Efficiently learning simple timed automata

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2009

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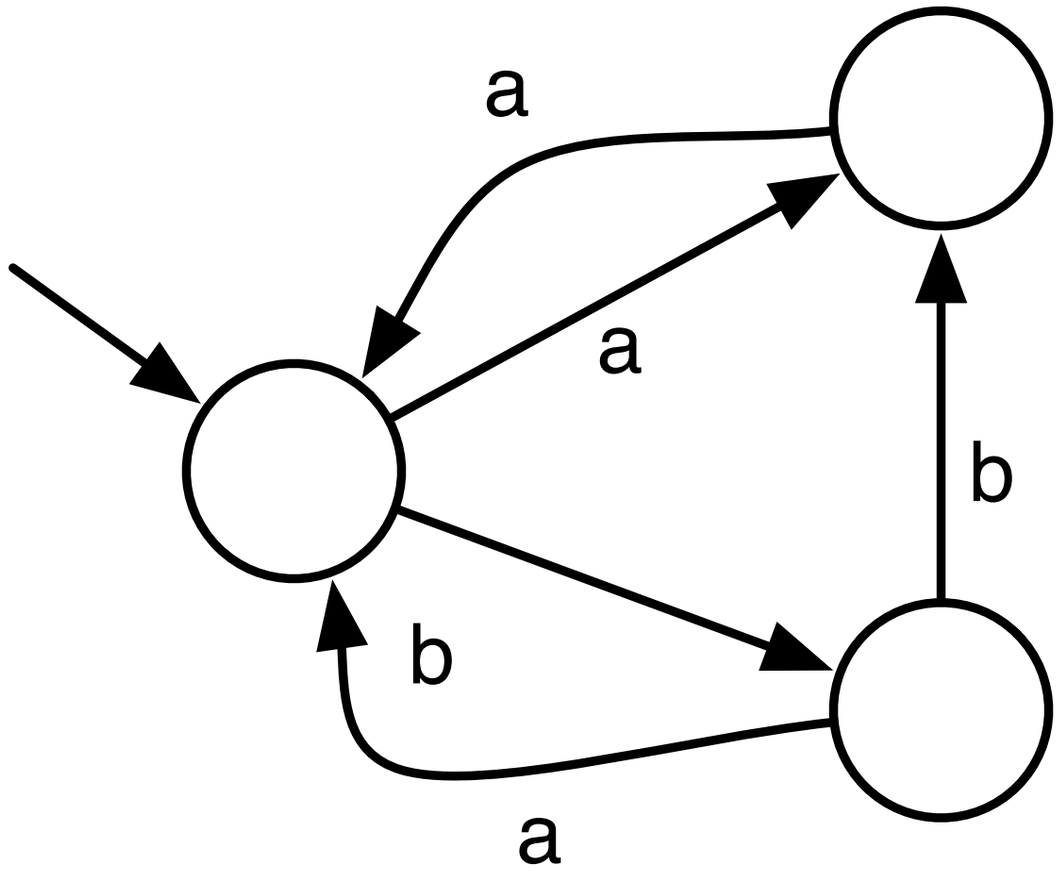
Overview

- Automata & how to learn them
- Real-time automata
- Learning real-time automata from labeled data
  - Results
- Learning real-time automata from unlabeled data
  - Results
- Conclusions

Automata



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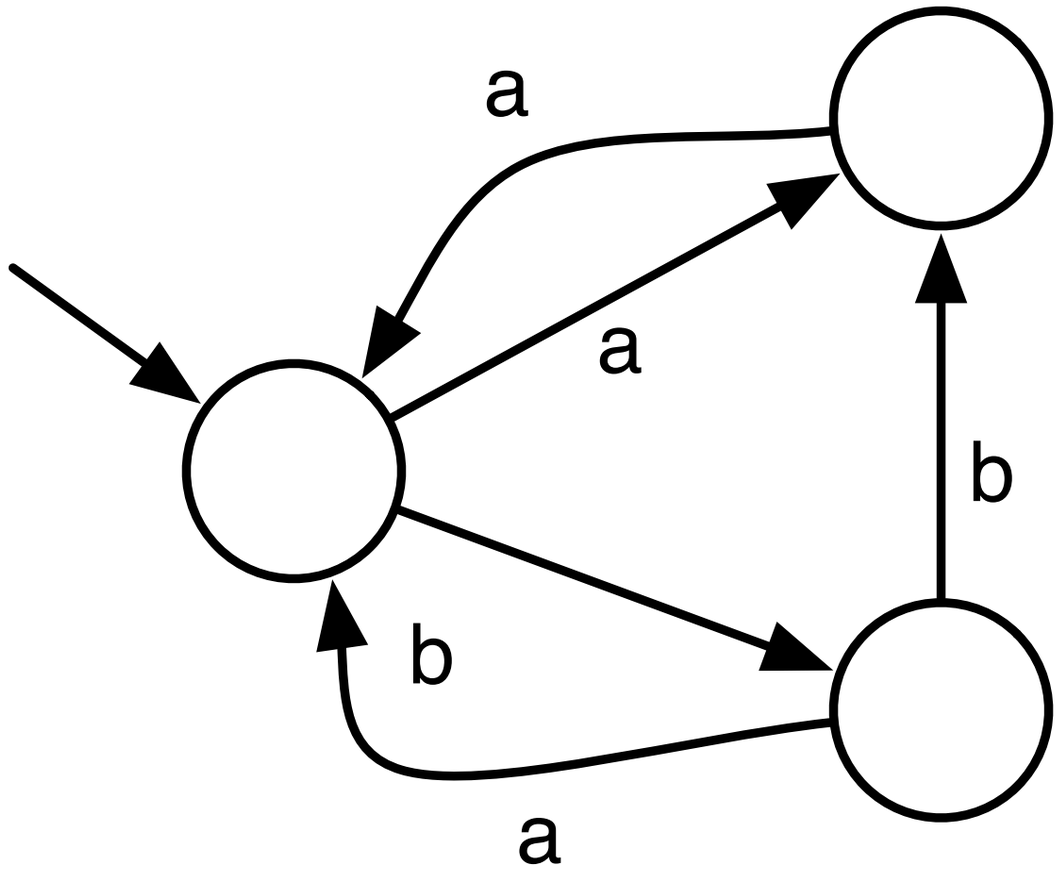
Automata and events



Produces strings: aa, aabb, aaaab, babba, bbab

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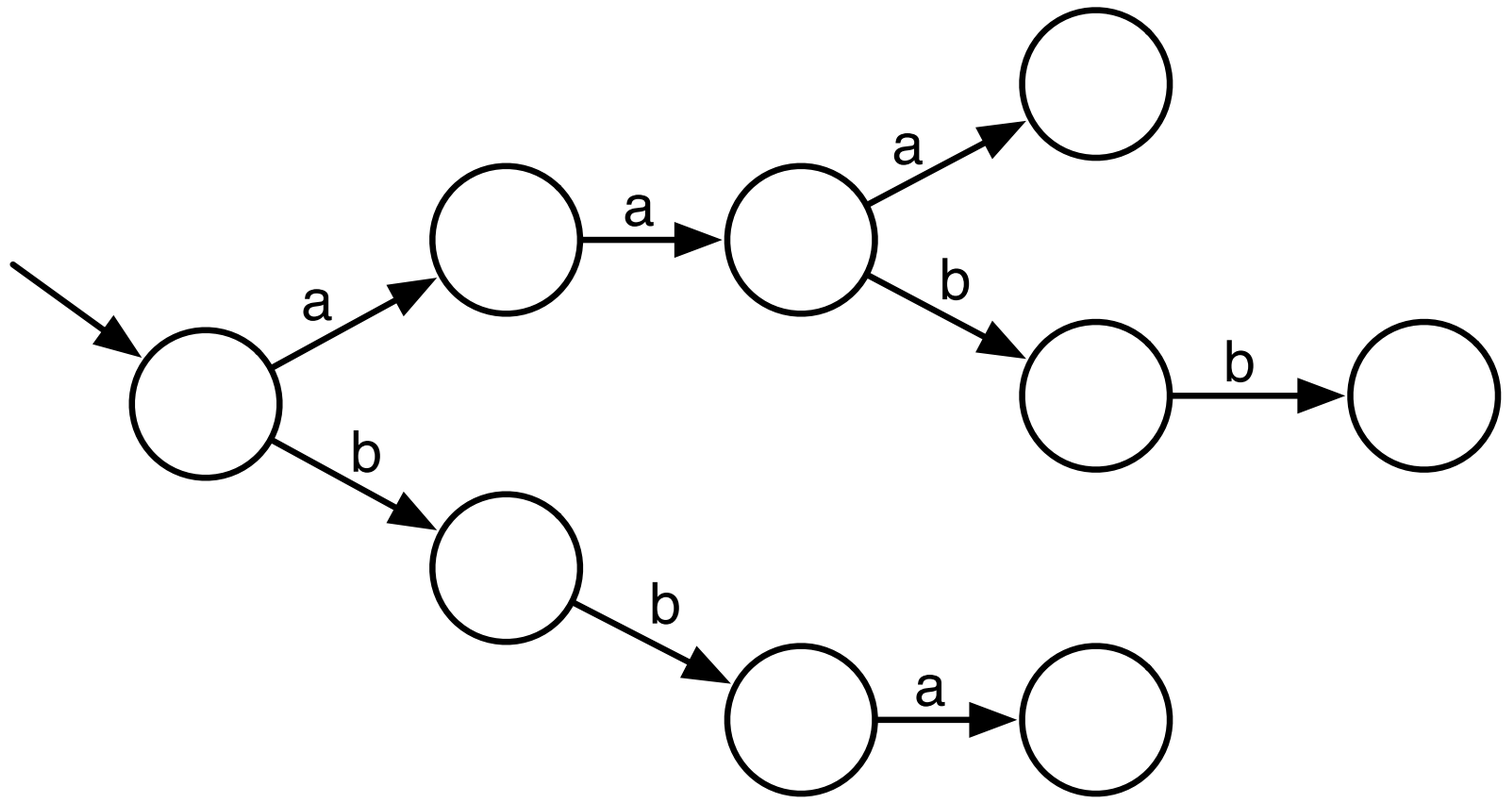
Learning DFA

- Input:
  - Data
- Goal:
  - Find the smallest DFA that is consistent with the data
- NP-hard but learnable in the limit from polynomial time and data

