QPEL — Quantum Program and Effect Language

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Introduction

QPEL (Quantum Program and Effect language) is a formal system for denoting:

- quantum programs
- effects ('fuzzy' quantum predicates) of quantum systems

It is intended for *reasoning* about quantum programs, and proving properties such as correctness.

State-and-Effect Triangles

These structures are used to give semantics to quantum programs:

- Hilbert spaces
- C^*-algebras
- W^*-algebras

State-and-Effect Triangles

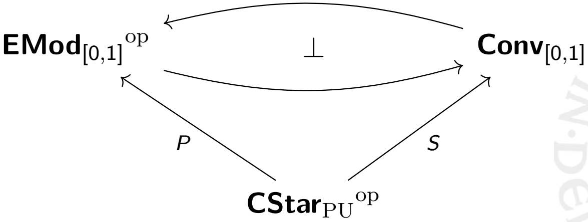
Many of these settings fit this pattern:

- A category whose objects represent *quantum systems*, and whose arrows represent *quantum programs*;
- An *effect algebra* E of probabilities (typically $[0, 1]$)
- A collection of *effects* (predicates) over each object, which form an *effect module* over E
- A collection of *states* for each object, which form a *convex set* over E

Let us call this a state-and-effect triangle. [Jac14]

State-and-Effect Triangles

Many of these settings fit this pattern:



Let us call this a state-and-effect triangle. [Jac14]

State-and-Effect Triangles

Many of these settings fit this pattern:

QPEL is the logic of state-and-effect triangles.

Effect Algebras

Definition

An effect algebra is a structure $(E, \ovee, (-)^\perp)$ where

- $\ovee : E^2 \rightharpoonup E$ (partial)
- $(-)^\perp : E \to E$

such that

- $x \ovee y \simeq y \ovee x$
- $x \ovee (y \ovee z) \simeq (x \ovee y) \ovee z$
- $x \ovee 0 = x$
- $x \ovee y = 0^\perp$ iff $y = x^\perp$
- If $x \perp 0^\perp$ then $x = 0$.

Let $1 = 0^\perp$

Examples:

- The set $\{0, 1\}$ under $x \ovee y = x + y$ if $x + y \leq 1$, $x^\perp = 1 - x$
- The set $[0, 1]$ under $x \ovee y = x + y$ if $x + y \leq 1$, $x^\perp = 1 - x$
- Any Boolean algebra with $x \ovee y = x \vee y$ if $x \wedge y = 0$, $x^\perp = \neg x$

Definition (Effect Monoid)

An effect monoid is an effect algebra E with a (total) operation $\cdot : E^2 \to E$ such that:

- $(x \ovee y) \cdot z \stackrel{\sim}{\to} (x \cdot z) \ovee (y \cdot z)$
- $x \cdot (y \ovee z) \stackrel{\sim}{\to} (x \cdot y) \ovee (x \cdot z)$
- $1 \cdot x = x \cdot 1 = x$
- $x \cdot (y \cdot z) = (x \cdot y) \cdot z$

Examples:

- $\{0, 1\}$ and $[0, 1]$ under multiplication.
- Any Boolean algebra under $\wedge$.

Definition (Effect Module)

An *effect module* over the effect monoid E is an effect algebra A with a (total) operation $\cdot : E \times A \to A$ such that:

- $r \cdot (x \varovee y) \stackrel{\sim}{\to} (r \cdot x) \varovee (r \cdot y)$
- $(r \varovee s) \cdot x \stackrel{\sim}{\to} (r \cdot x) \varovee (s \cdot x)$
- $(r \cdot s) \cdot x = r \cdot (s \cdot x)$
- $1 \cdot x = x$

Examples:

- The effects over a Hilbert space (positive operators less than I) form an effect module over $[0, 1]$.
- The effects in a C*-algebra (positive elements below 1) form an effect module over $[0, 1]$.

