# A likelihood-ratio test for identifying probabilistic deterministic real-time automata from positive data

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ICGI 2010

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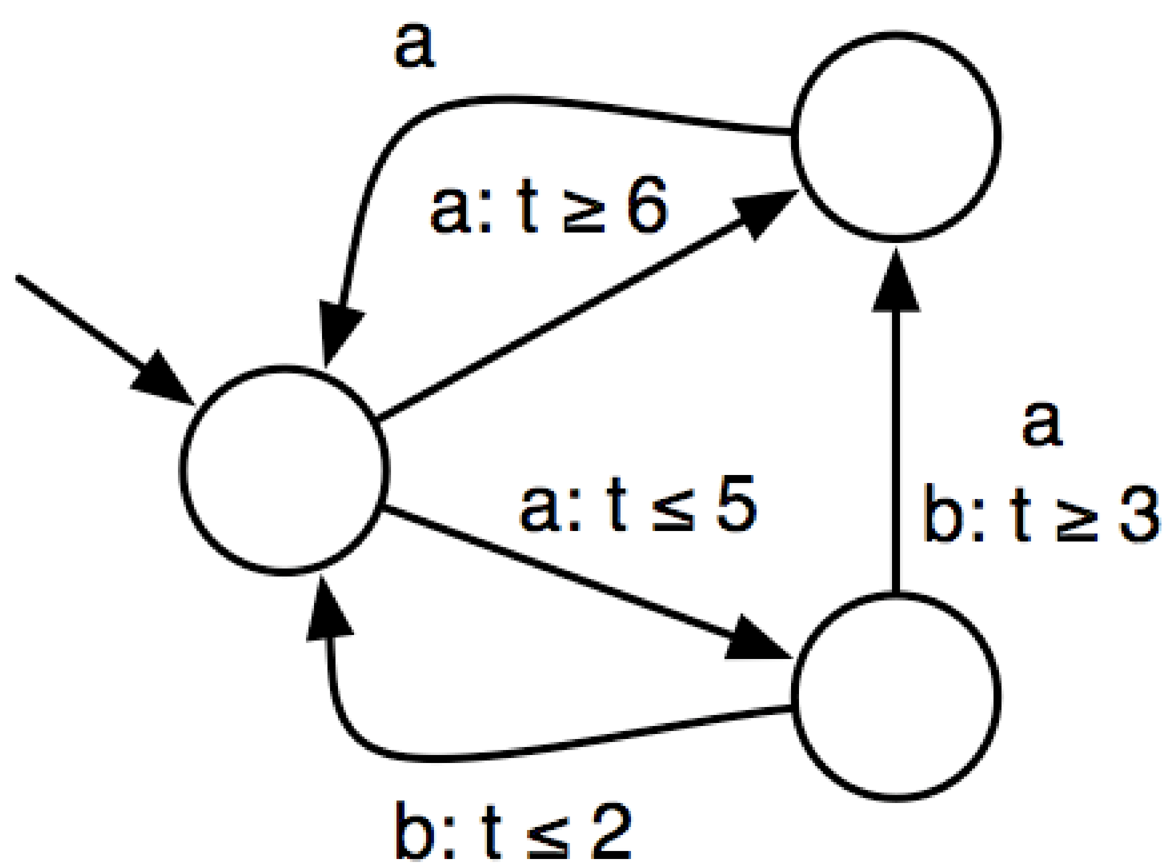
# Overview

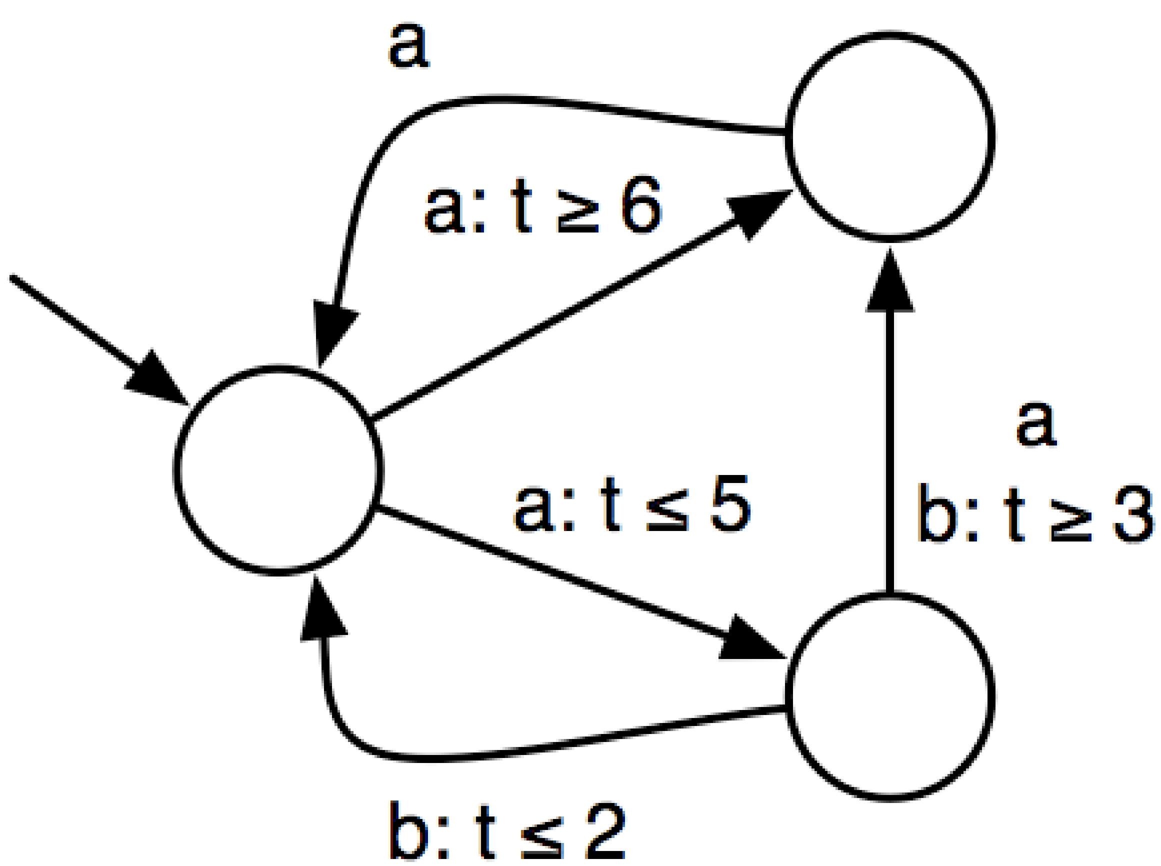
- Real-time automata
  - Why learn them?
- Learning real-time automata
  - Labeled
  - Unlabeled
- A likelihood-ratio test
- Results
- Conclusions

# Real-time automata (DRTAs)




Transitions contain guards on time values

# DRTAs and events




Produces timed strings:

(a,6)(a,2)(a,3); (a,2)(b,1)(a,1)(b,8)

# Why learn DRTAs?

- Suppose you observe:
  - (a,4)(b,7)(a,2)(a,10)…
- You can transform this into:

  - TTTTaTTTTTTTbTTaTTTTTTTTTTa..
  - where T represents a time tick

- Then apply a DFA learning algorithm
- The resulting DFA is language equivalent to the target DRTA

# Why learn DRTAs?

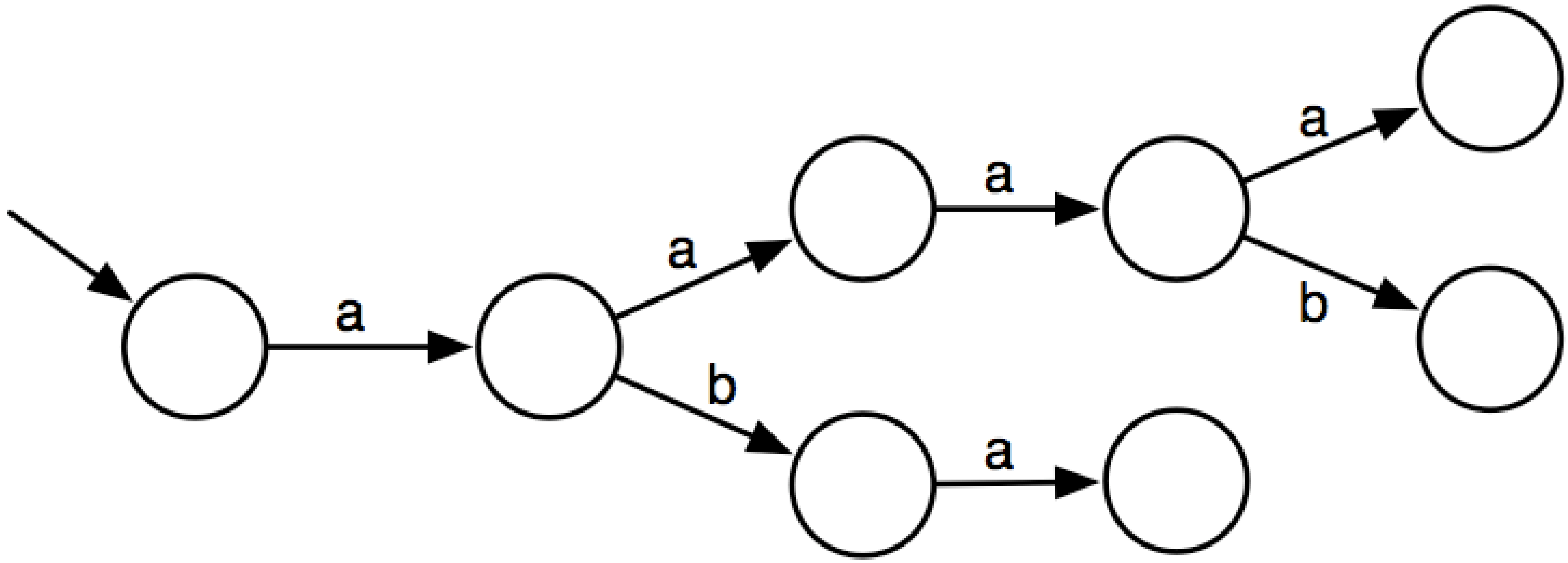
- The transformation goes from binary to unary
- Thus, the learned DFA is exponentially larger than the target DRTA
- DFAs are thus not efficiently learnable in this way from timed data

- DRTAs can be learned efficiently
- DRTAs result in compact representations

# Learning DRTAs

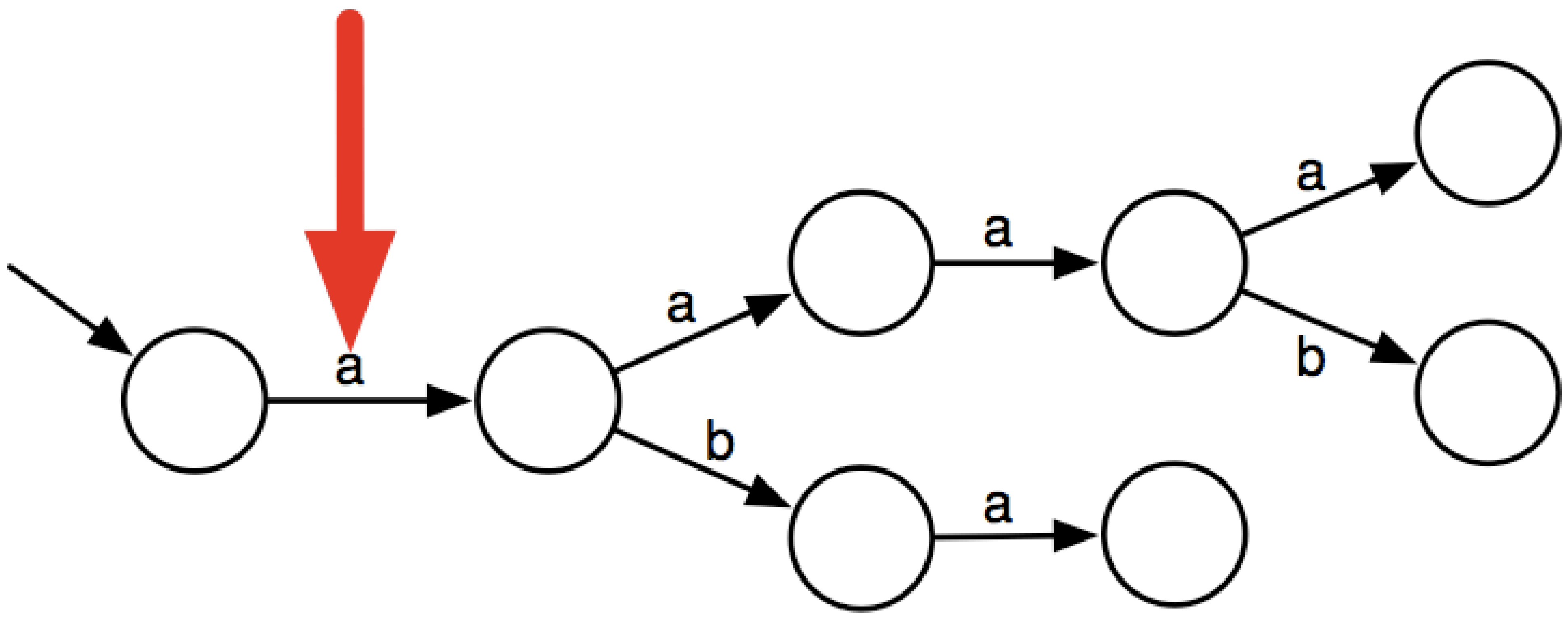
- State-merging and transition-splitting:
  - Start from a prefix tree
  - Try all possible merges and splits
  - If one scores good:
    - Perform the one that scores best
  - Else
    - break
  - Iterate