Learning DFAs



Some observations: a, aaa, aaa, aabb, b, bb, bb, bba
represented as a prefix tree

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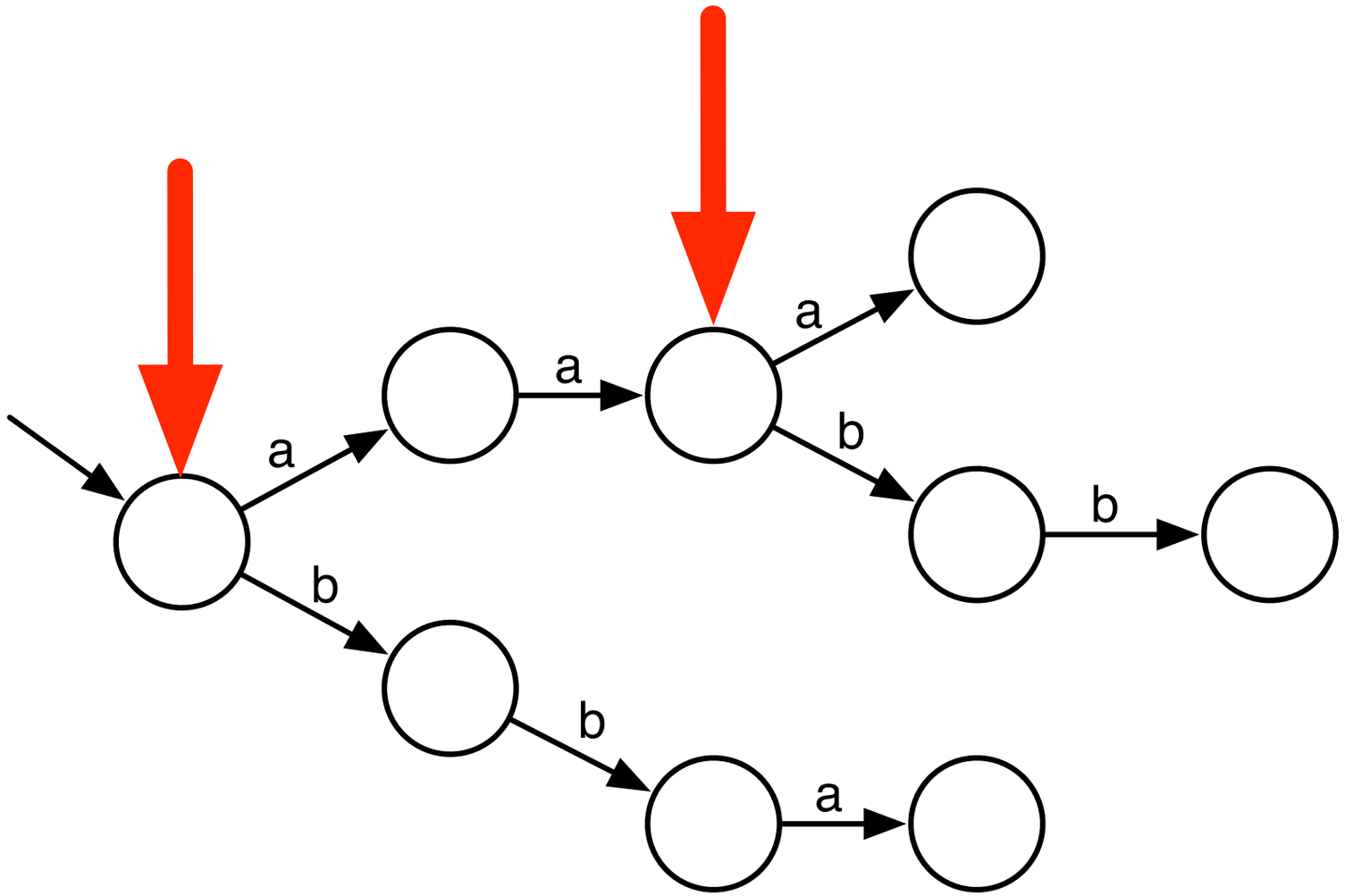
Learning DFAs



State merging:
select two nodes

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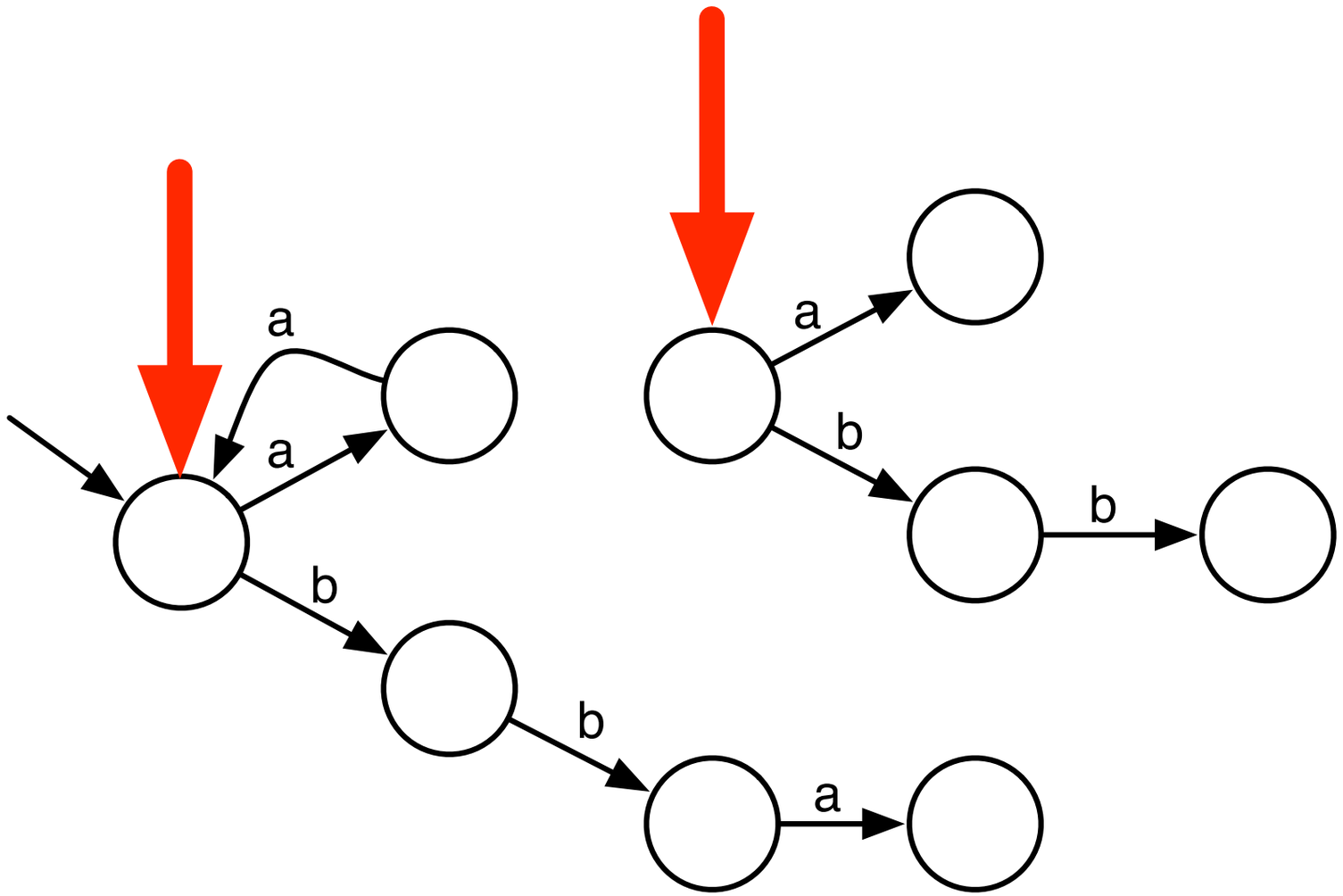
Learning DFAs



State merging:

move input transitions from one state to the other

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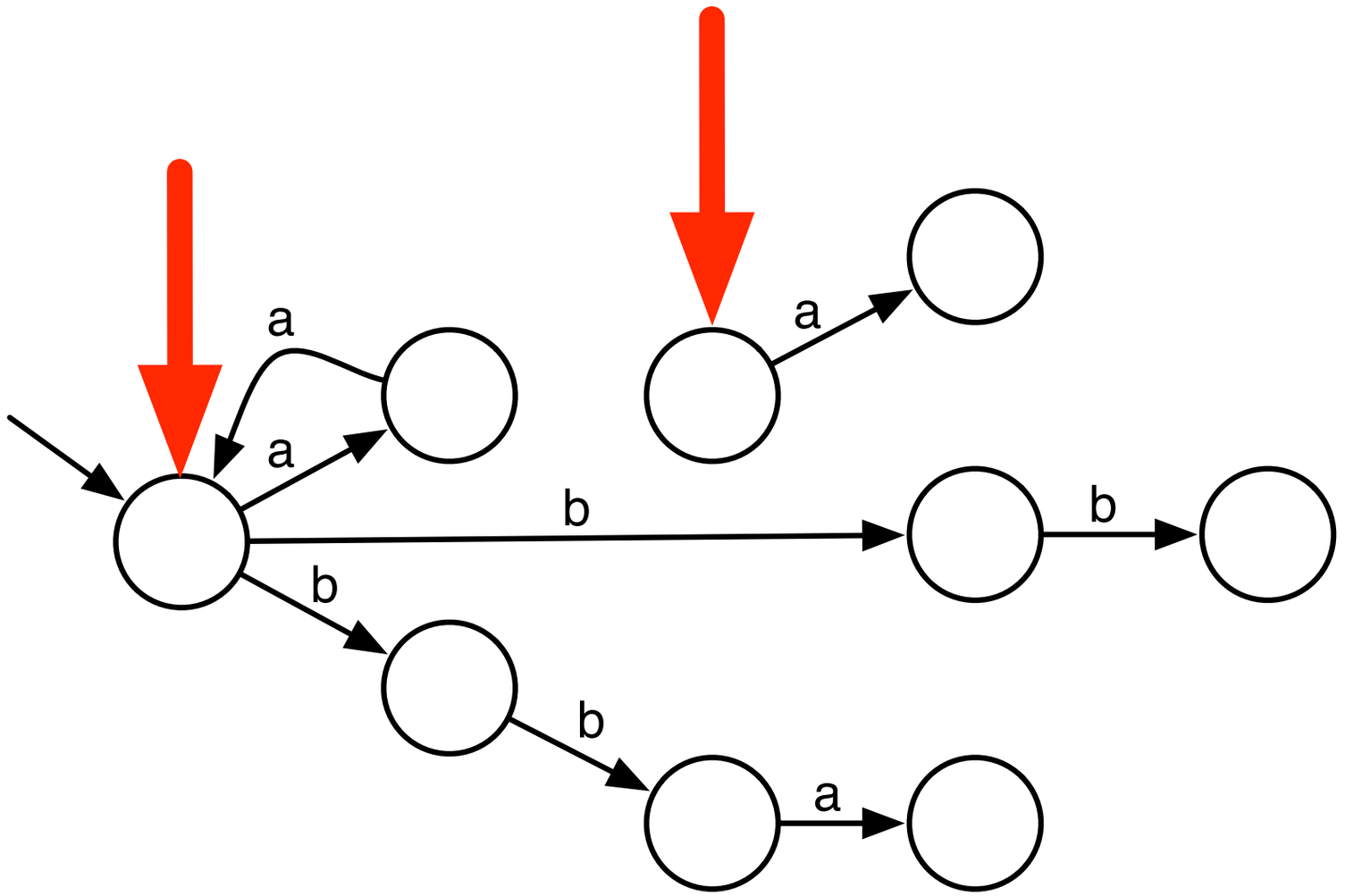
Learning DFAs