Convex Sets

Definition (Convex Set)

A convex set consists of a set X and an operation: given $r_1, \ldots, r_n \in E$ with

$$r_1 \oslash \cdots \oslash r_n = 1$$

and $x_1, \ldots, x_n \in X$, returns an element

$$r_1 x_1 + \cdots + r_n x_n \in X$$

such that certain equations hold.

Examples

- The density matrices over a Hilbert space form a convex set over $[0, 1]$.
- For A a C*-algebra, the positive unital maps $A \to \mathbb{C}$ form a convex set over $[0, 1]$.

State-and-Effect Triangles

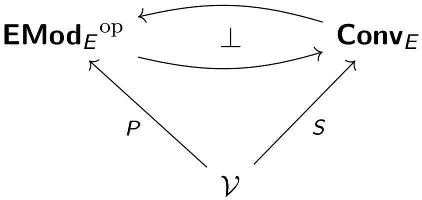
The functors

$$\mathbf{Conv}_E[-,E] : {\mathbf{Conv}_E}^{\mathrm{op}} \to \mathbf{EMod}_E$$
$$\mathbf{EMod}_E[-,E] : {\mathbf{EMod}_E}^{\mathrm{op}} \to \mathbf{Conv}_E$$

form an adjunction.

State-and-Effect Triangles

A *state-and-effect triangle* is a structure



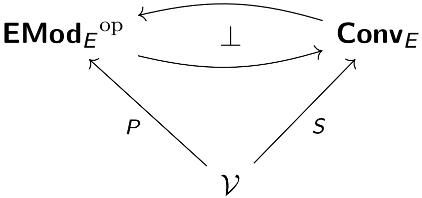
where:

- E is an effect monoid
- $\mathcal{V}$ is a symmetric monoidal category with binary coproducts that distribute over $\otimes$ such that the tensor unit I is terminal
- P preserves finite coproducts and the terminal object
- S is a symmetric monoidal functor

such that certain coherence conditions hold.

State-and-Effect Triangles

A *state-and-effect triangle* is a structure



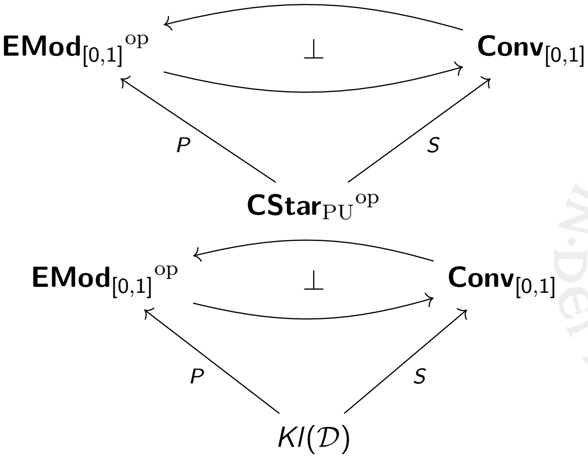
where:

- given $r_1 \ovee \cdots \ovee r_n = 1$ in PA, an arrow $\mathrm{meas}_A(r_1, \ldots, r_n) : A \to n \cdot I$ in $\mathcal{V}$
- natural transformations

$$\alpha : P \to \mathbf{Conv}_E[S-, E], \qquad \beta : S \to \mathbf{EMod}_E[P-, E]$$

such that certain coherence conditions hold.

Examples


$\mathbf{EMod}_{[0,1]}{}^{\mathrm{op}}$
$\perp$
$\mathbf{Conv}_{[0,1]}$
P
S
$\mathbf{CStar}_{\mathrm{PU}}{}^{\mathrm{op}}$
$\mathbf{EMod}_{[0,1]}{}^{\mathrm{op}}$
$\perp$
$\mathbf{Conv}_{[0,1]}$
P
S
$\mathcal{K}\ell(\mathcal{D})$

Syntax of QPEL

$$\text{Type } A ::= A \otimes A \mid I \mid A + B$$

- Terms $s, t, \ldots$ intended to represent quantum programs.
- Effects $\phi, \psi, \ldots$ intended to represent predicates on quantum states.