# Transition splitting



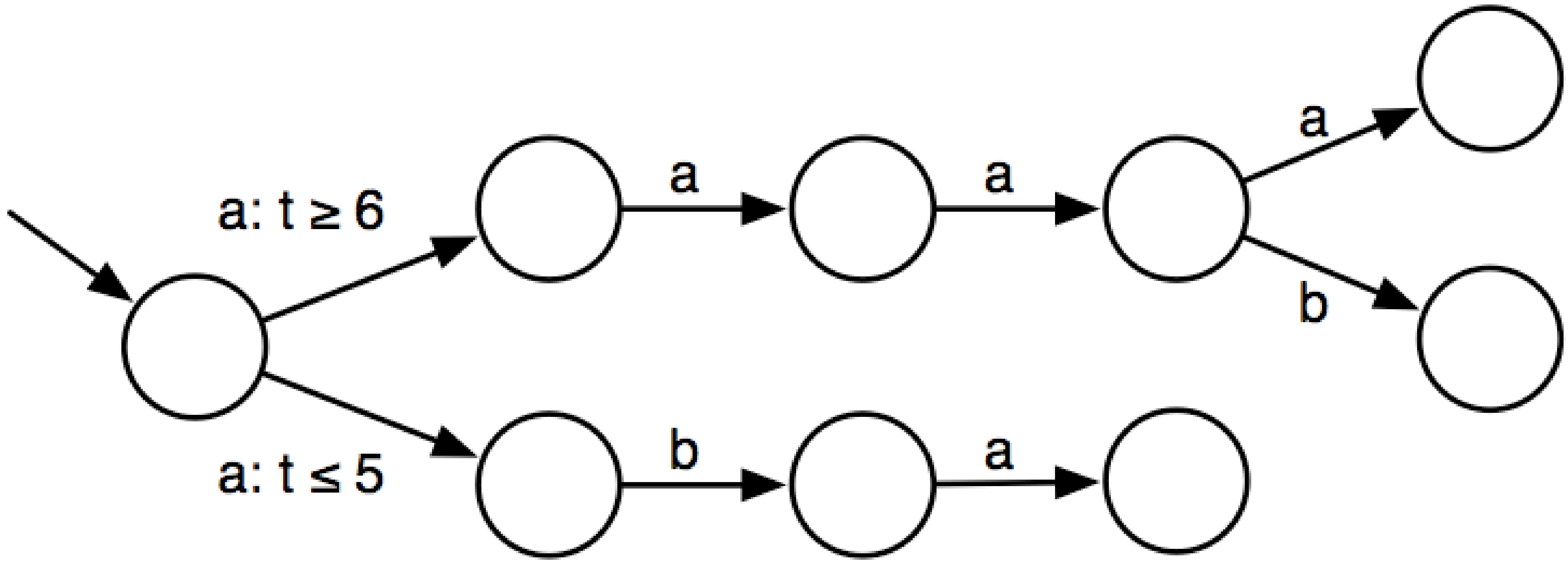

A prefix tree
all guards are set to true, [0, ∞)