State merging:

move output transitions from one state to the other

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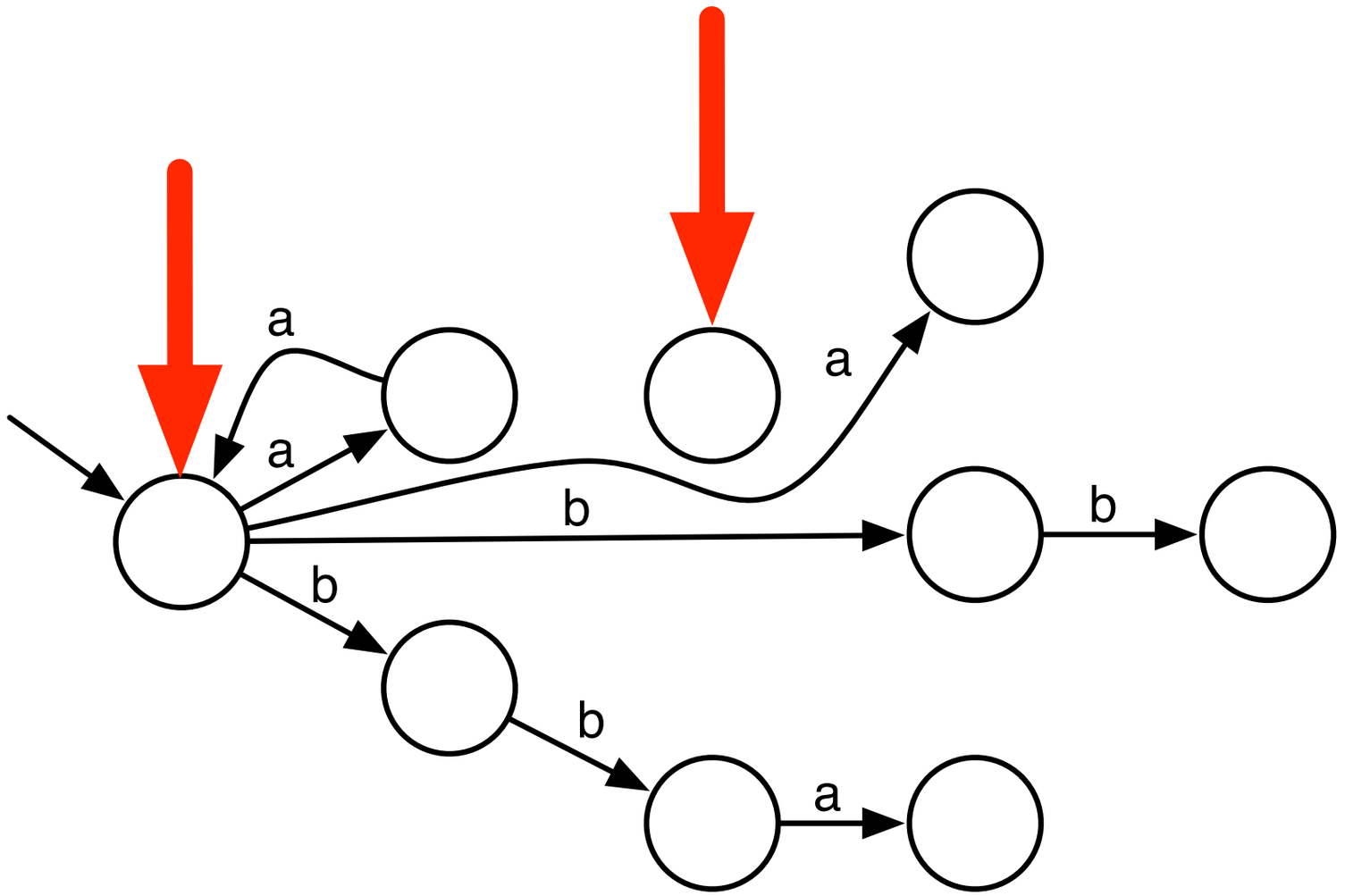
Learning DFAs



State merging:

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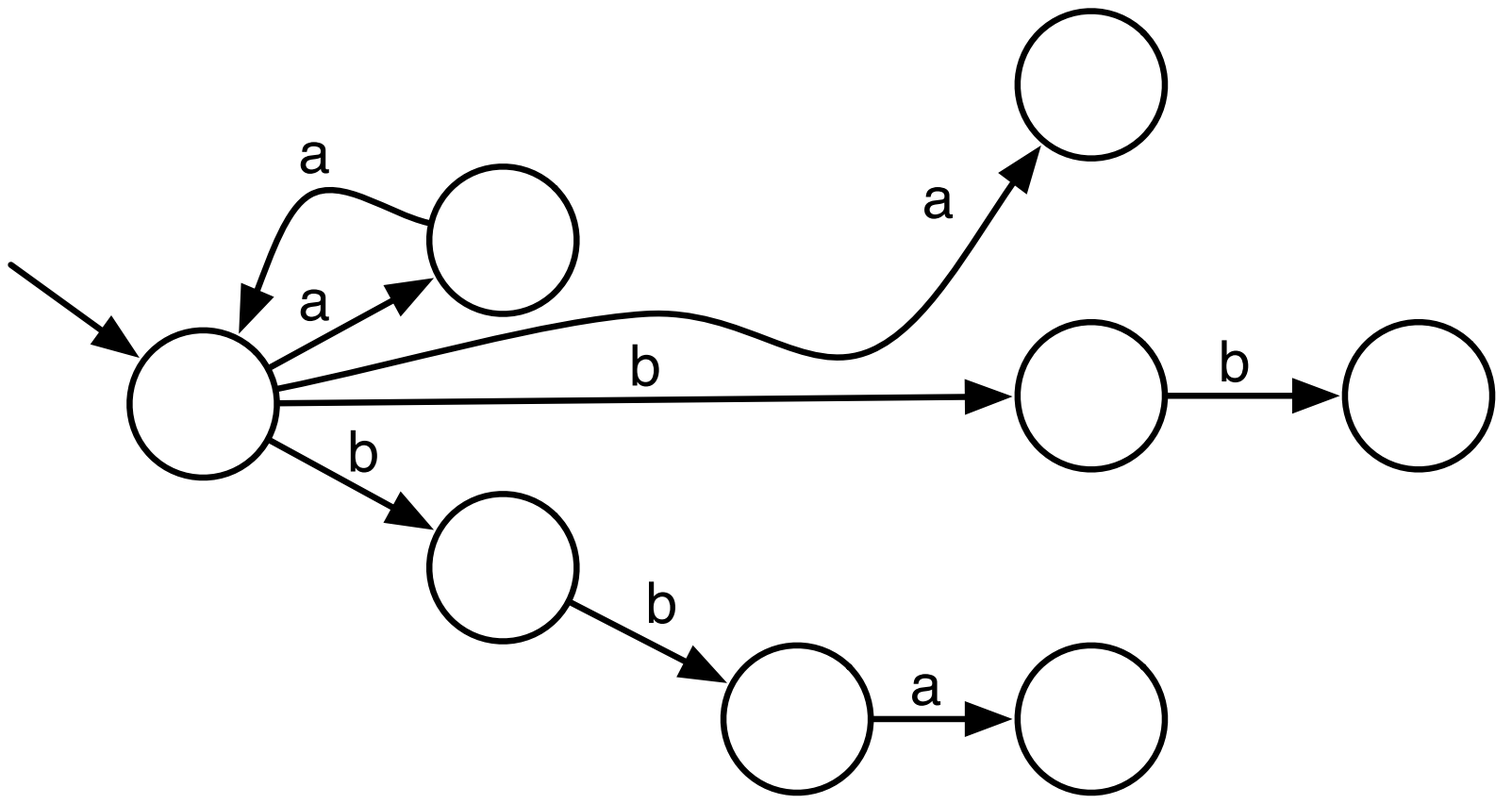
Learning DFAs