Judgement forms:

- $\Gamma \vdash t : A$
- $\Gamma \vdash s = t : A$
- $\Gamma \vdash \phi \text{ eff}$
- $\Gamma \vdash \phi \leq \psi$

This is a *linear* type system – no Contraction:

$$\frac{\Gamma, x : A, y : A \vdash t[x, y] : B}{\Gamma, x : A \vdash t[x, x] : B}$$

Allowing Contraction would violate the no-cloning theorem

Effects

Effect Formation

$$\frac{}{\Gamma \vdash 0 \text{ eff}} \qquad \frac{\Gamma \vdash \phi \text{ eff}}{\Gamma \vdash \phi^\perp \text{ eff}} \qquad \frac{\Gamma \vdash \phi \leq \psi^\perp}{\Gamma \vdash \phi \ovee \psi \text{ eff}}$$

$$\frac{\vdash \phi \text{ eff} \qquad \Gamma \vdash \psi \text{ eff}}{\Gamma \vdash \phi \cdot \psi \text{ eff}}$$

Derivability

$$\frac{\Gamma \vdash \phi \text{ eff}}{\Gamma \vdash \phi \leq \phi} \qquad \frac{\Gamma \vdash \phi \leq \psi \qquad \Gamma \vdash \psi \leq \chi}{\Gamma \vdash \phi \leq \chi} \qquad \frac{\Gamma \vdash \phi \leq \psi}{\Gamma \vdash \psi^\perp \leq \phi^\perp}$$

$$\frac{\Gamma \vdash \phi \text{ eff}}{\Gamma \vdash \phi \leq \phi^{\perp\perp}} \qquad \frac{\Gamma \vdash \phi \text{ eff}}{\Gamma \vdash \phi^{\perp\perp} \leq \phi}$$

The *scalars* are the effects in the empty context

Typing System

Structural Rules

$$\frac{\Gamma, x : A, y : B, \Delta \vdash \mathcal{J}}{\Gamma, y : B, x : A, \Delta \vdash \mathcal{J}} \qquad \frac{}{x : A \vdash x : A}$$

Tensor Products

$$\frac{\Gamma \vdash M : A \qquad \Delta \vdash N : B}{\Gamma, \Delta \vdash \langle M, N \rangle : A \otimes B}$$

$$\frac{\Gamma \vdash M : A \otimes B \qquad \Delta, x : A, y : B \vdash N : C}{\Gamma, \Delta \vdash \text{let } \langle x, y \rangle = M \text{ in } N : C}$$

Measurement

$$\frac{\Gamma \vdash 1 \leq \phi_1 \oslash \cdots \oslash \phi_n \qquad \Delta \vdash M_1 : A \qquad \cdots \qquad \Delta \vdash M_n : A}{\Gamma, \Delta \vdash \text{measure } \phi_1 \mapsto M_1 \mid \cdots \mid \phi_n \mapsto M_n : A}$$

Measurement

$$\frac{\Gamma \vdash 1 \leq \phi_1 \oslash \cdots \oslash \phi_n \qquad \Delta \vdash M_1 : A \qquad \cdots \qquad \Delta \vdash M_n : A}{\Gamma, \Delta \vdash \text{measure } \phi_1 \mapsto M_1 \mid \cdots \mid \phi_n \mapsto M_n : A}$$

$$\frac{\begin{array}{c}\Gamma \vdash 1 \leq \phi_1 \oslash \cdots \oslash \phi_n \\ \Delta \vdash M_1 : A \ \cdots \ \Delta \vdash M_n : A\end{array}}{\begin{array}{c}\Gamma, \Delta \vdash (\text{measure } \phi_1 \mapsto M_1 \mid \cdots \mid \phi_n \mapsto M_n) = \\ (\text{measure } \phi_{p(1)} \mapsto M_{p(1)} \mid \cdots \mid \phi_{p(n)} \mapsto M_{p(n)})\end{array}}$$