# Transition splitting



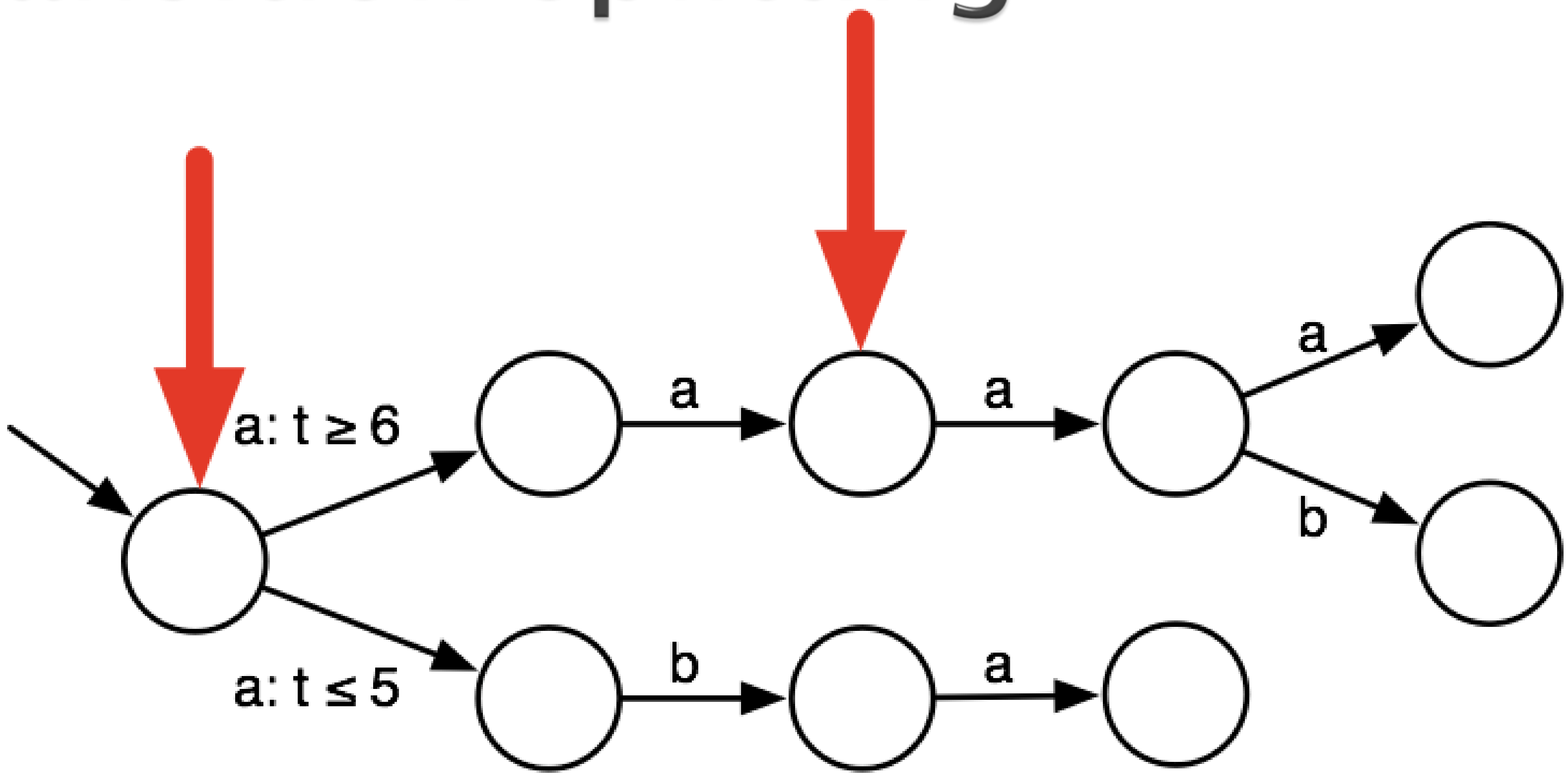

Transition splitting:
choose a transition

# Transition splitting



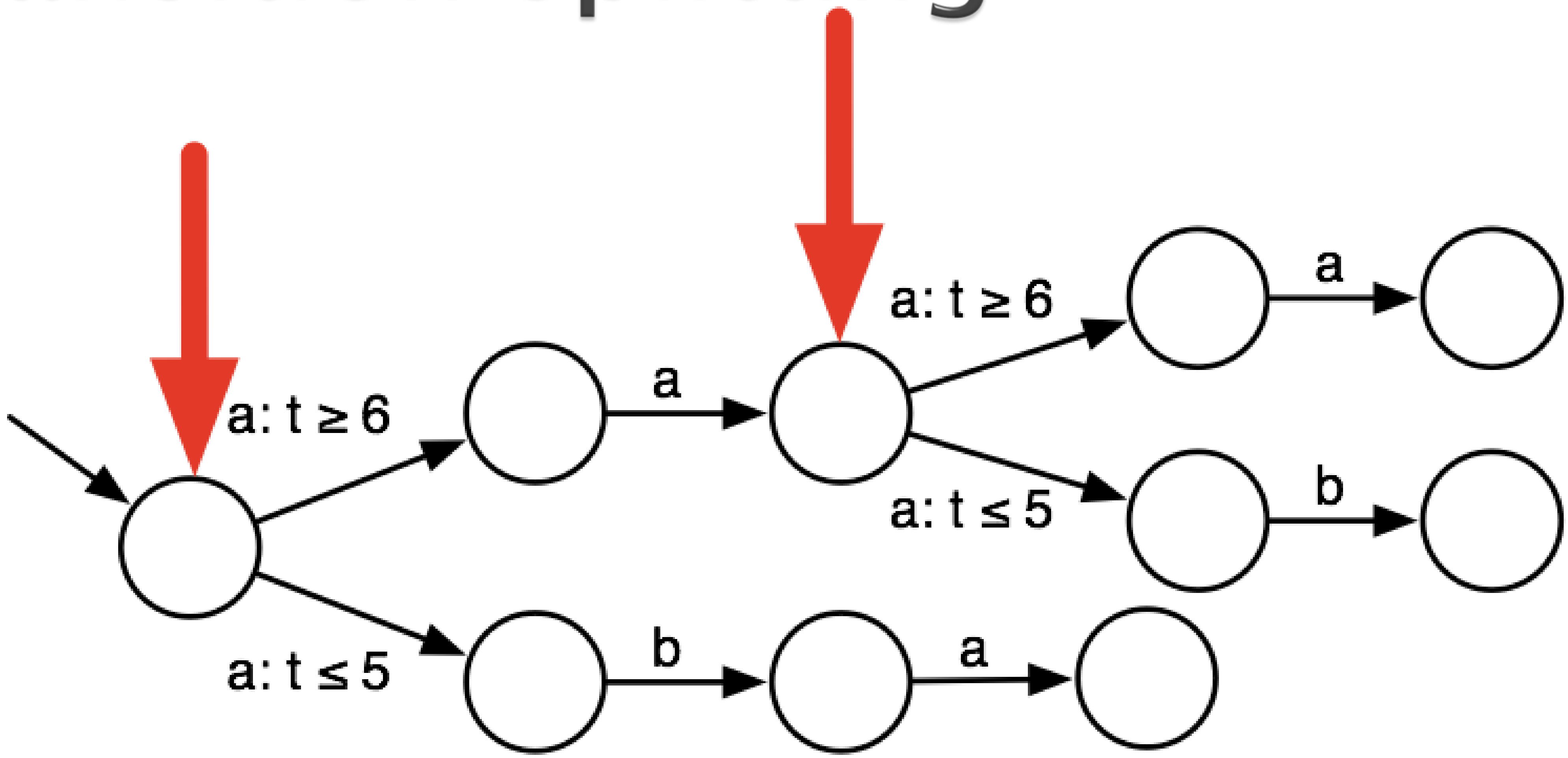

Split the transition and recalculate the subsequent part of the prefix tree

# Transition splitting




Later, when merging states

# Transition splitting

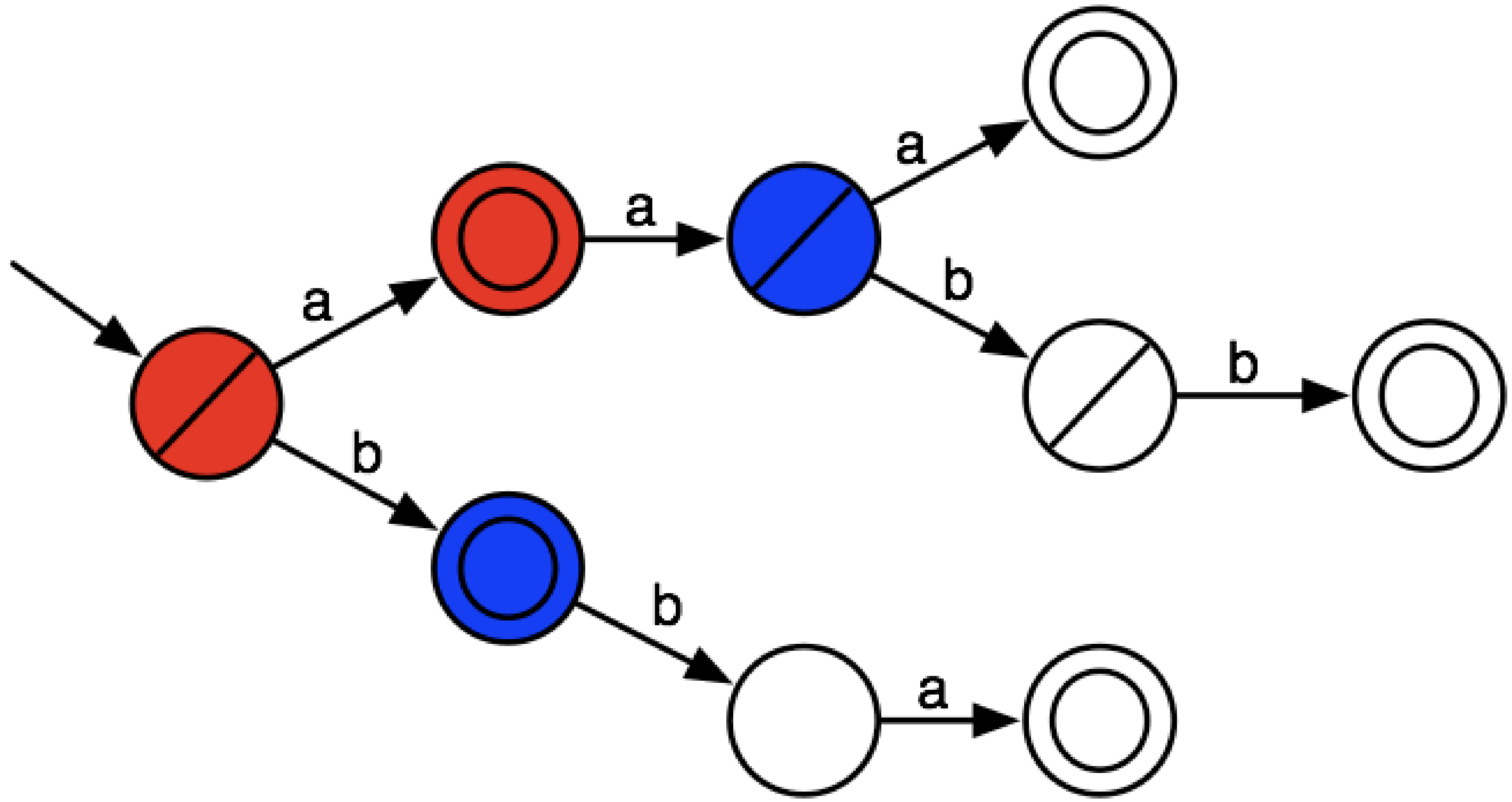



First split the transitions such that the guards match

# Labeled data

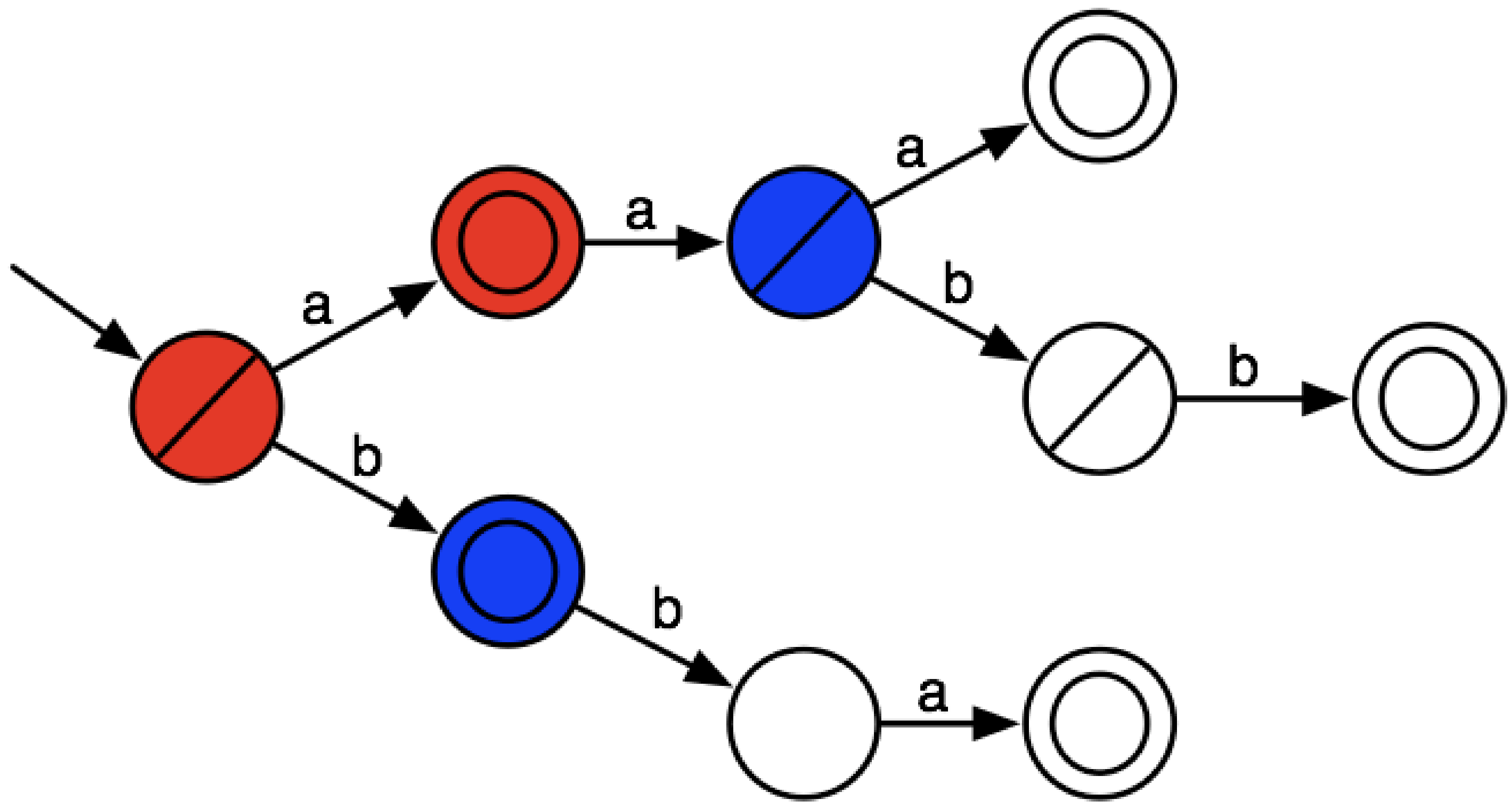
- Input: two sets S+ and S− of positive and negative timed strings
- Use the state merging and transition splitting algorithm, but
  - Allow merges between positive and negative states
  - Maintain a core of inferred states using the red-blue framework
  - Do not allow positive-negative merges in the core

# Labeled data



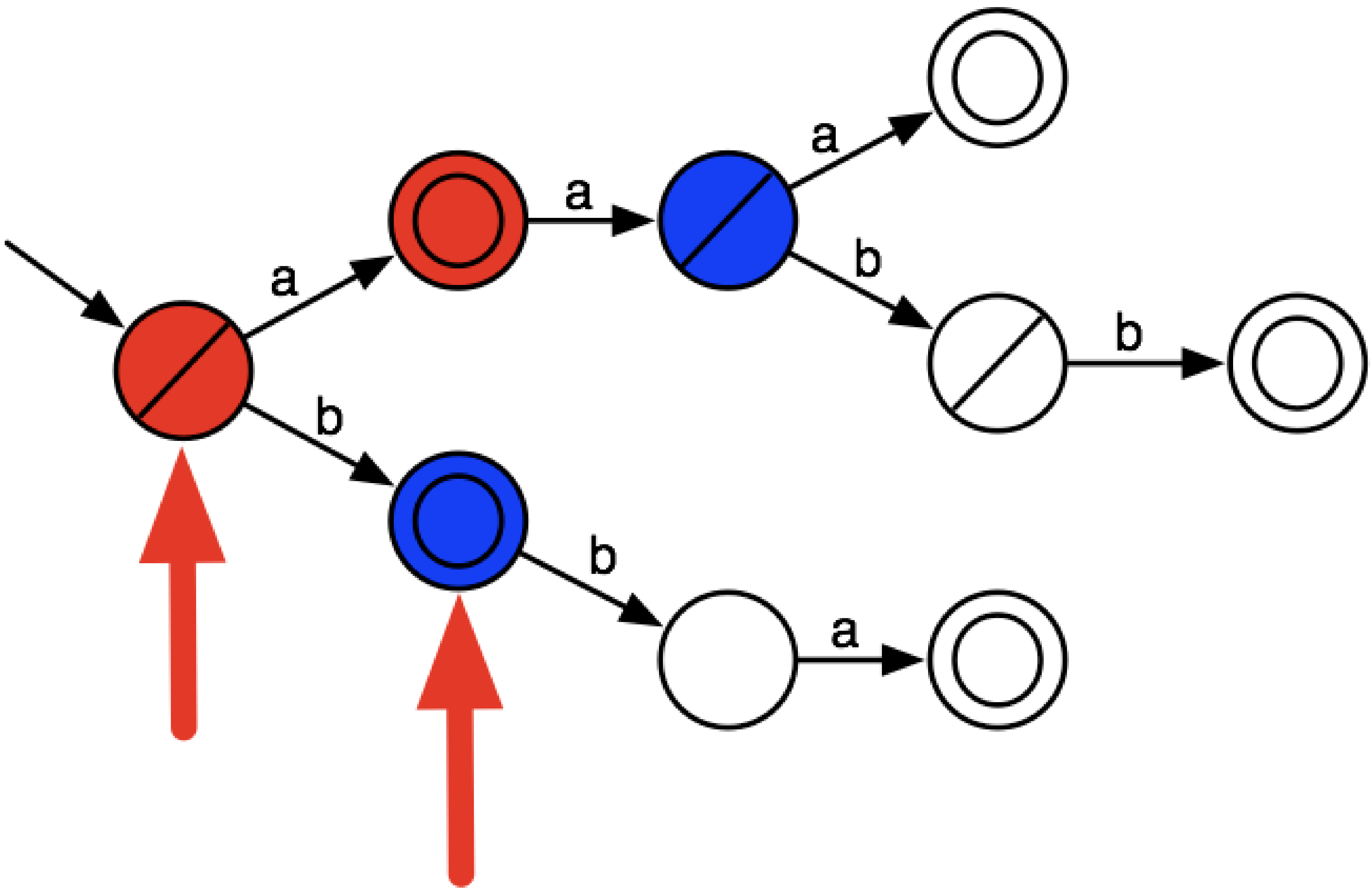

A core or red states, with a fringe of blue states
only allow merges or red and blue states
only split transitions to blue states