State merging:
delete the obsolete state

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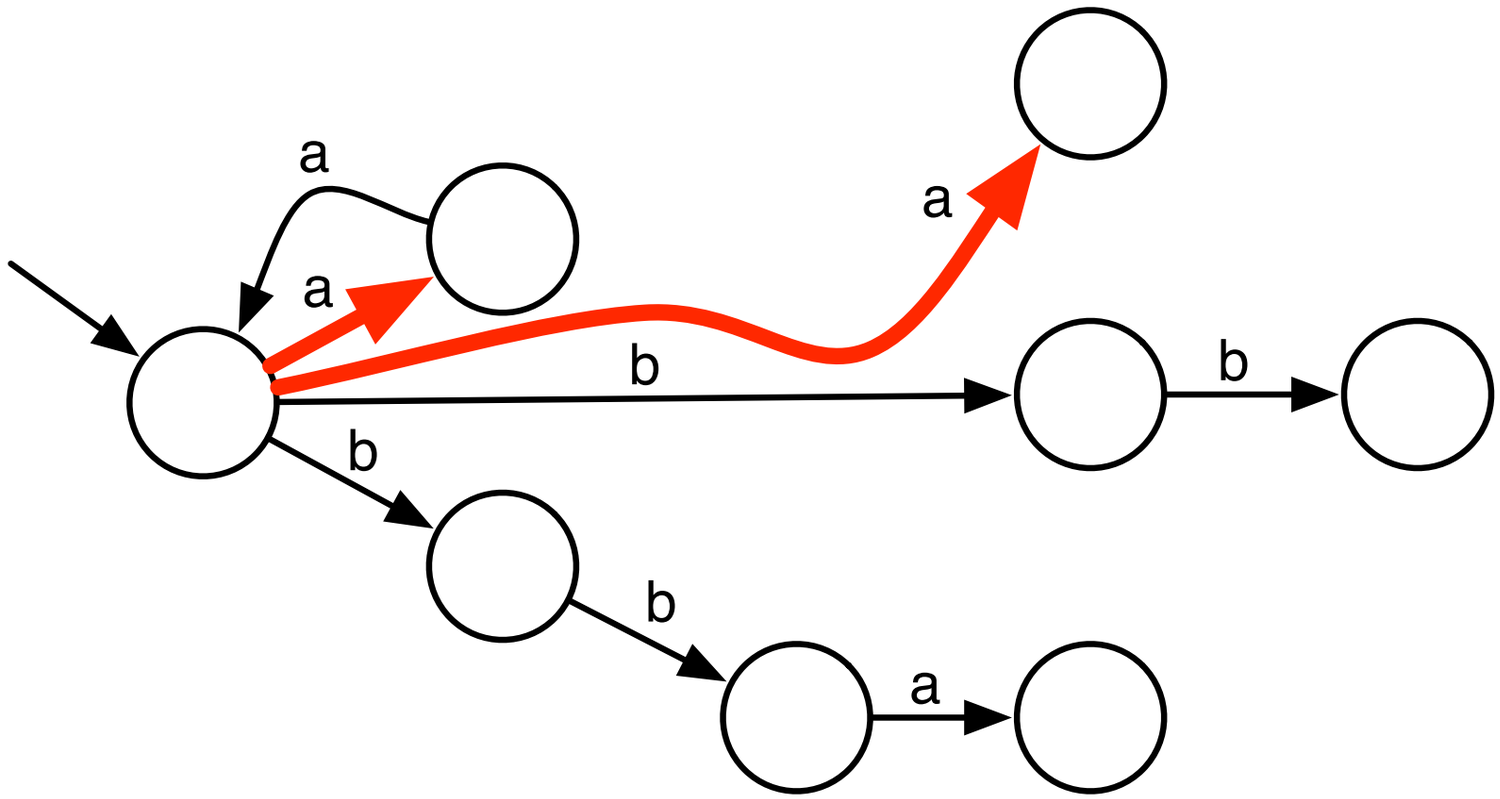
Learning DFAs



Determinization:

merge the targets of non-deterministic transitions

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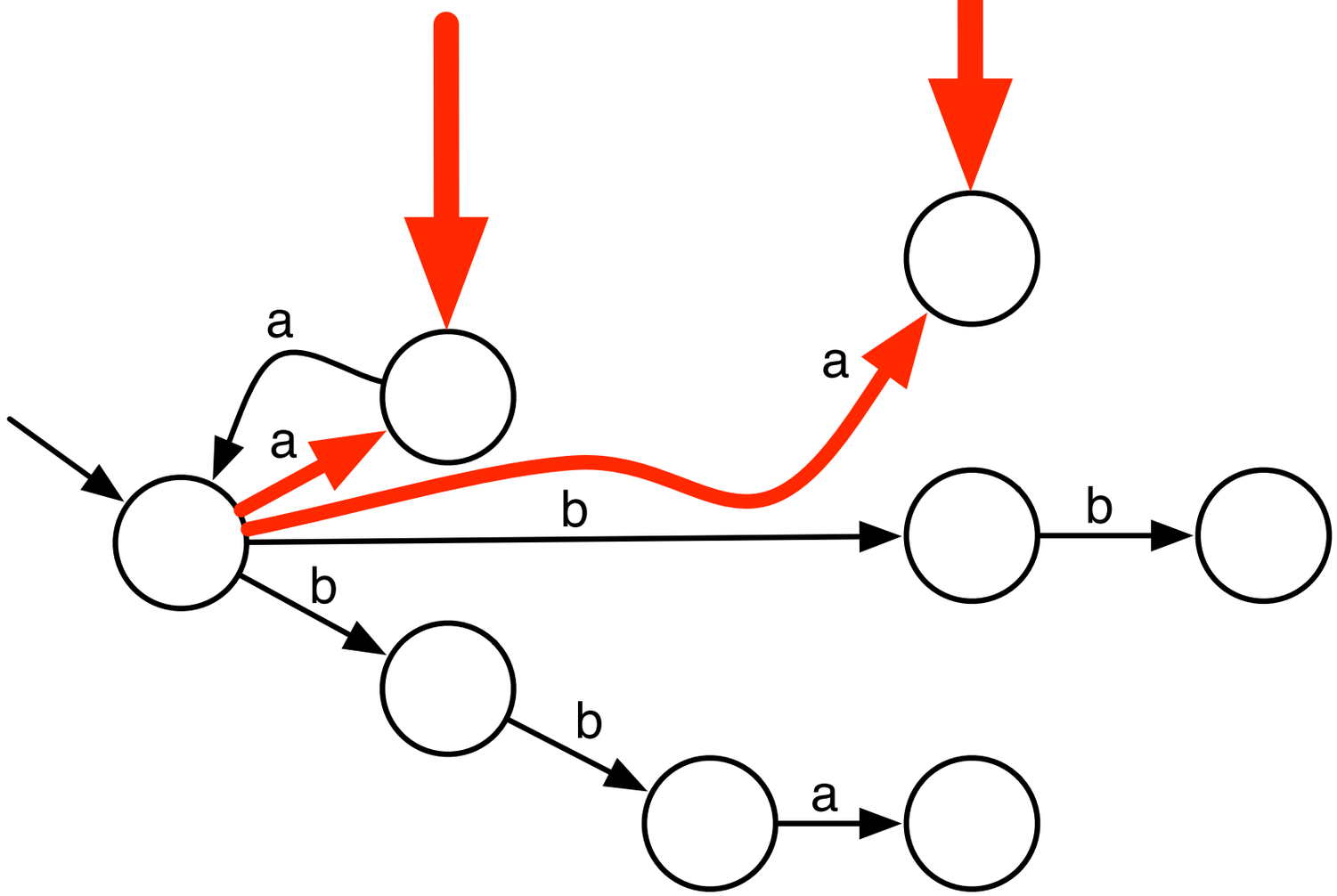
Learning DFAs



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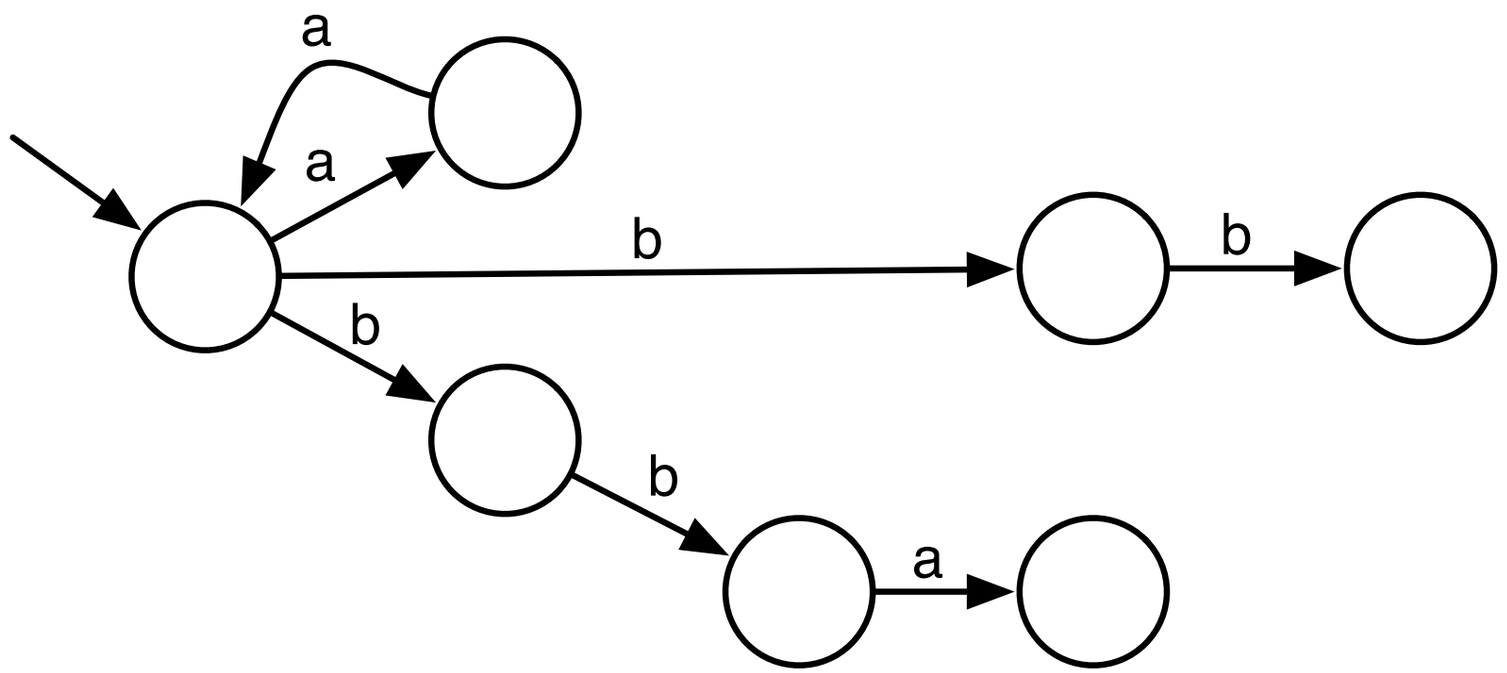

Determinization:

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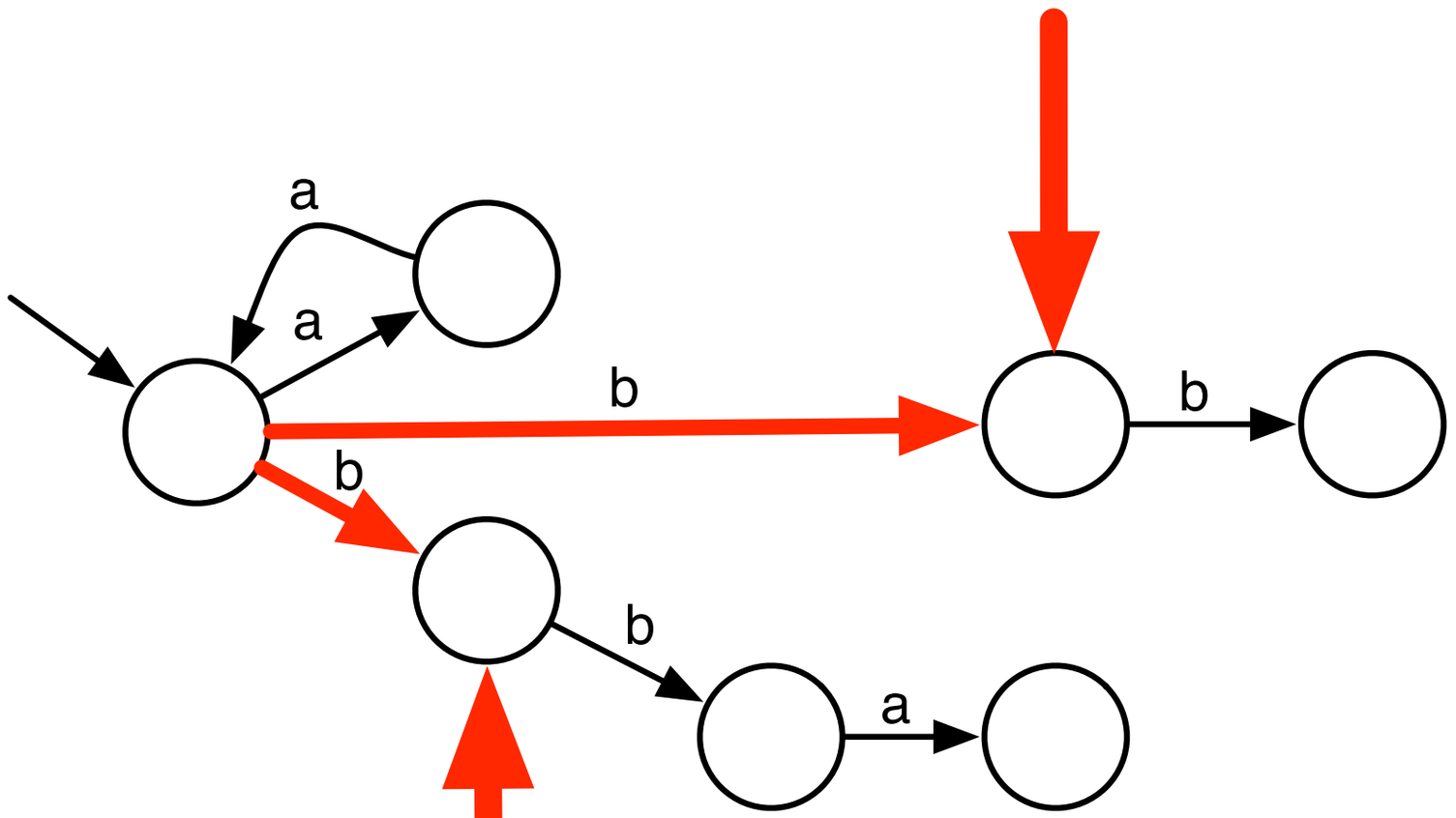

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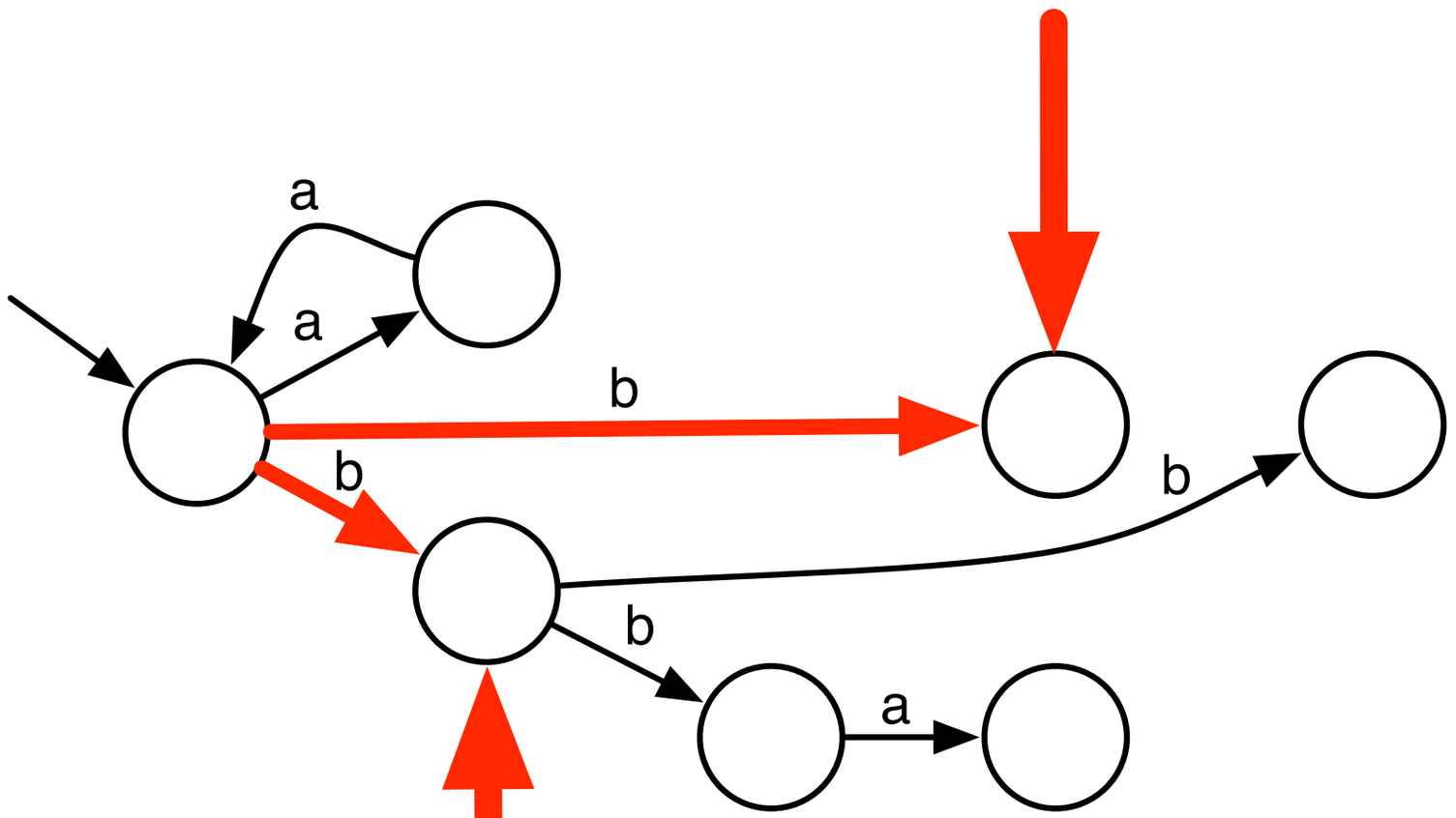

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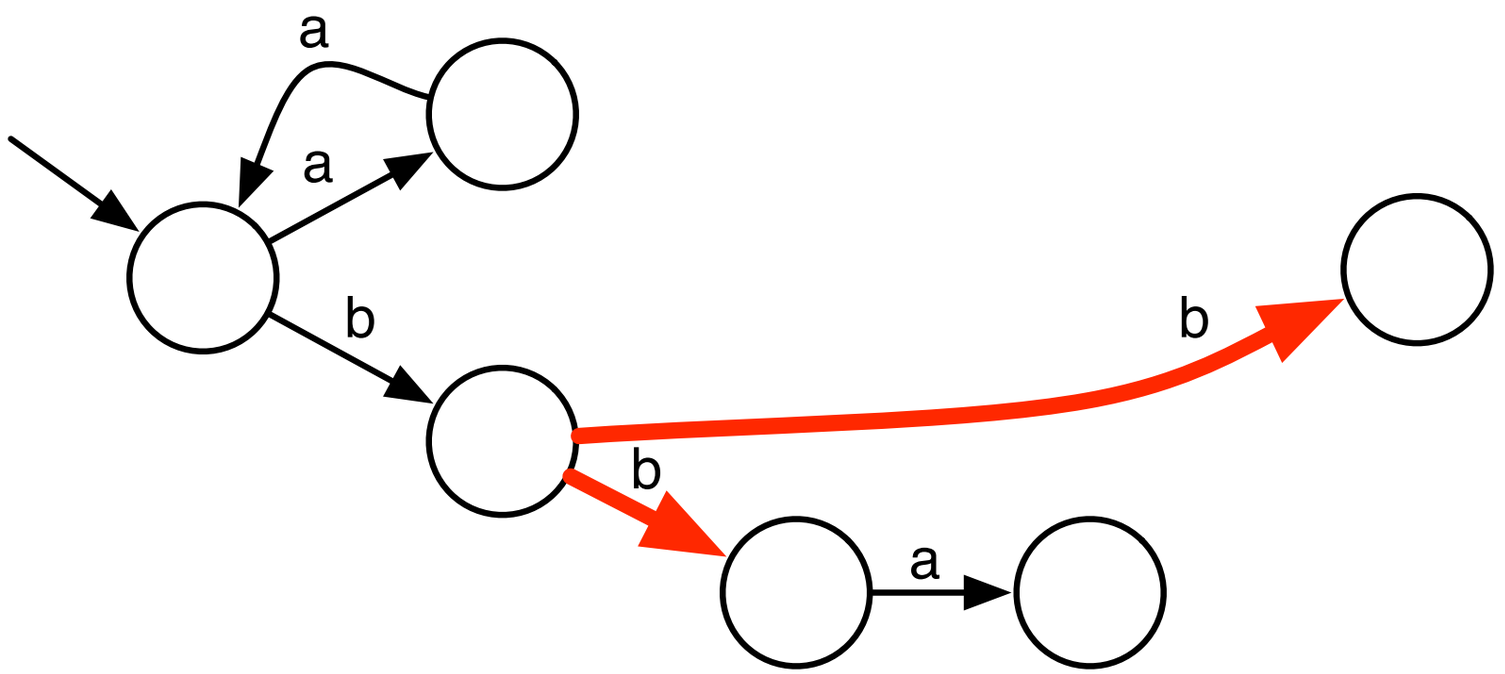
Learning DFAs