Measurement

$$\frac{\Gamma \vdash 1 \leq \phi_1 \varovee \cdots \varovee \phi_n \qquad \Delta \vdash M_1 : A \qquad \cdots \qquad \Delta \vdash M_n : A}{\Gamma, \Delta \vdash \text{measure } \phi_1 \mapsto M_1 \mid \cdots \mid \phi_n \mapsto M_n : A}$$

$$\frac{\begin{array}{c}\Gamma \vdash 1 \leq \phi_1 \varovee \cdots \varovee \phi_n \\ \Delta \vdash M_1 : A \ \cdots \ \Delta \vdash M_n : A\end{array}}{\begin{array}{c}\Gamma, \Delta \vdash (\text{measure } \phi_1 \mapsto M_1 \mid \cdots \mid \phi_n \mapsto M_n) = \\ (\text{measure } \phi_{p(1)} \mapsto M_{p(1)} \mid \cdots \mid \phi_{p(n)} \mapsto M_{p(n)})\end{array}}$$

$$\frac{\begin{array}{c}\Gamma \vdash 1 \leq \phi_1 \varovee \cdots \varovee \phi_n \\ \Delta \vdash M_1 : A \ \cdots \ \Delta \vdash M_{n+1} : A\end{array}}{\begin{array}{c}\Gamma \vdash (\text{measure } \phi_1 \mapsto M_1 \mid \cdots \mid \phi_n \mapsto M_n \mid 0 \mapsto M_{n+1}) = \\ \text{measure } \phi_1 \mapsto M_1 \mid \cdots \mid \phi_n \mapsto M_n : A\end{array}}$$

Measurement

$$\frac{\Gamma \vdash M : A}{\Gamma \vdash (\text{measure } 1 \mapsto M) = M : A}$$

Measurement

$$\frac{\Gamma \vdash M : A}{\Gamma \vdash (\text{measure } 1 \mapsto M) = M : A}$$

$$\frac{\vdash 1 \leq \phi \varovee \psi \varovee \chi_1 \varovee \cdots \varovee \chi_n \qquad \Gamma \vdash M : A \qquad \Gamma \vdash P_1 : A \qquad \cdots}{\begin{array}{l}\Gamma \vdash (\text{measure } \phi \varovee \psi \mapsto M \mid \chi_1 \mapsto P_1 \mid \cdots \mid \chi_n \mapsto P_n) \\ = (\text{measure } \phi \mapsto M \mid \psi \mapsto M \mid \chi_1 \mapsto P_1 \mid \cdots \mid \chi_n \mapsto P_n)\end{array}}$$

Semantics

Define:

- an object $[\![A]\!] \in \mathcal{V}$ for each type A
- an object $[\![\Gamma]\!] \in \mathcal{V}$ for each context Γ
- an arrow $[\![M]\!] : [\![\Gamma]\!] \to [\![A]\!]$ for each term $\Gamma \vdash M : A$
- an element $[\![\phi]\!] \in P\,[\![\Gamma]\!]$ for each effect $\Gamma \vdash \phi\ \mathrm{eff}$
- an element $(\![\phi]\!) \in E$ for each effect $\vdash \phi\ \mathrm{eff}$.

Semantics

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- an arrow $[\![M]\!] : [\![\Gamma]\!] \to [\![A]\!]$ for each term $\Gamma \vdash M : A$
- an element $[\![\phi]\!] \in P\,[\![\Gamma]\!]$ for each effect $\Gamma \vdash \phi \text{ eff}$
- an element $(\![\phi]\!) \in E$ for each effect $\vdash \phi \text{ eff}$.