# Labeled data



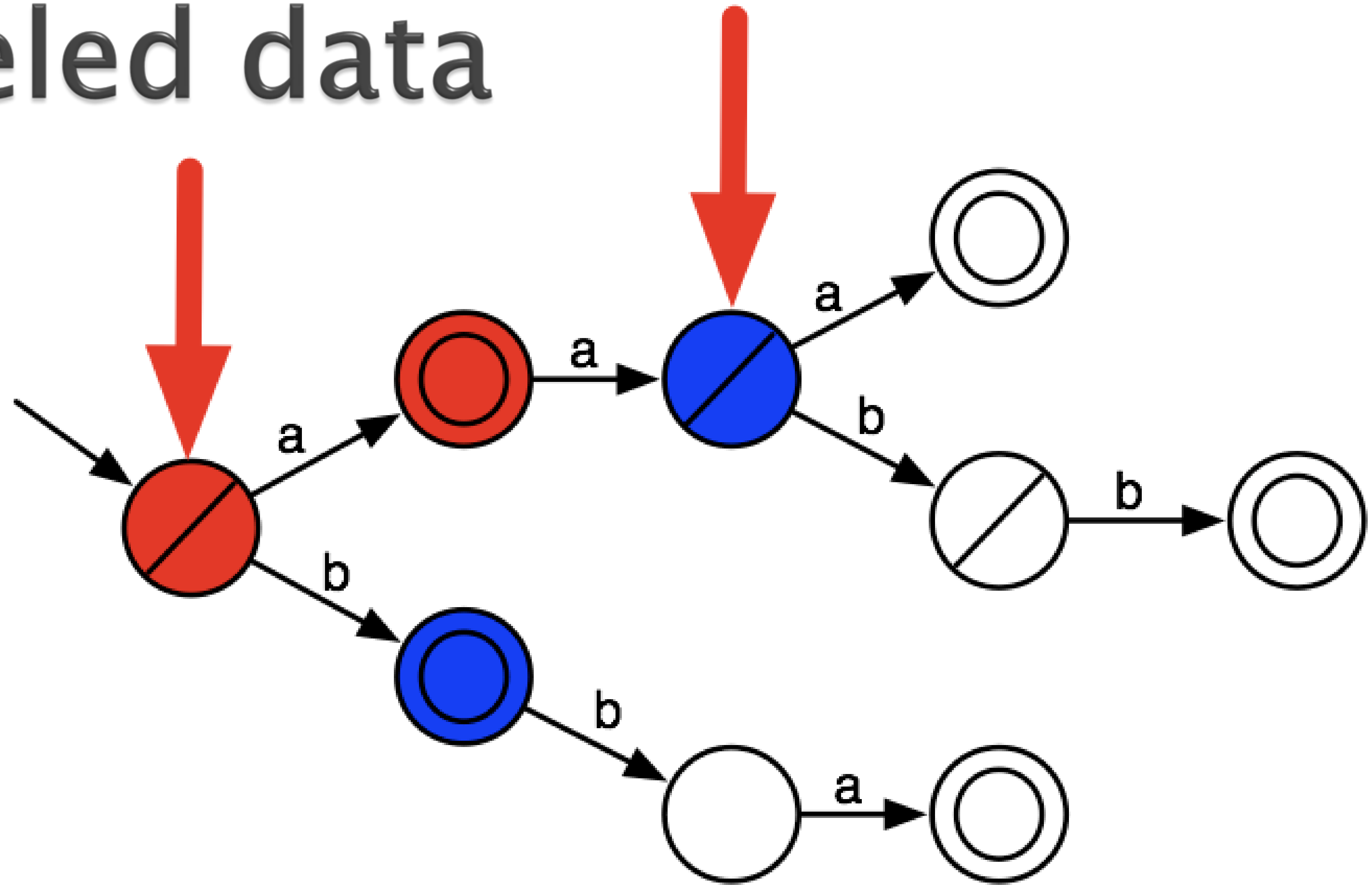

When a blue state cannot be merged or split, this blue state is colored red

# Labeled data



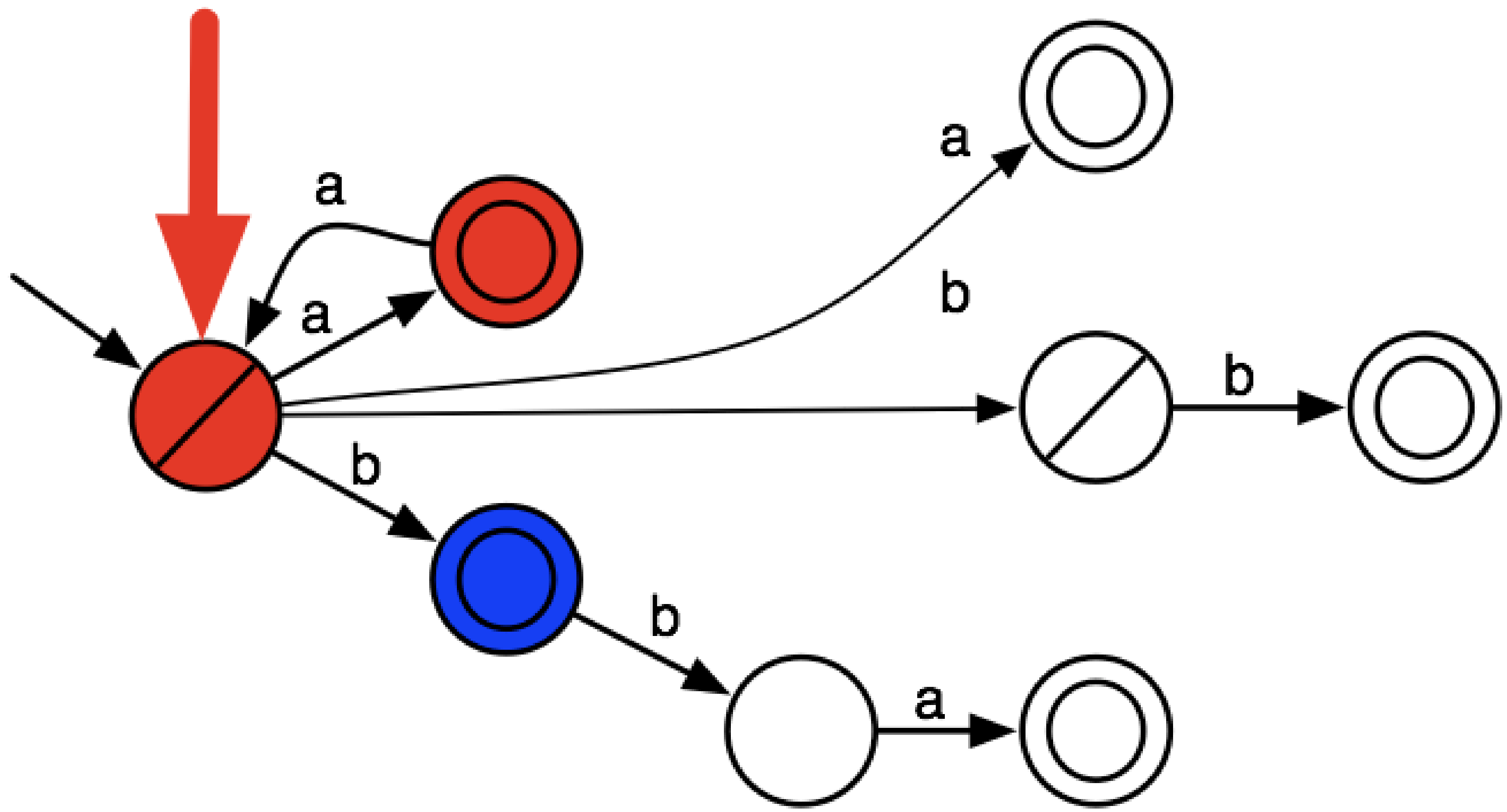

Results in a positive-negative merge in the core

# Labeled data



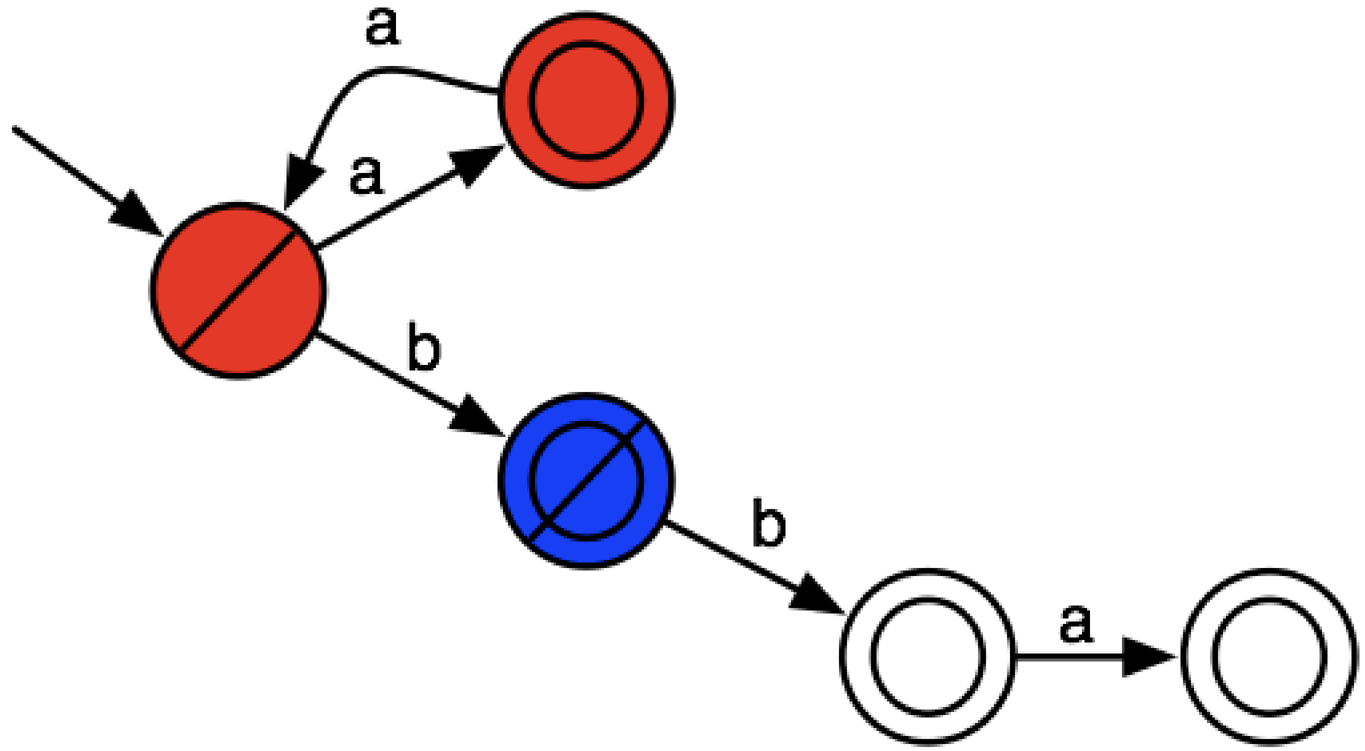

Does not results in a positive-negative merge in the core

# Labeled data



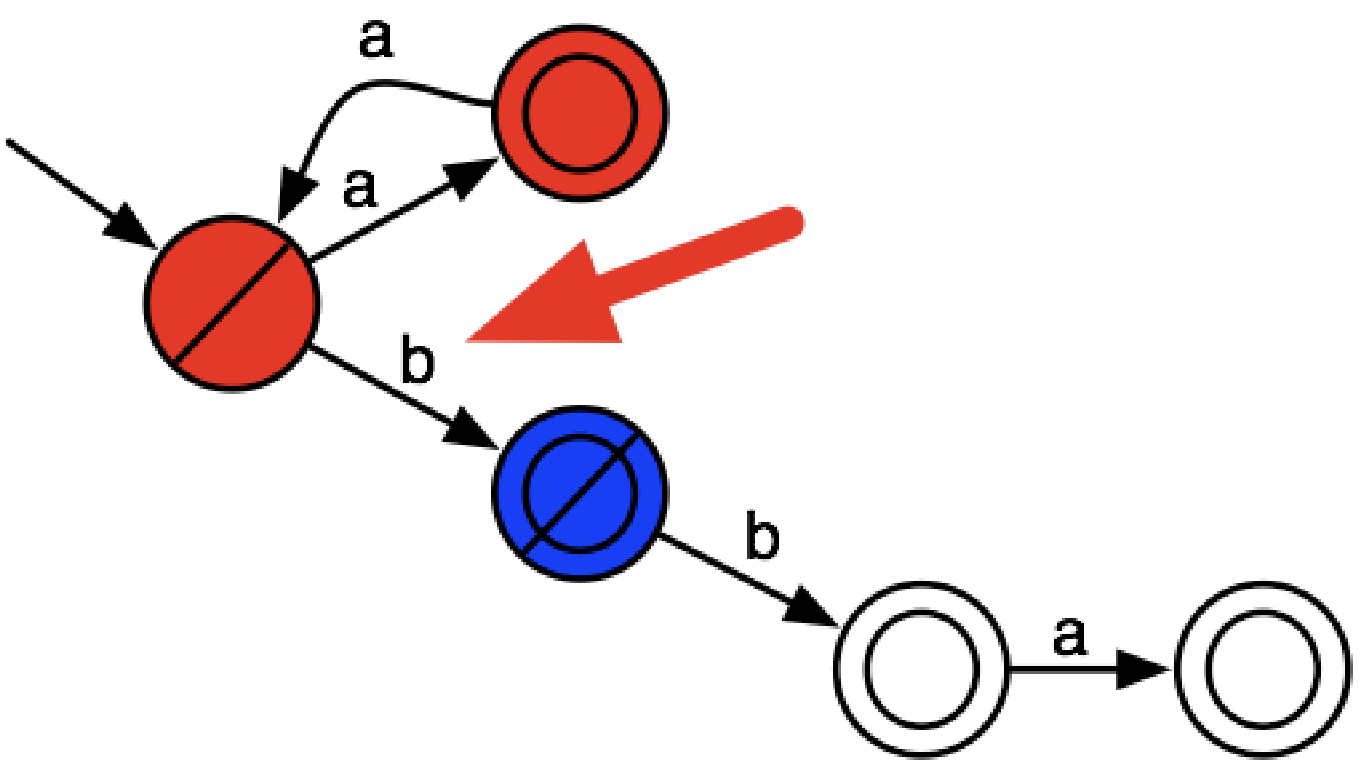

Does not results in a positive-negative merge in the core

# Labeled data



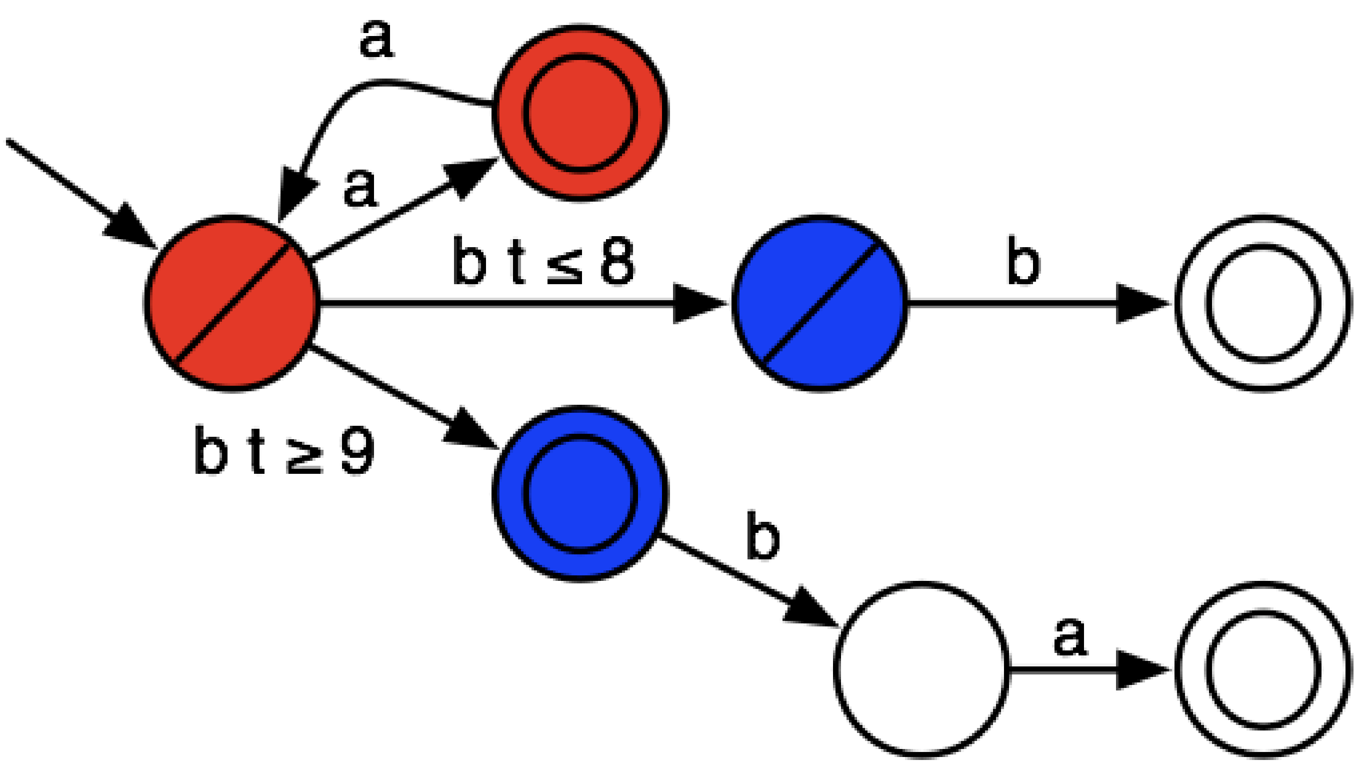