Determinization:

merge the targets of non-deterministic transitions

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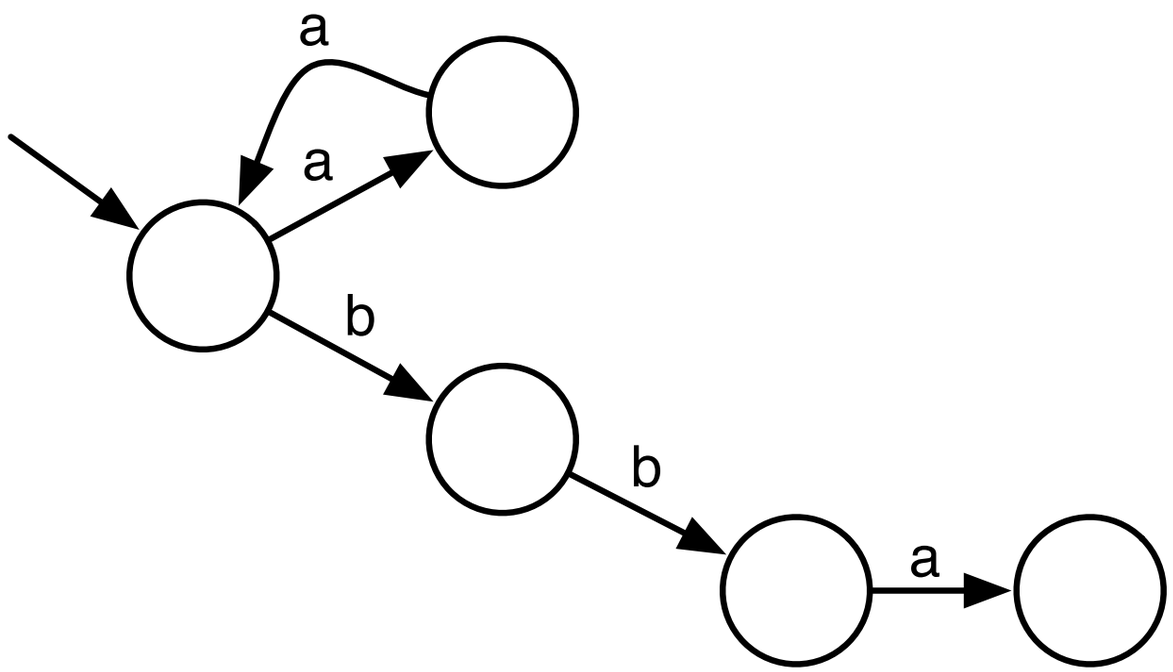
Learning DFAs



Select two new nodes to merge and iterate

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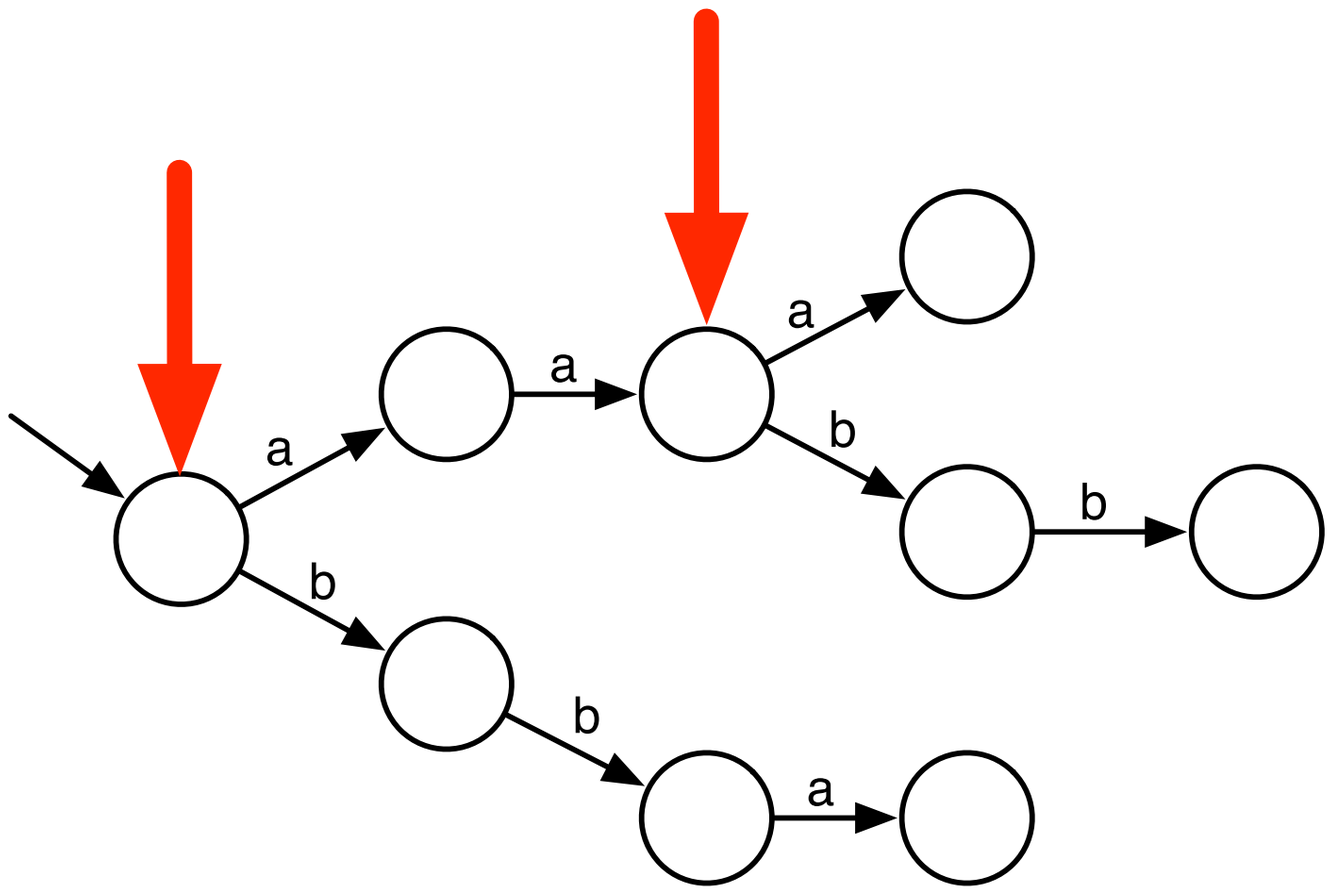
Learning DFAs

- State merging:
  - Start from a tree
  - Try all possible merges, including determinization
  - Perform the one that scores best
  - Iterate

Learning DFAs



The score of a merge can be determined using
labels or statistics

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EDSM

- For every string s it is known whether s is an element of the language or not, it is positive or negative
- Evidence driven state merging (EDSM):
  - Initially, the states *q* of the prefix tree are labeled according to positive/negative strings that end in *q*
  - It is not possible to merge positively labeled states with negatively labeled states
  - Score = #positive merges + # negative merges

ALERGIA

- It is not known whether strings are in the language or not
- ALERGIA:
  - Use a norm or statistic, like L_∞ or chi squared
  - Define a bound *b*, for the similarity between states
  - It is not possible to merge states for which the norm or statistical dissimilarity is greater than *b*
  - Score = value of norm or statistical difference

Algorithms for learning DFA

- State merging using EDSM performs best for labeled data
- The idea in ALERGIA has been used in many other algorithms, also in PAC learning DFA distributions
- Both algorithms converge in the limit to the correct DFA
- For both algorithms it is possible to compute the amount of data necessary to converge with sufficient probability

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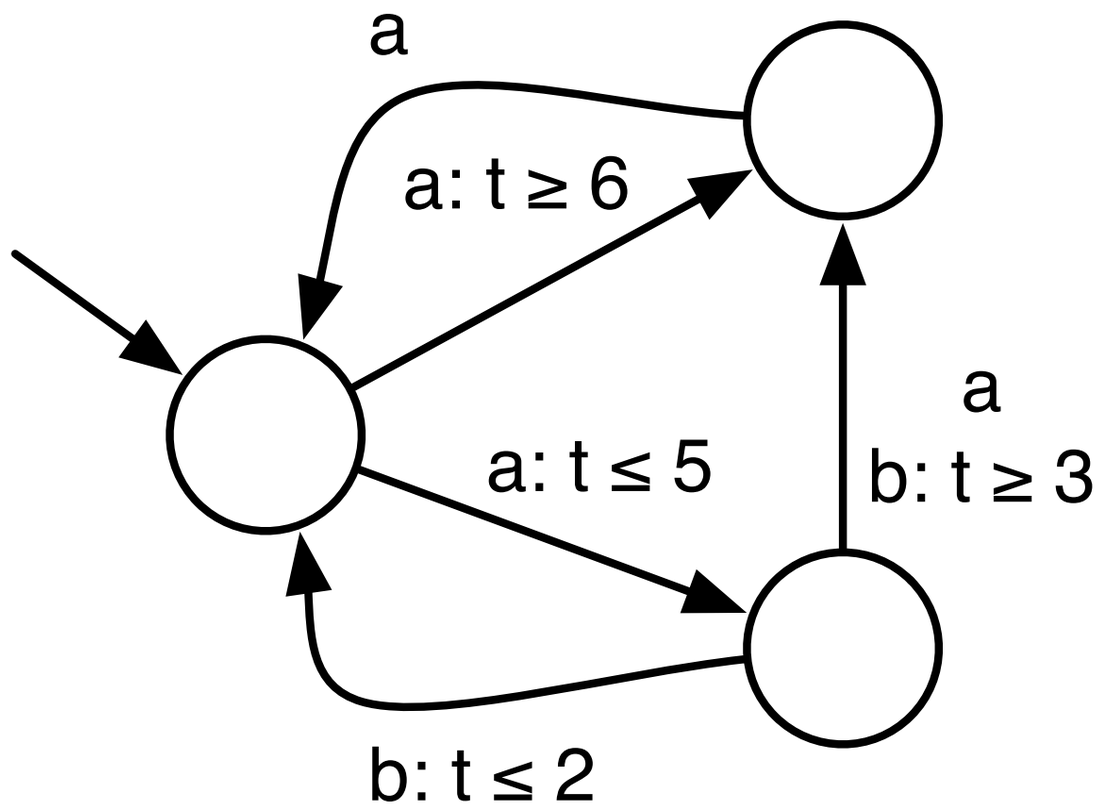
Real-time automata (DRTAs)