Example: If $\Gamma \vdash \phi_i \text{ eff}$ and $\Delta \vdash M_i : A$, then
$[\![\Gamma, \Delta \vdash \text{measure } \phi_1 \mapsto M_1 \mid \cdots \mid \phi_n \mapsto M_n : A]\!]$ is

$$[\![\Gamma]\!] \otimes [\![\Delta]\!] \xrightarrow{\text{meas}_A([\![\phi_1]\!], \ldots, [\![\phi_n]\!]) \otimes 1} n \cdot [\![\Delta]\!] \xrightarrow{[[\![M_1]\!], \ldots, [\![M_n]\!]]} [\![A]\!]$$

Completeness Theorem

Theorem (Soundness)

Any derivable judgement is true in any state-and-effect triangle.

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Any judgement that is true in every state-and-effect triangle is derivable.

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Theorem (Completeness)

Any judgement that is true in every state-and-effect triangle is derivable.

Proof.

Define a state-and-effect triangle as follows.
The category $\mathcal{V}$ is the category with objects the types, and arrows $A \to B$ the terms M such that $x : A \vdash M : B$, quotiented by: $M = N$ iff $x : A \vdash M = N : B$. □

Qubits

Based on the rules of the Measurement Calculus [DKPP09].
These can be interpreted in $\mathbf{FdHilb}_{\mathrm{Un}}$, $\mathbf{CStar}^{\mathrm{op}}$ and $\mathbf{WStar}^{\mathrm{op}}$, but not in an arbitrary state-and-effect triangle.
Extend the system with:

$$\text{Type } A ::= \cdots \mid \mathbf{qbit}$$

$$\frac{}{\vdash |0\rangle : \mathbf{qbit}}$$

$$\frac{\Gamma \vdash t : \mathbf{qbit}}{\Gamma \vdash Xt : \mathbf{qbit}} \qquad \frac{\Gamma \vdash t : \mathbf{qbit}}{\Gamma \vdash Zt : \mathbf{qbit}} \qquad \frac{\Gamma \vdash s : \mathbf{qbit} \qquad \Delta \vdash t : \mathbf{qbit}}{\Gamma, \Delta \vdash Est : \mathbf{qbit} \otimes \mathbf{qbit}}$$

$$\frac{\Gamma \vdash t : \mathbf{qbit}}{\Gamma \vdash (t = |+_\alpha\rangle) \ \mathrm{eff}} \ (0 \leq \alpha < 2\pi)$$

Define:

$$|1\rangle = Z|0\rangle \qquad (x = |1\rangle) = (x = |+_0\rangle)^\perp$$

Equations for Qubits

$$E(Xs)t = \text{let } \langle x, y \rangle = Est \text{ in } \langle Xx, Zy \rangle$$
$$E(Zs)t = \text{let } \langle x, y \rangle = Est \text{ in } \langle Zx, y \rangle$$
$$(Xt = |+_\alpha\rangle) = (t = |+_{-\alpha}\rangle)$$
$$(Zt = |+_\alpha\rangle) = (t = |+_{\alpha-\pi}\rangle)$$
$$X(Xt) = t$$
$$Z(Zt) = t$$
$$(t = |+_\alpha\rangle)^\perp = (t = |+_{-\alpha}\rangle)$$
$$(X(Zt) = |+_\alpha\rangle) = (Z(Xt) = |+_\alpha\rangle)$$

Superdense Coding

Alice prepares two entangled qubits in the state $|b_1\rangle = 1/\sqrt{2}(|00\rangle + |11\rangle)$ and sends one to Bob.
She wishes to send an integer $1 \leq i \leq 4$ to Bob.
She performs an operation on her qubit:

i	1	2	3	4
Operation	I	X	Z	XZ

She then sends this qubit to Bob.
Bob measures the pair of qubits in the basis $\{|b_1\rangle, |b_2\rangle, |b_3\rangle, |b_4\rangle\}$ and learns the value of i.