Does not results in a positive-negative merge in the core

# Labeled data




The positive-negative merge can be resolved using a split

# Labeled data

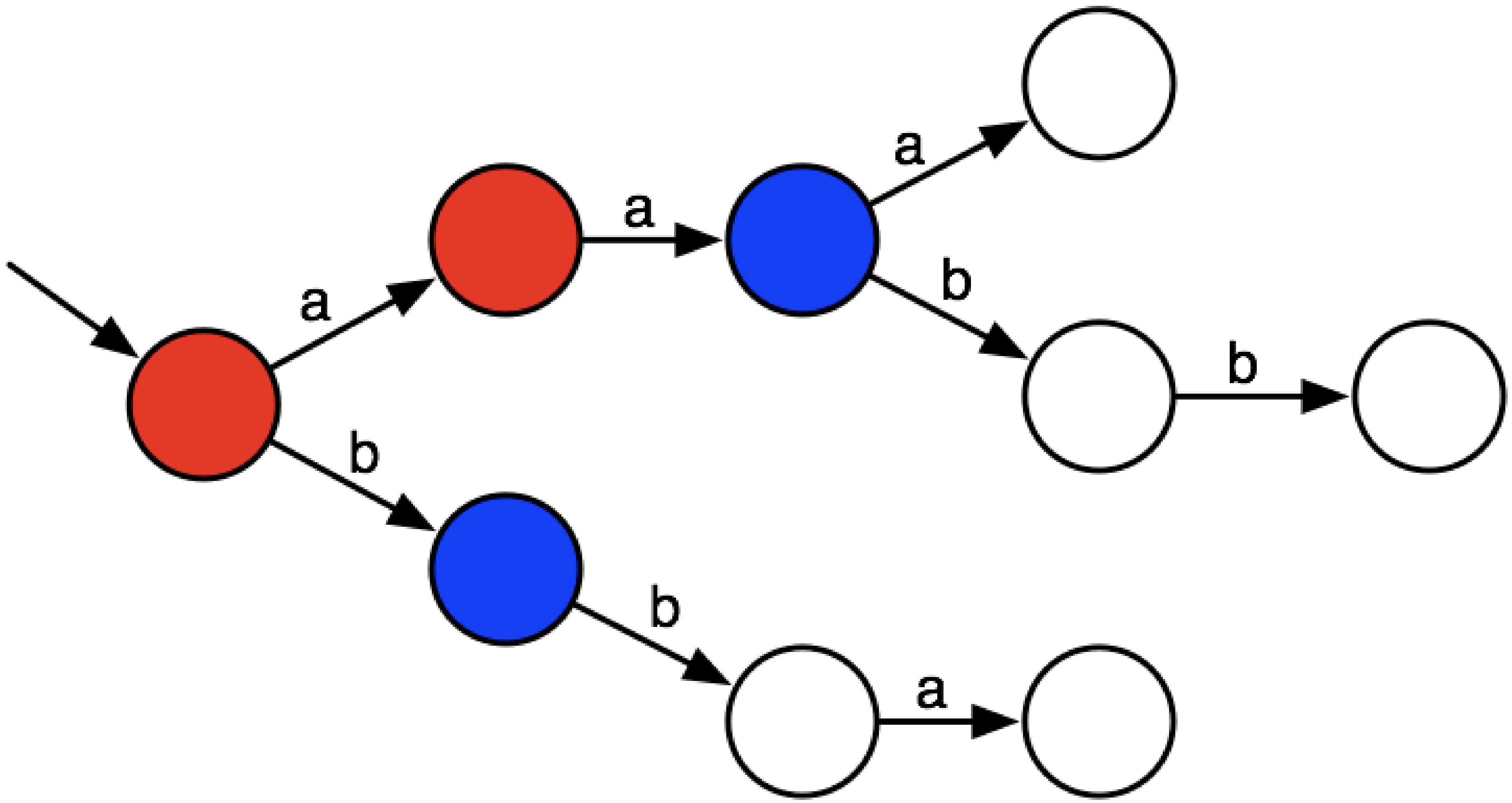



The positive-negative merge can be resolved using a split

# Unlabeled data

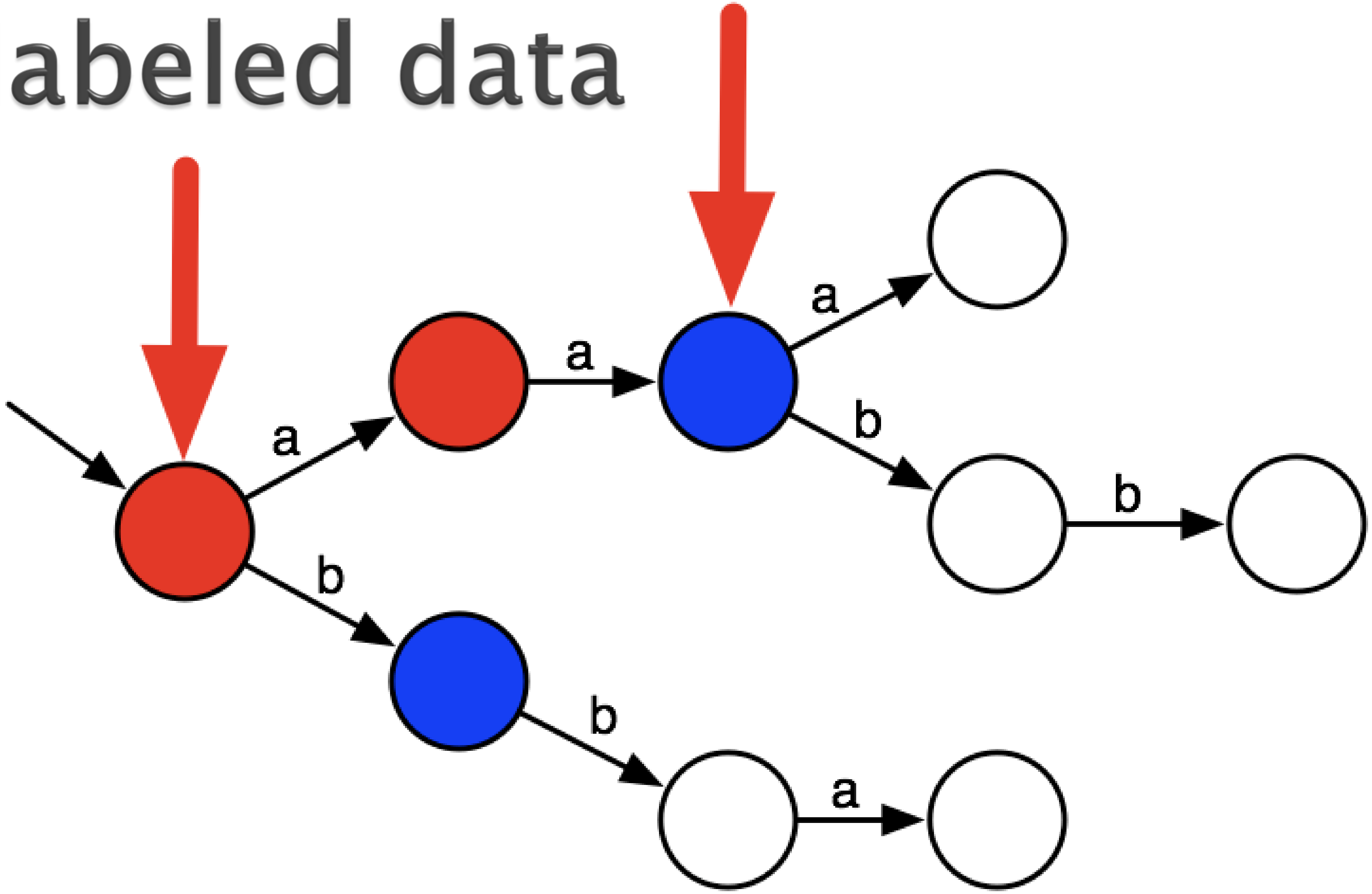
- Input: one sets S+ of positive timed strings
- Use the state merging and transition splitting algorithm, but
  - Maintain a core of inferred states using the red-blue framework
  - Only perform splits/merges of the transition/state that is visited by the most examples from S+