Transitions contain guards on time values

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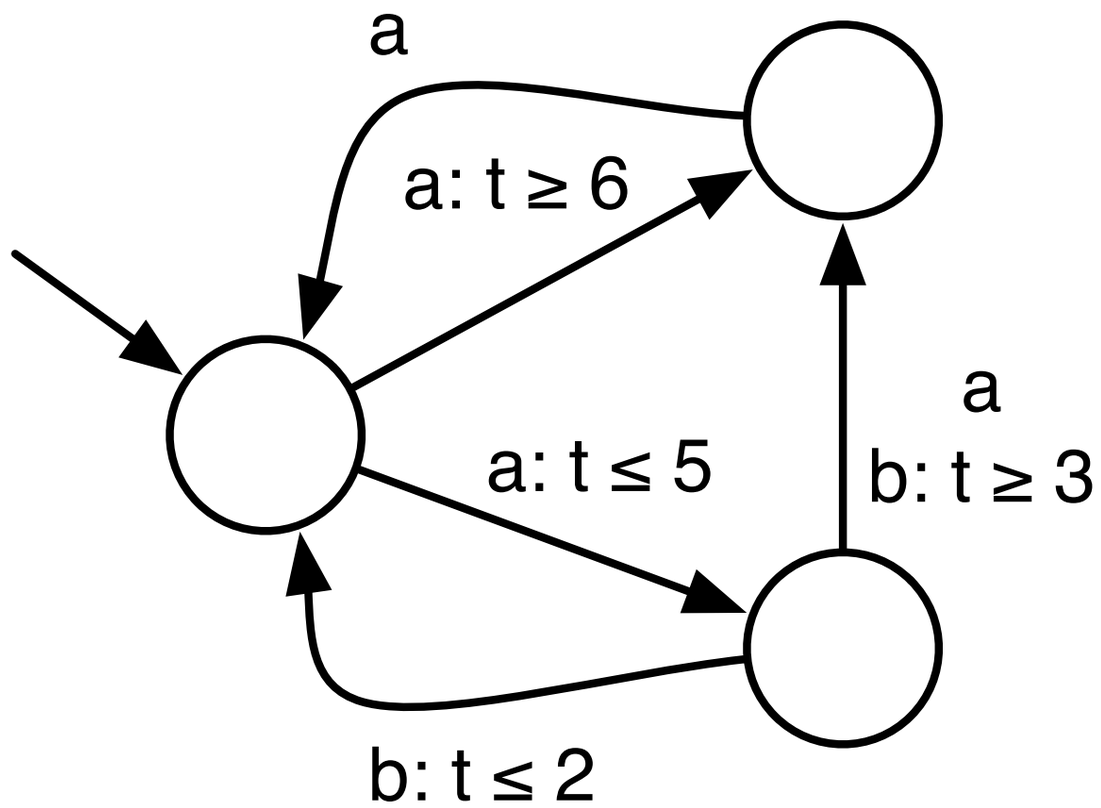
DRTAs and events



Produces timed strings:

(a,6)(a,2)(a,3); (a,2)(b,1)(a,1)(b,8)

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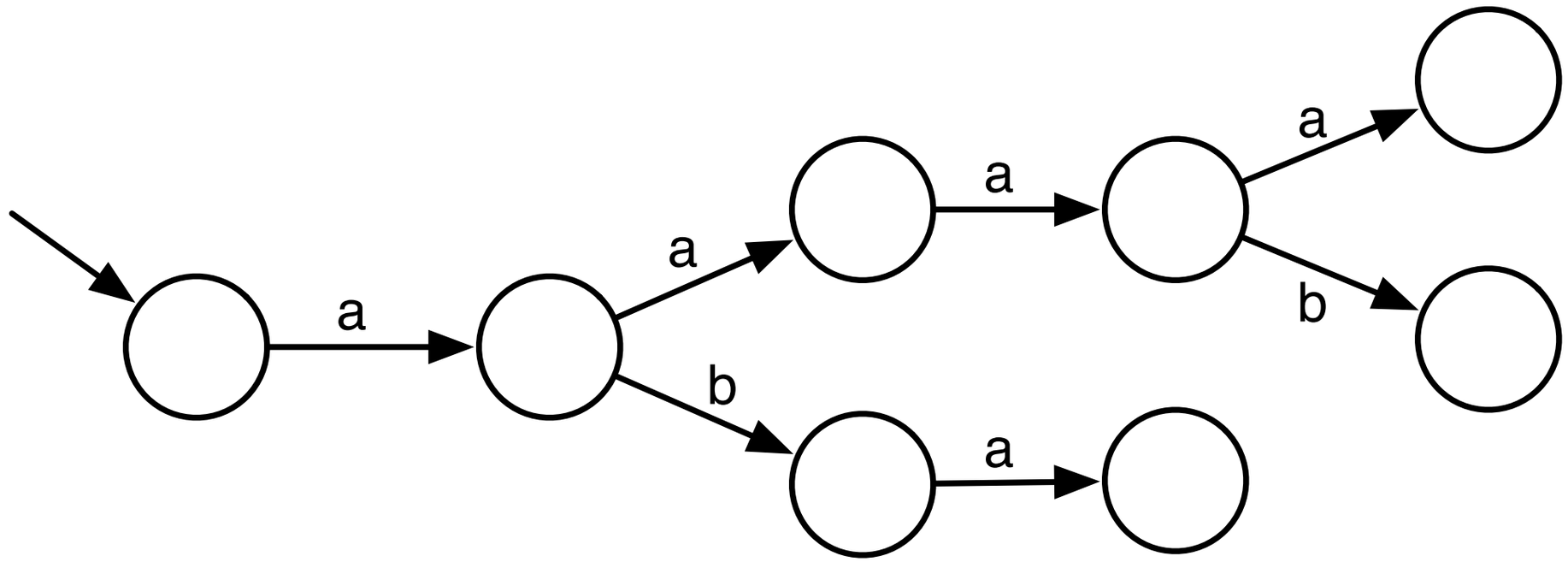
Learning DRTAs



A prefix tree
all guards are set to true, [0, ∞)

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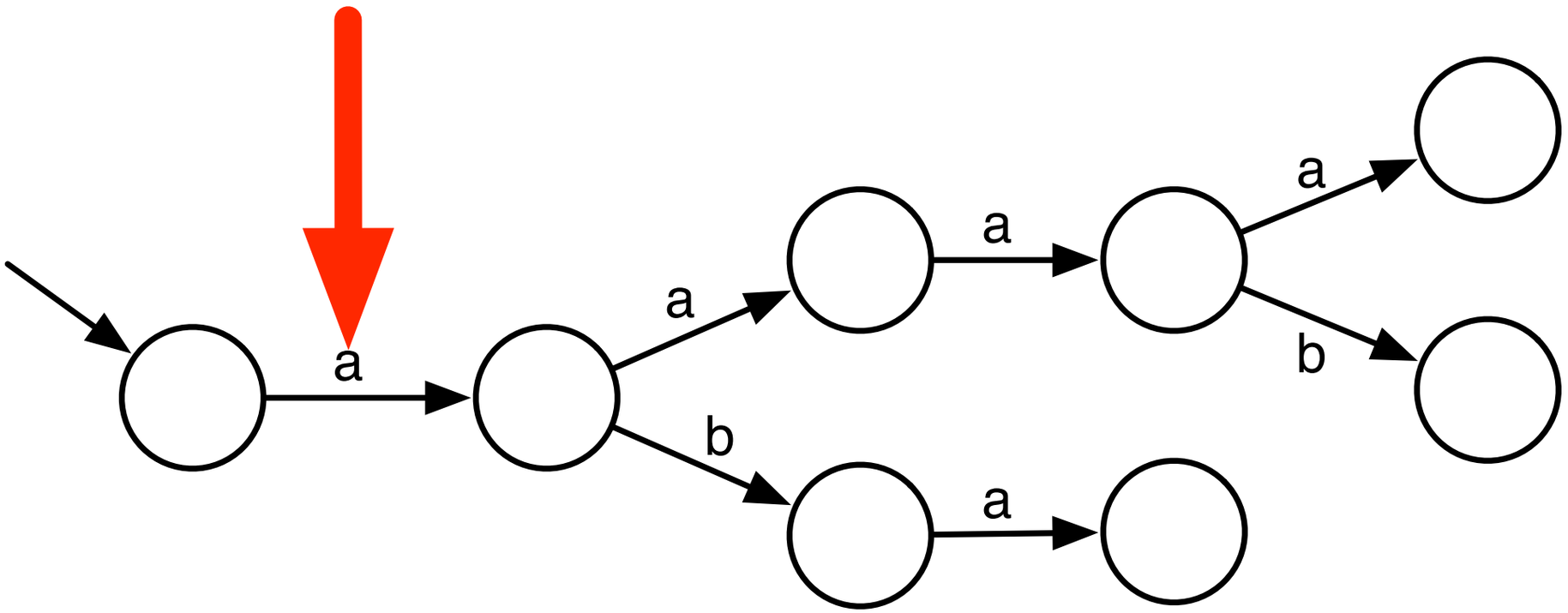
Learning DRTAs