Let

$$\begin{aligned} Ht = &\text{let } \langle x, y \rangle = Et|1\rangle \text{ in} \\ &\text{measure} \\ &x = |0\rangle \mapsto Xy \mid \\ &x = |1\rangle \mapsto y \end{aligned}$$

$$\begin{aligned} CNOT\ s\ t = &\text{ let } \langle x, y \rangle = Es(Ht) \text{ in} \\ &\langle x, Hy \rangle \end{aligned}$$

Add the axioms:

$$\begin{aligned} H(Ht) &= t \\ \text{let} \langle x, y \rangle = \text{CNOT}\ \ s\ t & \\ \text{in}\ \ \text{CNOT}\ \ x\ y &= \langle s, t \rangle \end{aligned}$$

Axioms for **qbit** $\otimes$ **qbit**

Let

$$|e_1\rangle = |11\rangle, |e_2\rangle = |10\rangle, |e_3\rangle = |01\rangle, |e_4\rangle = |00\rangle \quad .$$

Add the axioms:

$$q : \mathbf{qbit} \otimes \mathbf{qbit} \vdash 1 \leq q = |e_1\rangle \varovee q = |e_2\rangle \varovee q = |e_3\rangle \varovee q = |e_4\rangle$$

$$\vdash 1 \leq |e_i\rangle = |e_i\rangle$$

$$\vdash (|e_i\rangle = |e_j\rangle) \leq 0 \qquad (i \neq j)$$

$$(\langle X(Zs), t\rangle = |e_i\rangle) = (\langle Z(Xs), t\rangle = |e_i\rangle)$$

$$z : I + I + I + I \vdash sdc(z) : I + I + I + I$$

$$\begin{aligned}
sdc(z) \equiv & \text{let } \langle x, y \rangle = \text{CNOT}(H\,|1\rangle)(|1\rangle) \text{ in} \\
& \text{let } t_A \; = \text{case } z \text{ of} \\
& 1 \mapsto \langle x, y \rangle \\
& 2 \mapsto \langle Xx, y \rangle \\
& 3 \mapsto \langle Zx, y \rangle \\
& 4 \mapsto \langle XZx, y \rangle \text{ in} \\
& \text{measure} \\
& t_A = |b_1\rangle \mapsto 1 \mid \\
& t_A = |b_2\rangle \mapsto 2 \mid \\
& t_A = |b_3\rangle \mapsto 3 \mid \\
& t_A = |b_4\rangle \mapsto 4
\end{aligned}$$

Here

$$\begin{aligned}
|b_1\rangle &= \mathrm{CNOT}(H|1\rangle)|1\rangle \\
|b_2\rangle &= \mathrm{CNOT}(H|1\rangle)|0\rangle \\
|b_3\rangle &= \mathrm{CNOT}(H|0\rangle)|1\rangle \\
|b_4\rangle &= \mathrm{CNOT}(H|0\rangle)|0\rangle
\end{aligned}$$

Let

$$\begin{aligned}
(q = |b_i\rangle) \equiv &(\text{let } \langle x, y \rangle = q \text{ in} \\
&\text{let } \langle x, y \rangle = \mathrm{CNOT}\, x \ y \text{ in} \\
&\langle Hx, y \rangle) = |e_i\rangle
\end{aligned}$$

Then

$$\begin{aligned}
q : qbit \otimes qbit &\vdash 1 \leq q = |b_1\rangle \varovee \cdots \varovee q = |b_4\rangle \\
&\vdash 1 \leq (|b_i\rangle = |b_i\rangle) \\
&\vdash (|b_i\rangle = |b_j\rangle) \leq 0 \qquad (i \neq j)
\end{aligned}$$

$$
\begin{aligned}
sdc(3) = & \text{let } \langle x, y \rangle = \text{CNOT}(H\,|1\rangle)(|1\rangle) \text{ in} \\
& \text{let } t_A = \langle Zx, y \rangle \text{ in} \\
& \text{measure} \\
& t_A = |b_1\rangle \mapsto 1 \mid \\
& t_A = |b_2\rangle \mapsto 2 \mid \\
& t_A = |b_3\rangle \mapsto 3 \mid \\
& t_A = |b_4\rangle \mapsto 4
\end{aligned}
$$