# Unlabeled data



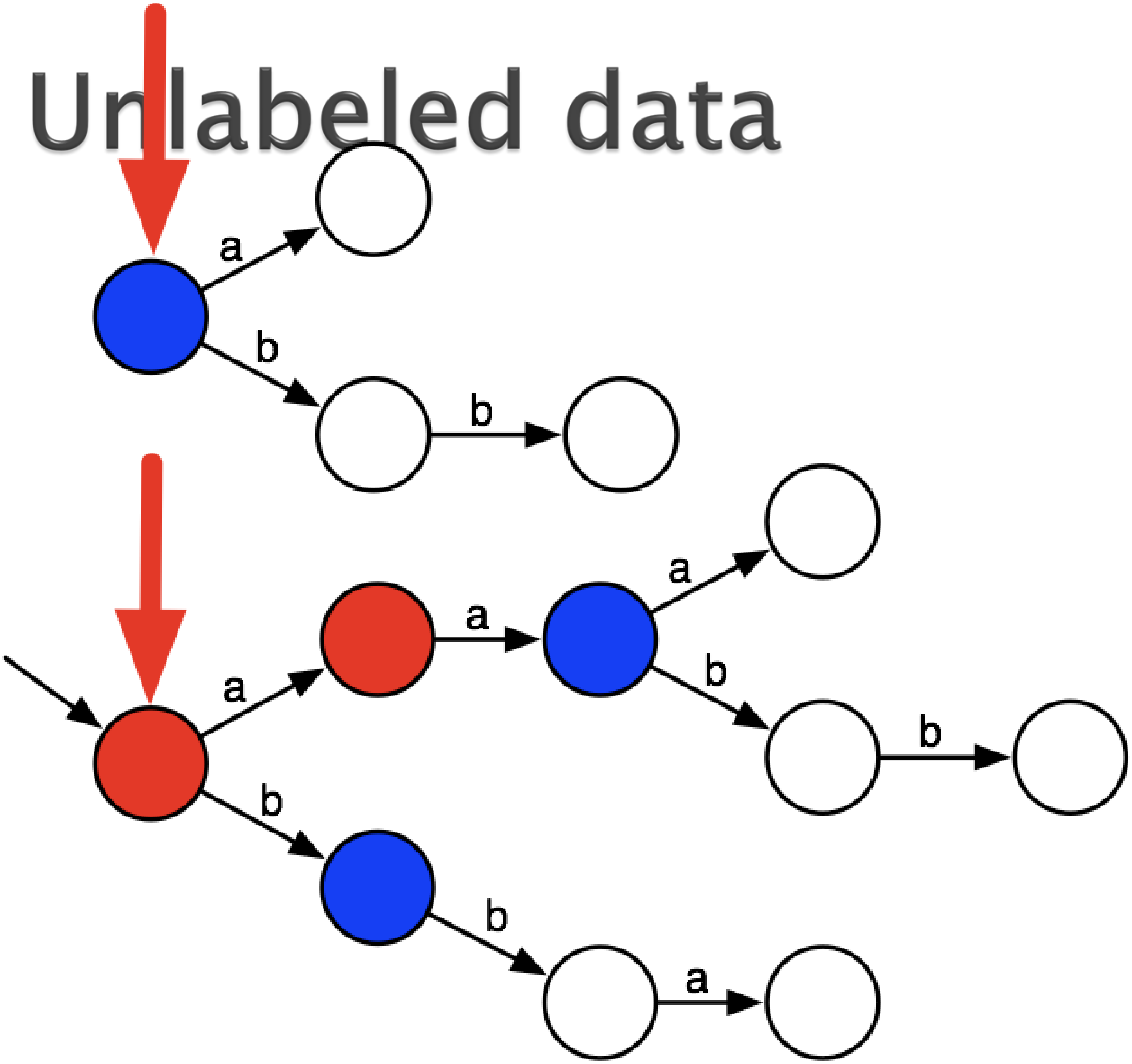

There are only positive states

# Unlabeled data



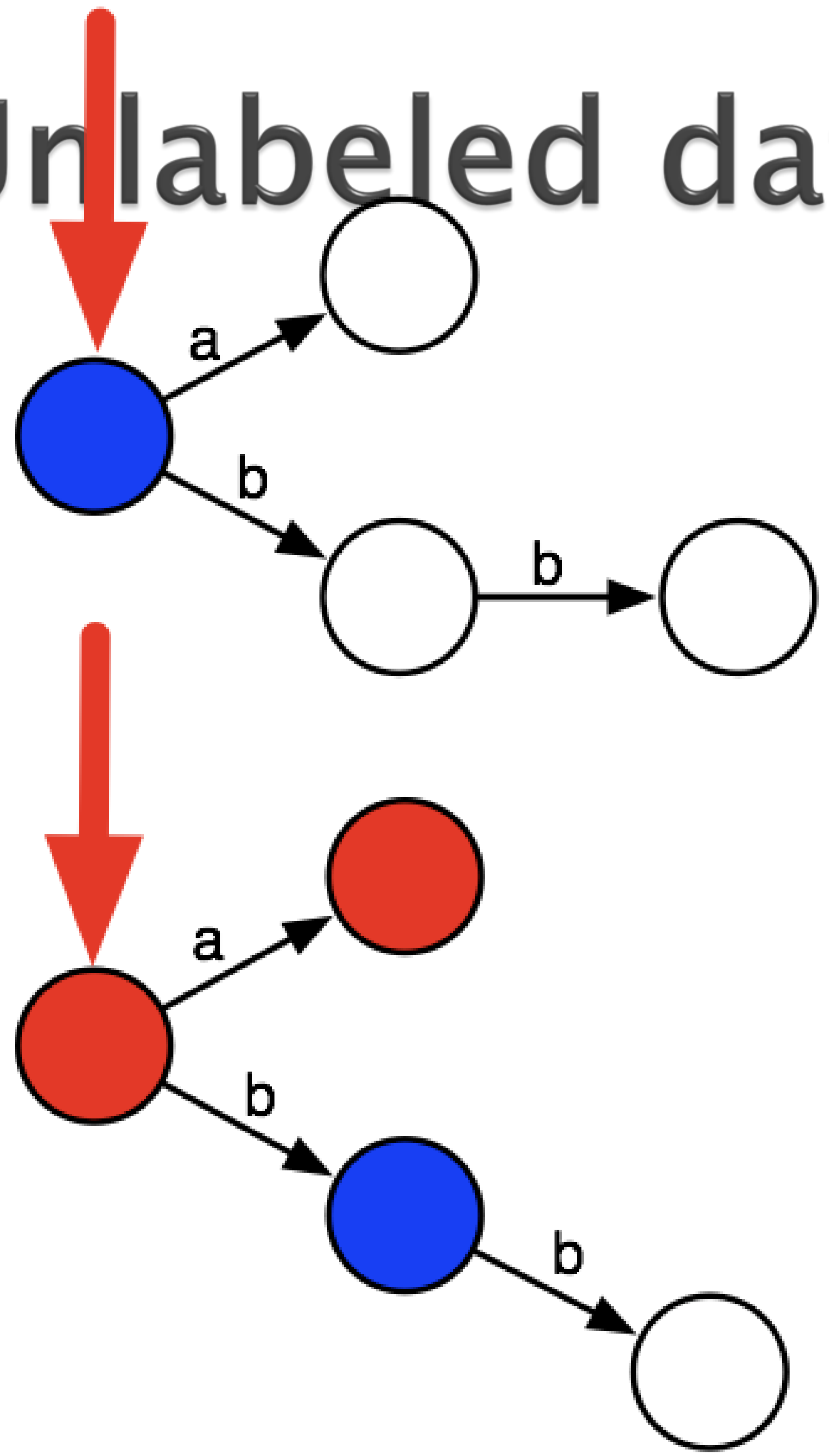

States should be merged if their future behavior is similar

# Unlabeled data



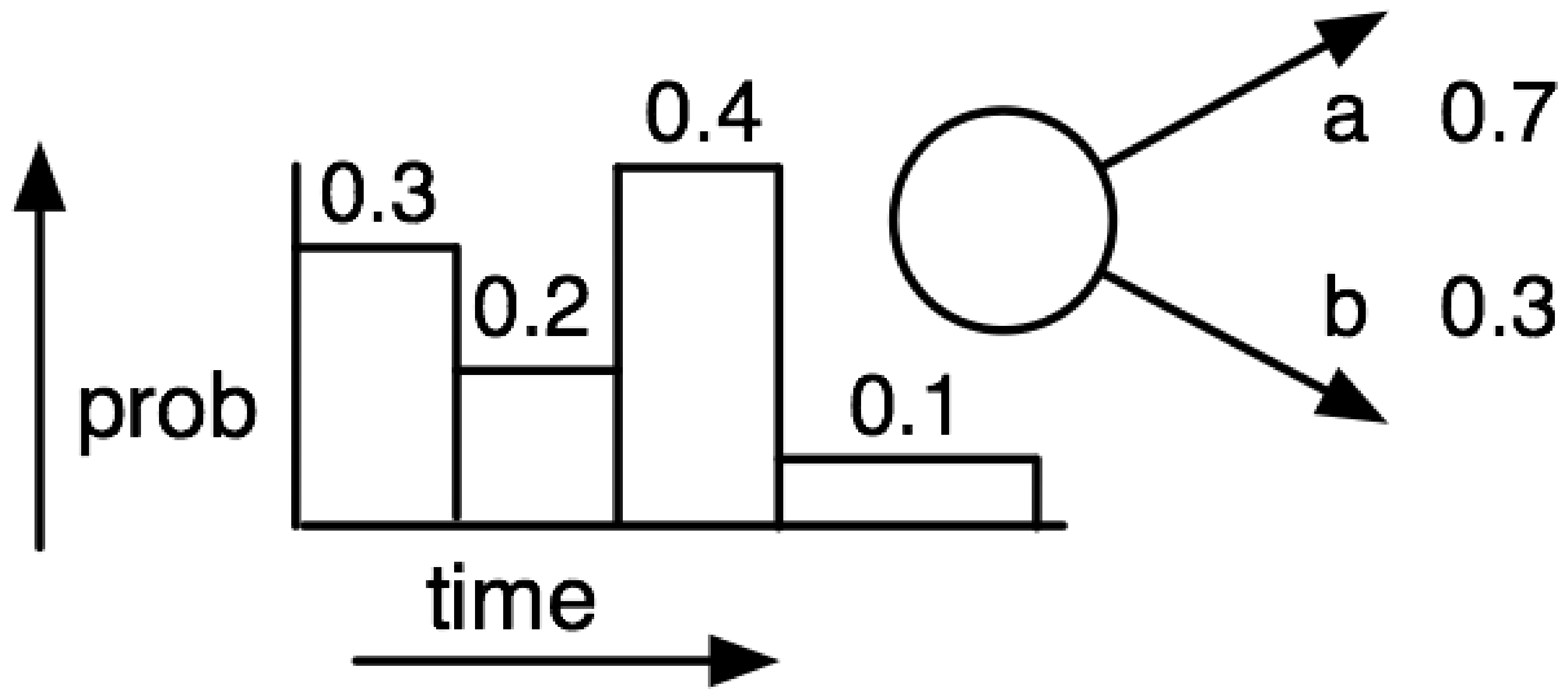

This behavior is determined by the trees rooted by the two states

# Unlabeled data




We are only interested in the overlapping behavior, i.e., the behavior for which we have data

# DRTA Distributions




A multinomial symbol distribution, and
a histogram (uniform in every bin) time distribution
this is a semi-Markovian model

# DRTA Distributions

- The time and symbol distributions are independent
- The parameters are set by counting occurrences
- Bins are pooled for low occurrence counts

- Multiplying all time and symbol probabilities gives a distribution over timed strings

# Similarity using likelihood ratio

- Two states and their future states define a distribution over timed strings
- Two states are similar if merging results in
  - a significantly smaller model,
  - with respect to the likelihood of S+

- This is a likelihood ratio test

# Similarity using likelihood ratio

- The likelihood ratio test:

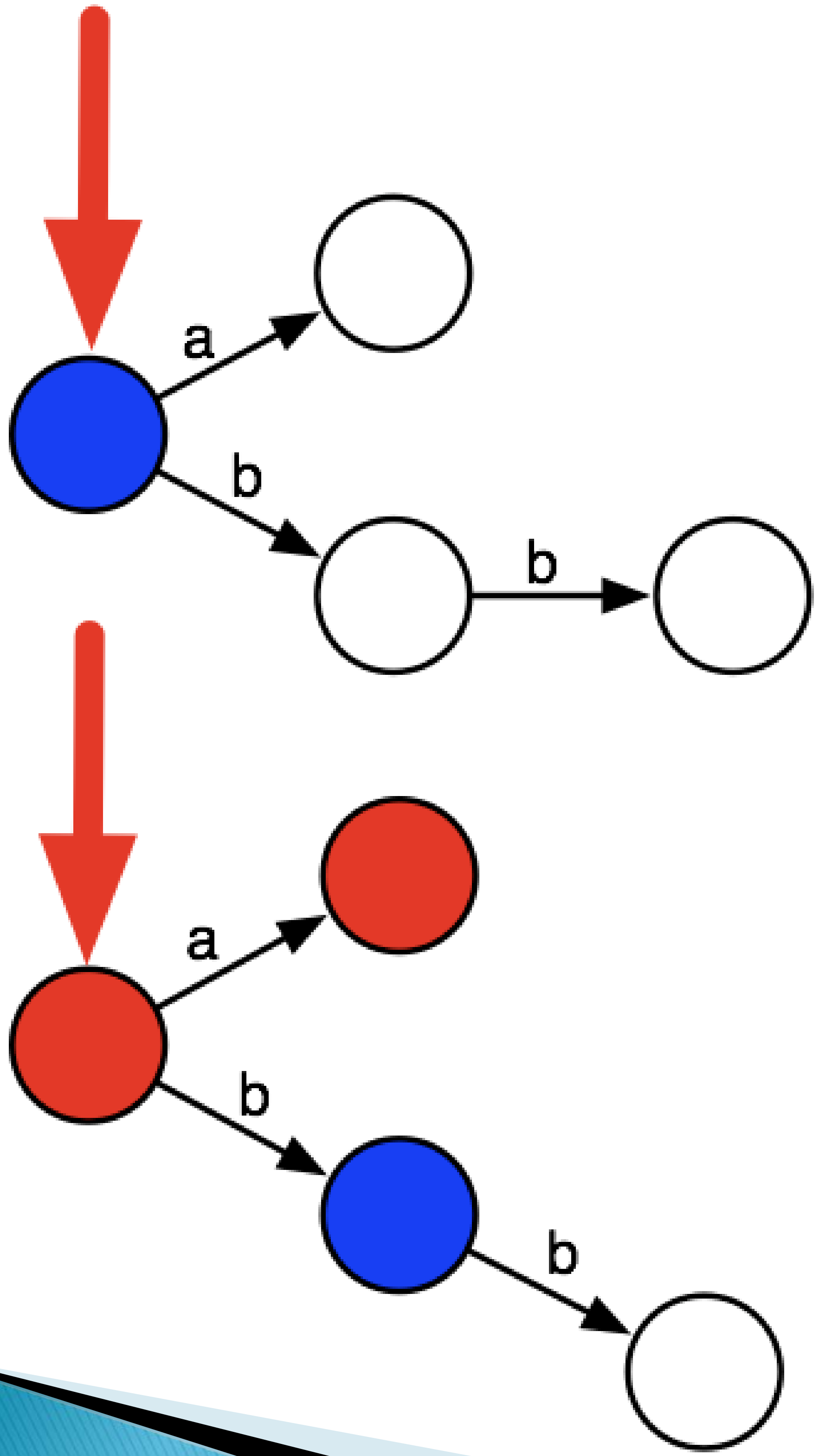
  Given two nested hypotheses H and H', then

  LR = likelihood(S+, H) / likelihood(S+, H')

  is compared to a chi-squared distribution with n' – n degrees of freedom

- This tests whether the increase in likelihood (LR) is significant for n' – n more parameters
- Fits nicely into state-merging since the models are always nested!

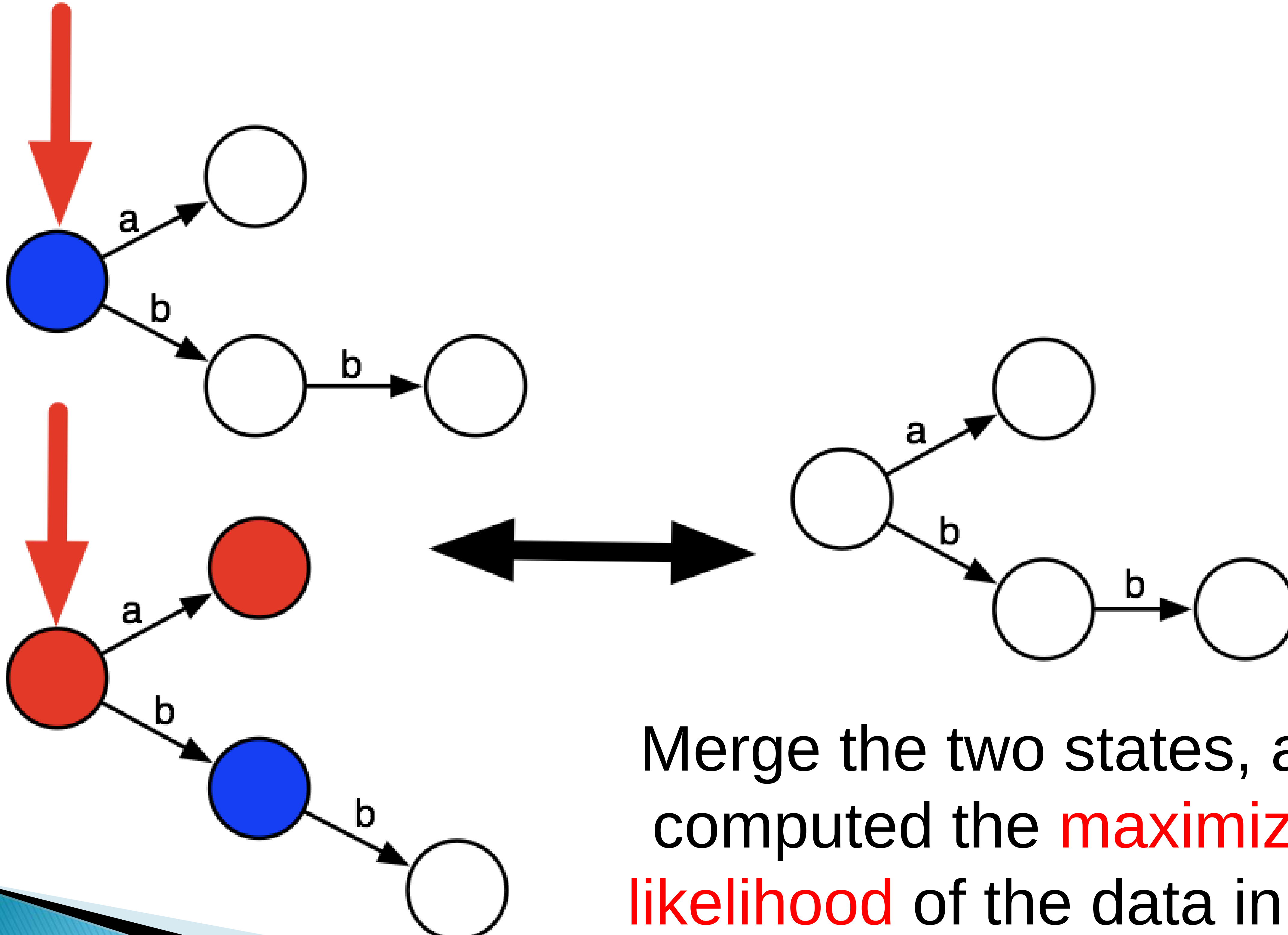
# Likelihood ratio




In the two states and the future states:

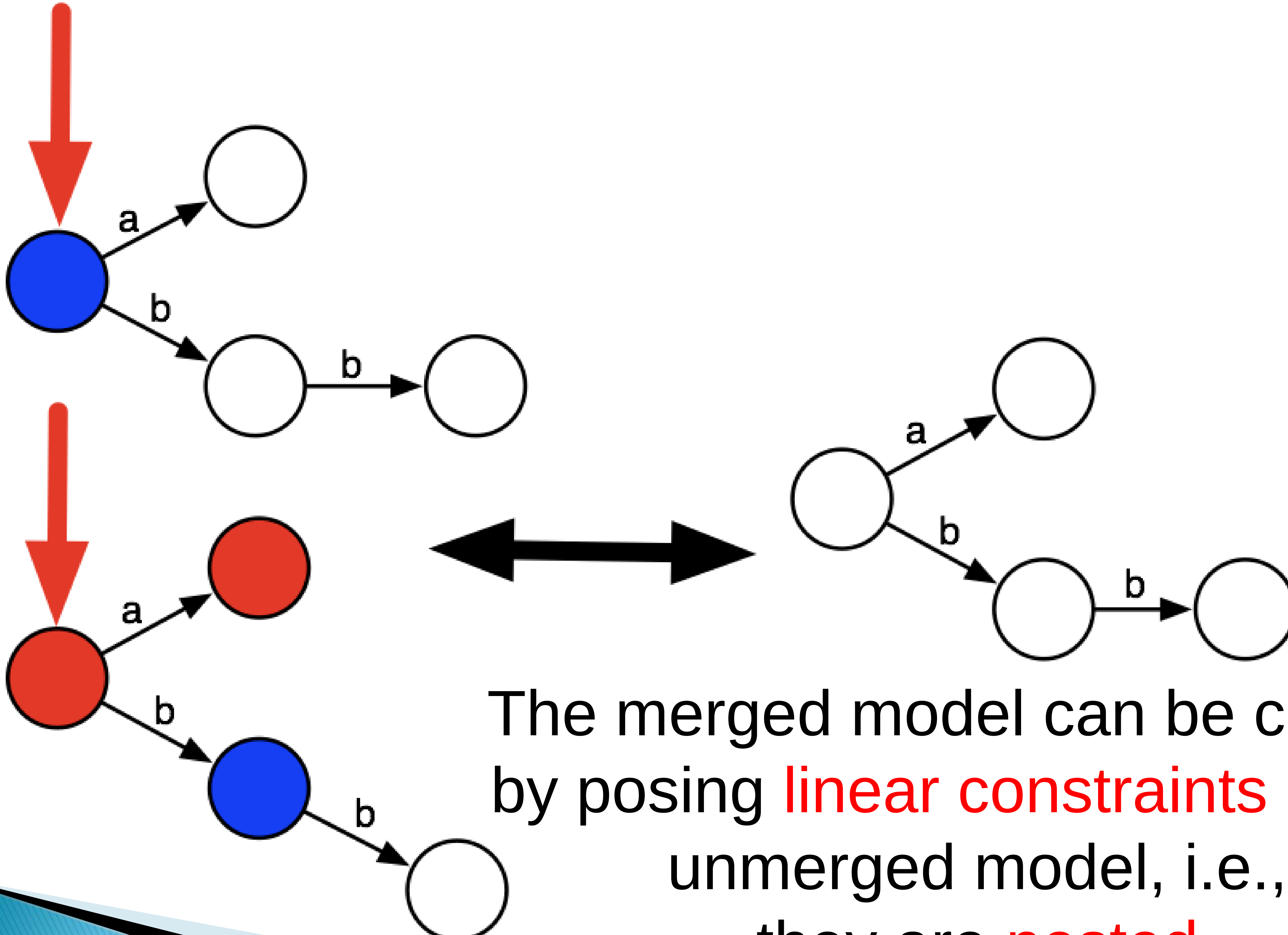
Determine the maximized likelihood of the data

# Likelihood ratio



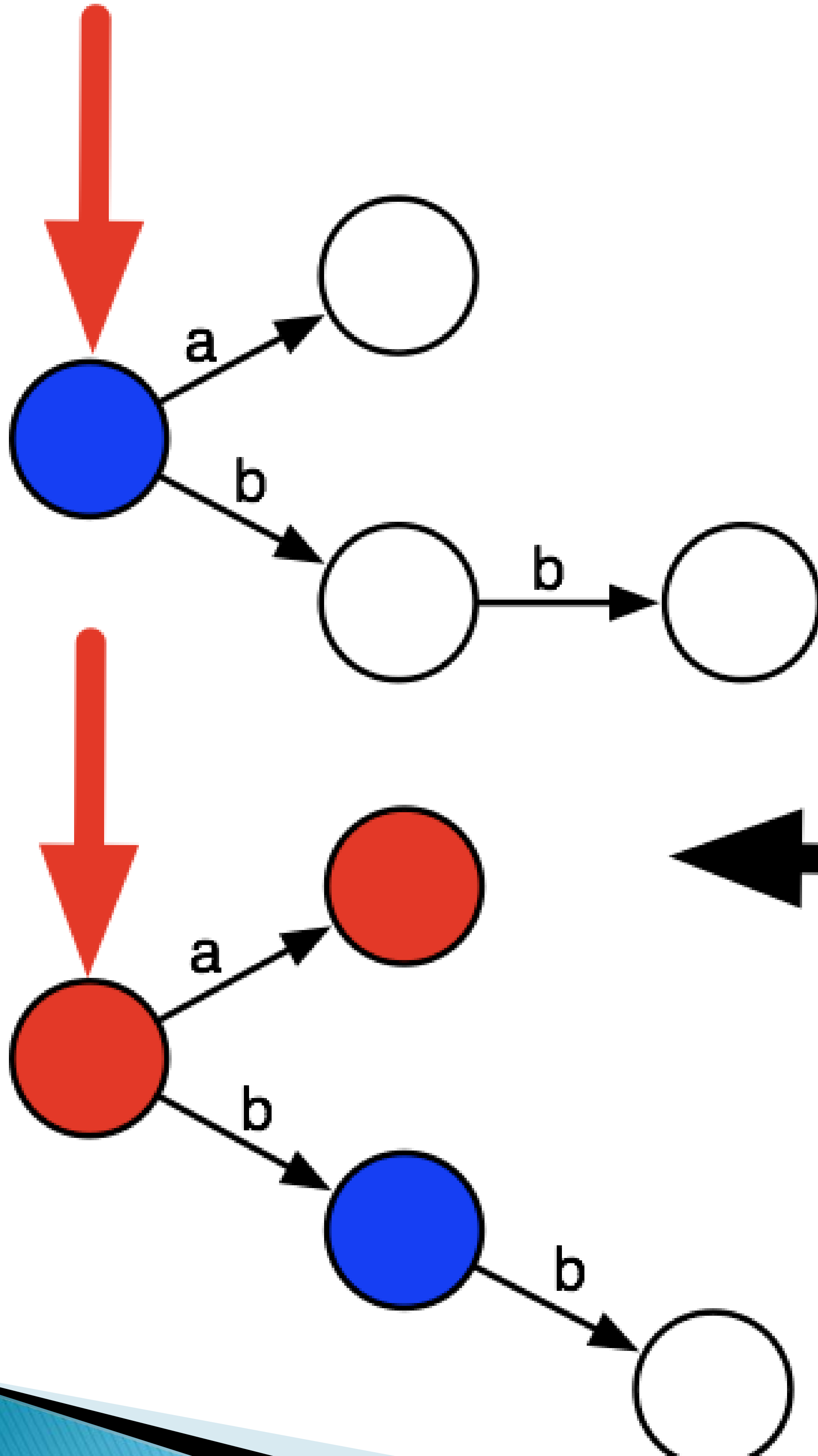

Merge the two states, and computed the maximized likelihood of the data in the merged PDRTA

