\
  \text{in}_1(M_3) = 2, \text{in}_2(M_3) = 3, \text{out}(A_1) = 10, \text{out}(A_2) = 12, \\
  \text{out}(M_2) = 5 \}
  \]

- New diagnoses: \( D'_1 = \{M_1, M_2, M_3\} \), \( D'_2 = \{M_1, M_2, A_2\} \), 
  \( D'_3 = \{M_2, M_3, A_1\} \), \( D'_4 = \{M_2, A_1, A_2\} \); note supertsets 
  of the old diagnoses, e.g., \( D_1, D_3 \subseteq D'_1 \) (case 3)
Definition of diagnosis revisited

Example two inverters $I_k, k = 1, 2$:

\[
\begin{array}{c}
0 \\
\downarrow \quad I_1 \quad \downarrow I_2 \\
\downarrow \quad 0 \\
\end{array}
\]

\[
\begin{align*}
SD &= \{ \forall x ((\text{INV}(x) \land \neg \text{Ab}(x)) \rightarrow \neg (\text{out}(x) = \text{in}(x))) , \\
&\quad \text{INV}(I_1) , \quad \text{INV}(I_2) \} \\
OBS &= \{ \text{in}(I_1) = 0 , \text{out}(I) = 1 \} \\
\text{Diagnoses:} &\quad \{I_1\} \quad \text{and} \quad \{I_2\} \\
\text{However,} &\quad \{I_1, I_2\} \quad \text{might also be a diagnosis (only excluded because of definition using subset-minimality condition)}
\end{align*}
\]
Faul models

Knowledge about behaviour for $\text{Ab}(c)$ is called fault model

- $\{I_1\}$ and $\{I_2\}$ still diagnoses,
- $\{I_1, I_2\}$ no longer a diagnosis, because

$$SD \cup \text{OBS} \cup \{\text{Ab}(I_1), \text{Ab}(I_2)\} \models \bot$$
Conclusions

- Consistency-based diagnosis popular for troubleshooting of equipment and devices
- Extensions: temporal behaviour and continuous behaviour
- Diagnoses may be ranked using probability theory and entropy (General Diagnostic Engine, GDE)

Software:
- GDE/ATMS (Palo Alto Research Center): http://www2.parc.com/spl/members/dekleer/
- Leancop (University Darmstad/Potsdam): http://www.leancop.de/
- AILog!