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$$
\begin{aligned}
sdc(3) = &\text{let } \langle x, y \rangle = \mathrm{CNOT}(H\,|1\rangle)(|1\rangle) \text{ in} \\
&\text{measure} \\
&\langle Zx, y \rangle = |b_i\rangle \mapsto i
\end{aligned}
$$

$$
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\begin{aligned}
sdc(3) = & \text{let } \langle x, y \rangle = \mathrm{CNOT}(H|1\rangle)|1\rangle \text{ in} \\
& \text{measure} \\
& \text{let } \langle x, y \rangle = CNOTxy \text{ in} \\
& \text{measure} \\
& \langle H(Zx), y \rangle = |e_i\rangle \mapsto i
\end{aligned}
$$

$$sdc(3) = \text{measure } \langle H(Z(H|1\rangle)), |1\rangle\rangle = |e_i\rangle \mapsto i$$

$$sdc(3) = \text{measure}\ \langle H(H|0\rangle), |1\rangle\rangle = |e_i\rangle \mapsto i$$

$$sdc(3) = \text{measure}$$
$$\langle |0\rangle, |1\rangle\rangle = |e_i\rangle \mapsto i$$

$$
\begin{aligned}
sdc(3) = &\text{measure} \\
&\langle |0\rangle, |1\rangle\rangle = \langle |1\rangle, |1\rangle\rangle \mapsto 1 \\
&\langle |0\rangle, |1\rangle\rangle = \langle |1\rangle, |0\rangle\rangle \mapsto 2 \\
&\langle |0\rangle, |1\rangle\rangle = \langle |0\rangle, |1\rangle\rangle \mapsto 3 \\
&\langle |0\rangle, |1\rangle\rangle = \langle |0\rangle, |0\rangle\rangle \mapsto 4
\end{aligned}
$$

$$
\begin{aligned}
sdc(3) = &\text{measure} \\
&0 \mapsto 1 \\
&0 \mapsto 2 \\
&1 \mapsto 3 \\
&0 \mapsto 4
\end{aligned}
$$

$$sdc(3) = \text{measure}$$
$$1 \mapsto 3$$

$$sdc(3) = 3$$

$$z : I + I + I + I \vdash sdc(z) = z : I + I + I + I$$

Related Work

- Baltag and Smets [BS04, B14] work on LQP (Logic of Quantum Programs)
  - Based on propositional dynamic logic
  - Includes $[P]\phi$ — 'after P, ϕ is true'
  - Language for terms/states is an underspecification language.

Related Work

- Baltag and Smets [BS04, B14] work on LQP (Logic of Quantum Programs)
  - Based on propositional dynamic logic
  - Includes $[P]\phi$ — 'after P, ϕ is true'
  - Language for terms/states is an underspecification language.
- d'Hondt, Panangaden and Ying [dP06, Yin11] give a Floyd-Hoare logic for quantum programs.
  - includes $\{\phi\}P\{\psi\}$ — 'if ϕ is true before P is run, then ψ will be true after'
  - Syntax for quantum programs
  - No syntax for logic — predicates are operators on Hilbert spaces

Conclusion

QPEL is a sound and complete system for state-and-effect triangles.

Within its framework, we can give a theory of qubits and reason about quantum programs.

For the future:

- Complete axiomatization of qubits.

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- Incorporate $[\phi?]\psi$ and $\langle\phi?\rangle\psi$ into logic.

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- Incorporate $[\phi?]\psi$ and $\langle\phi?\rangle\psi$ into logic.
- Make more use of the state space — three judgement forms?

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QPEL is a sound and complete system for state-and-effect triangles.

Within its framework, we can give a theory of qubits and reason about quantum programs.

For the future:

- Complete axiomatization of qubits.
- Incorporate $[\phi?]\psi$ and $\langle\phi?\rangle\psi$ into logic.
- Make more use of the state space — three judgement forms?
- Other triangles — classical logic, probabilistic logic, ...

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