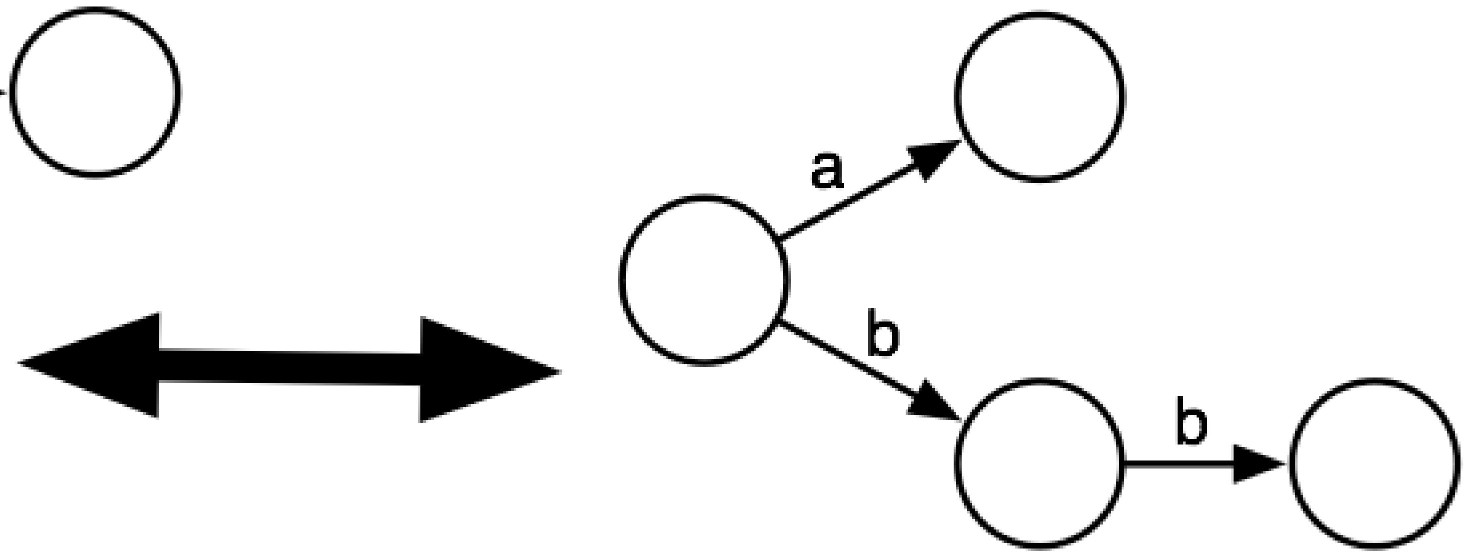
# Likelihood ratio




The merged model can be created by posing linear constraints on the unmerged model, i.e., they are nested

# Likelihood ratio




Compare the amount of reduced model parameters with the decrease in likelihood using LR

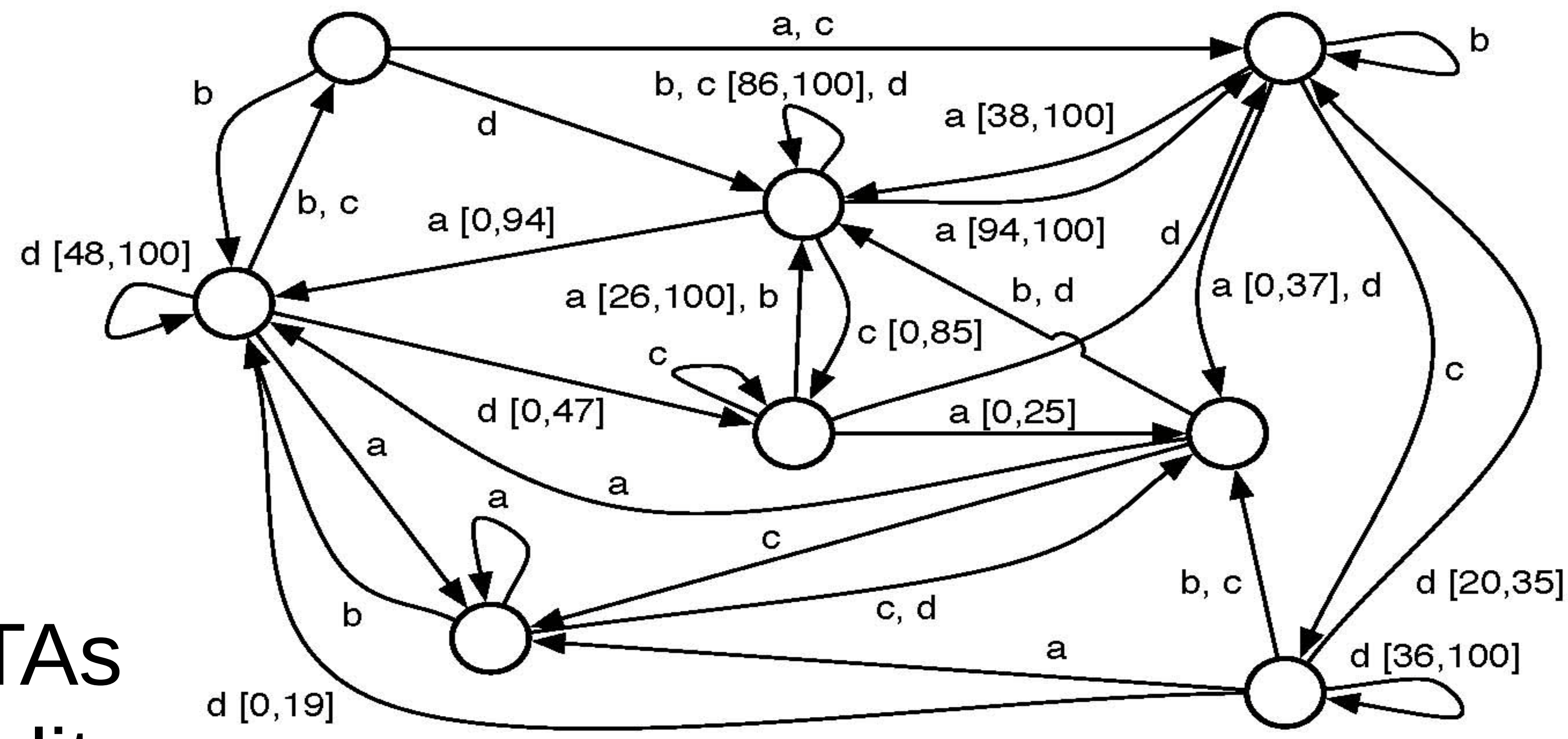
# Using similarity values

- Try all possible merges and splits of the transition to a blue state, compute their similarity p-values
- If the smallest split p-value is less than 0.05:
  - perform the split with the smallest p-value
- Else if the largest merge p-value is greater than 0.05:
  - perform the merge with the largest p-value
- Else color the blue state red

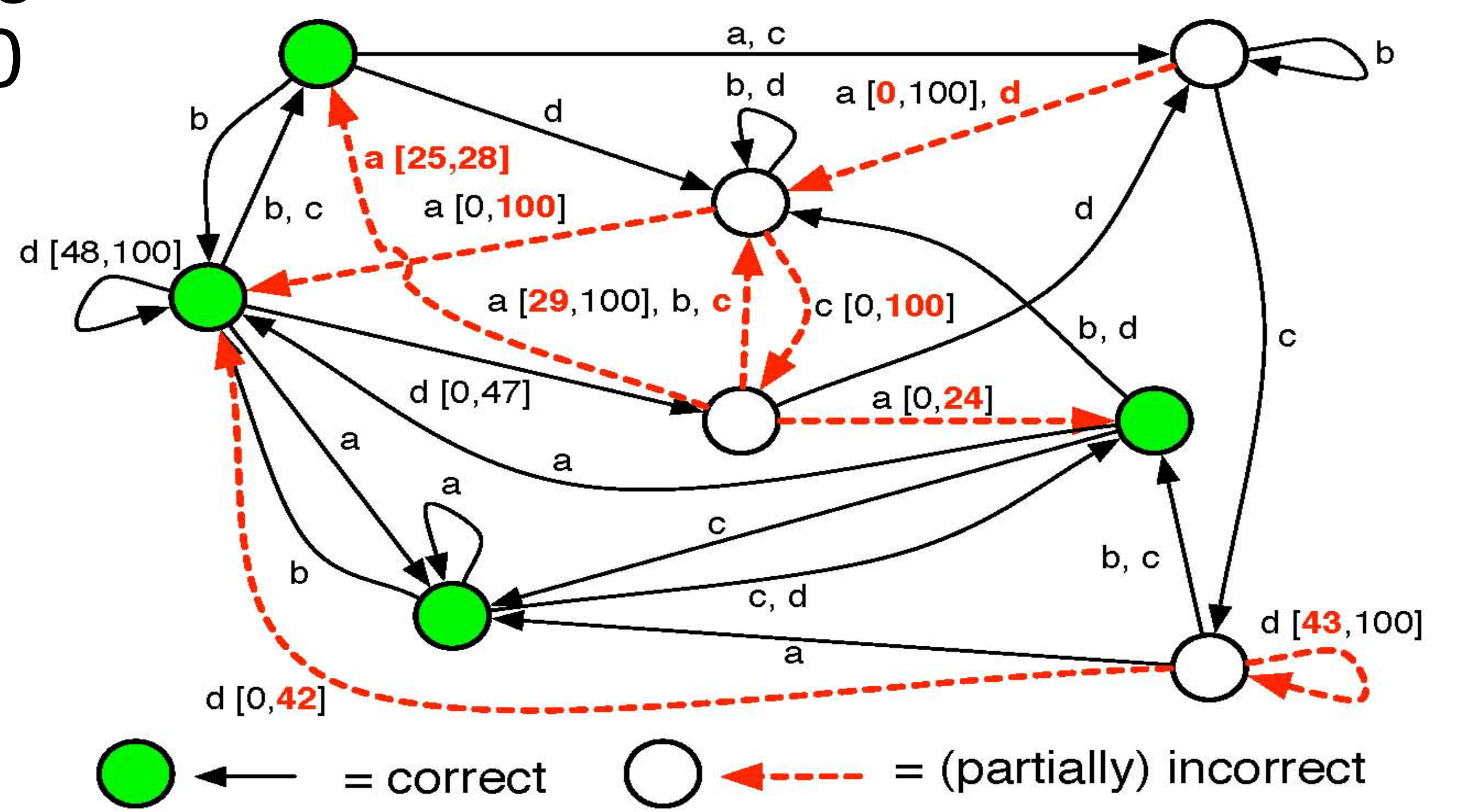
# Results

Can learn target DRTAs with 8 states and 8 splits from 2000 timed strings of average length 10

Originally generated random DRTA




Identified using RTI+ with the likelihood ratio test



# Conclusions

- We have an efficient algorithm for learning deterministic real-time automata (DRTA)s
- The algorithm works both on labeled and unlabeled data
- The algorithm is based on the best known algorithm for learning DFAs, state-merging
- It shows promising results on artificial data

# Future work

- Prove convergence:
  - Is it efficient in the limit?
  - What is the convergence speed of the test?
- Try the likelihood ratio test to identify probabilistic DFAs
- Apply the algorithm in practice (more)
  - Code is available on my homepage