Transition splitting:
choose a transition

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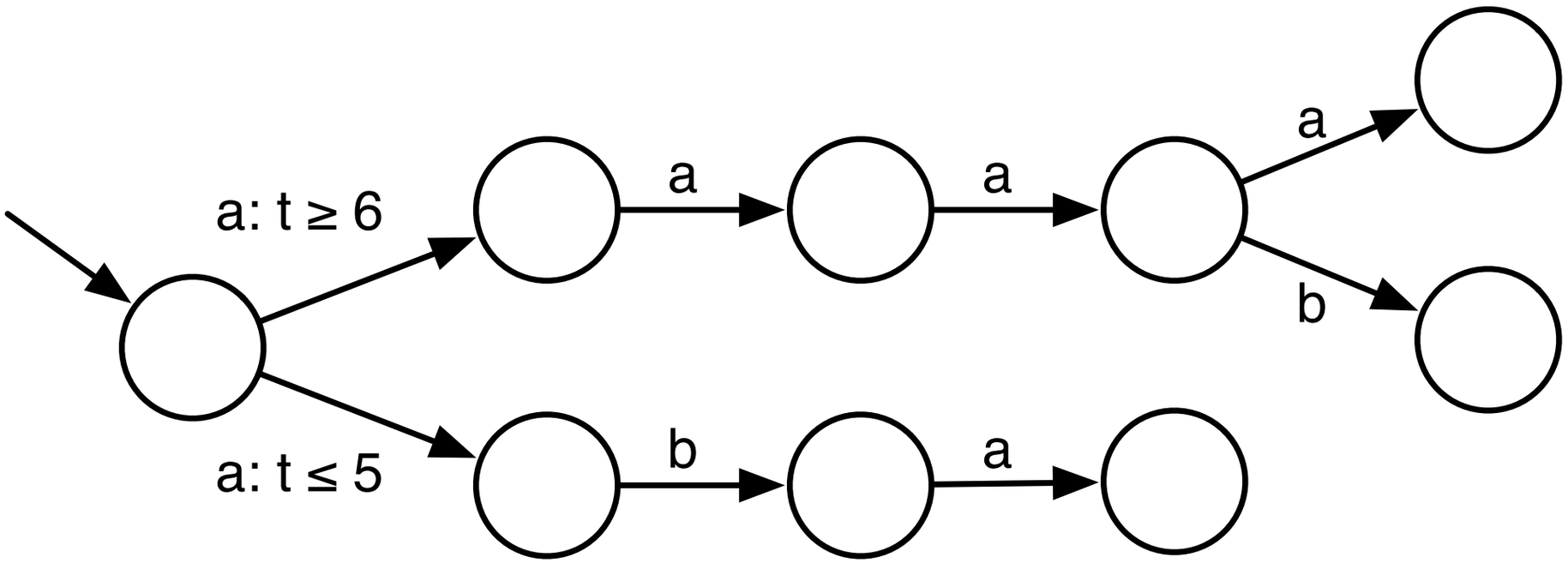

Split the transition and recalculate the subsequent part of the prefix tree

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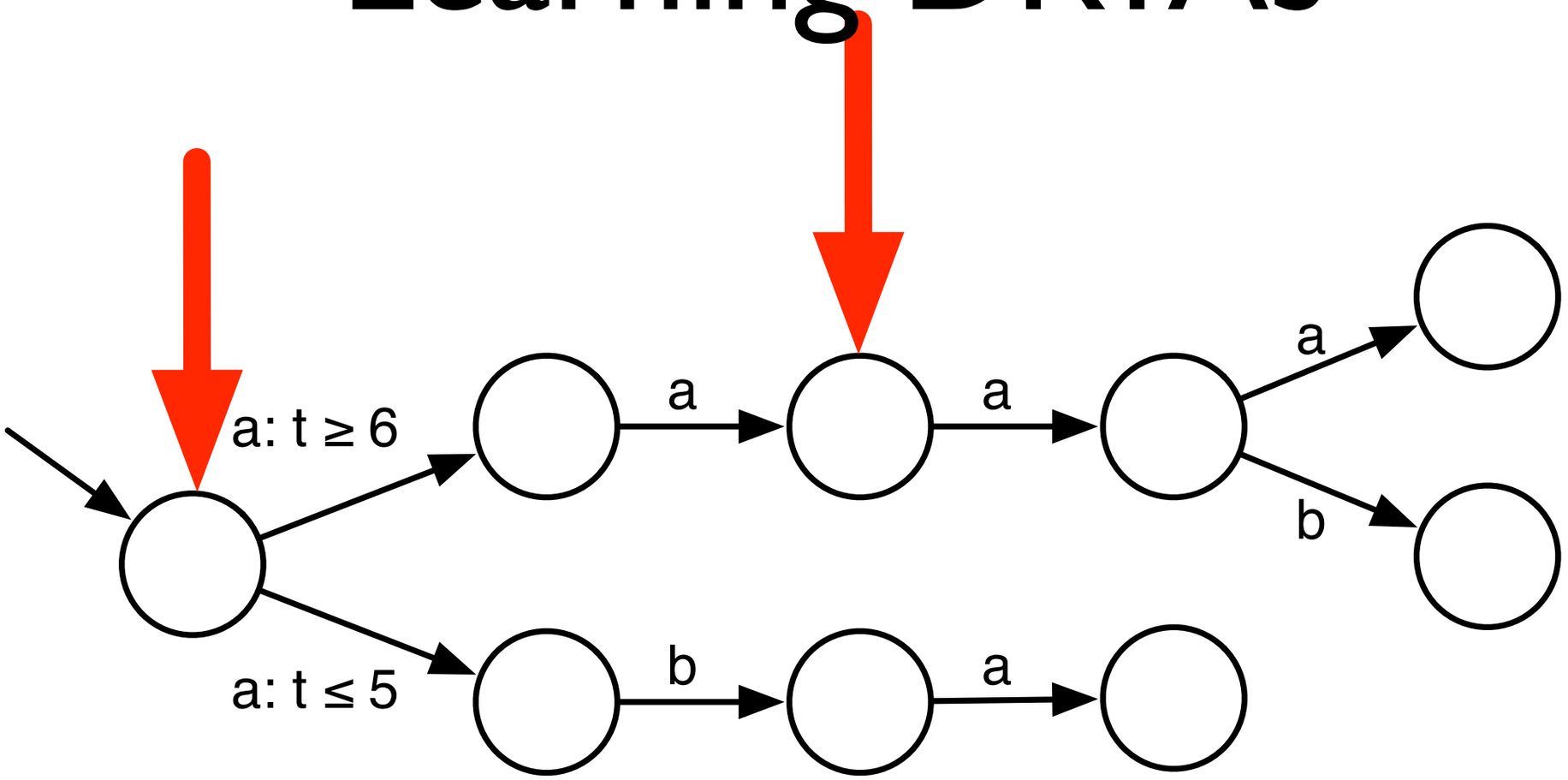

Later, when merging states

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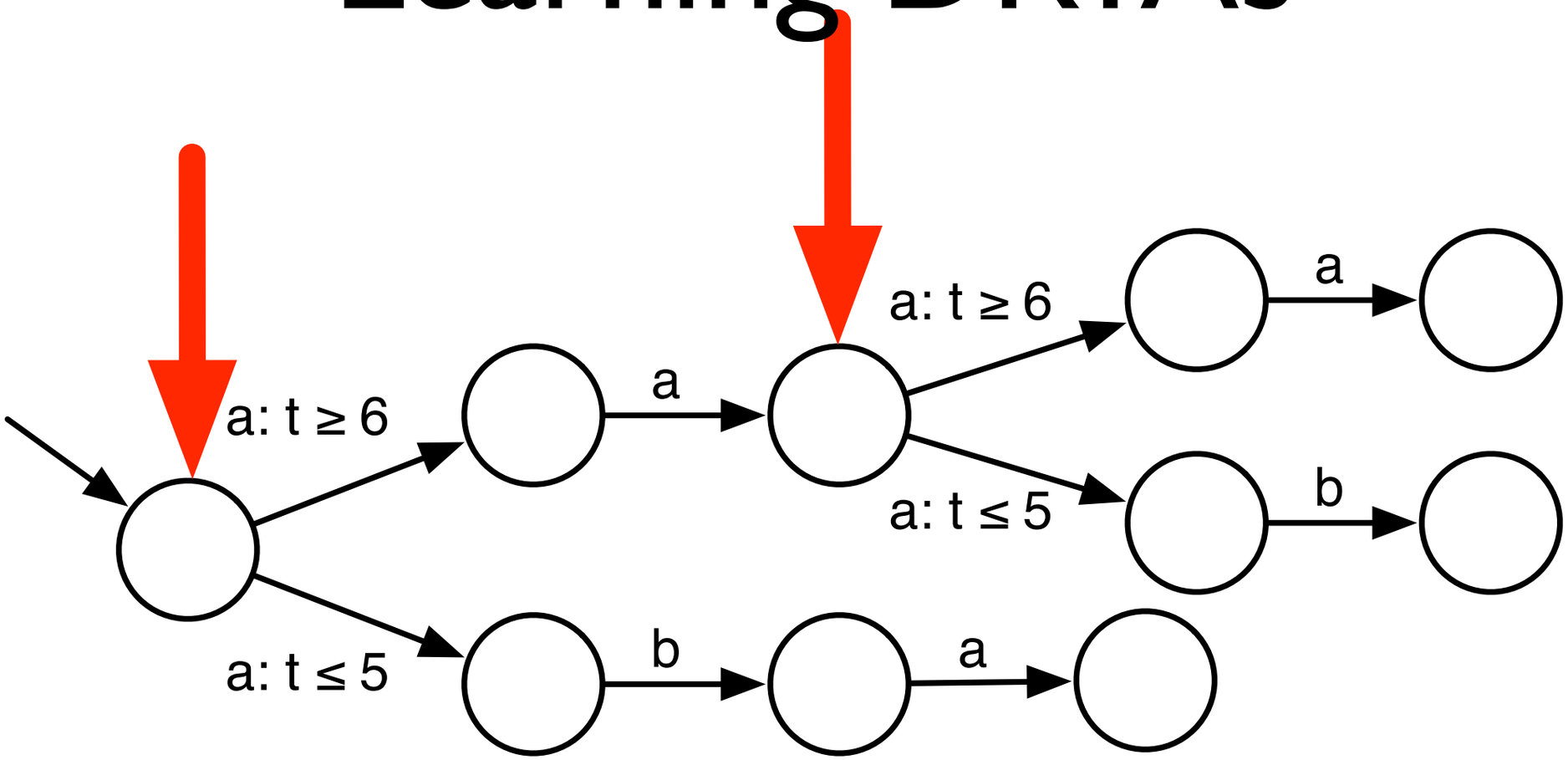
Learning DRTAs



First split the transitions such that the guards match

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Learning DRTAs

- State merging and transition splitting:
  - Start from a tree
  - Try all possible merges and splits
  - If one scores good:
    - Perform the one that scores best
  - Else
    - break
  - Iterate

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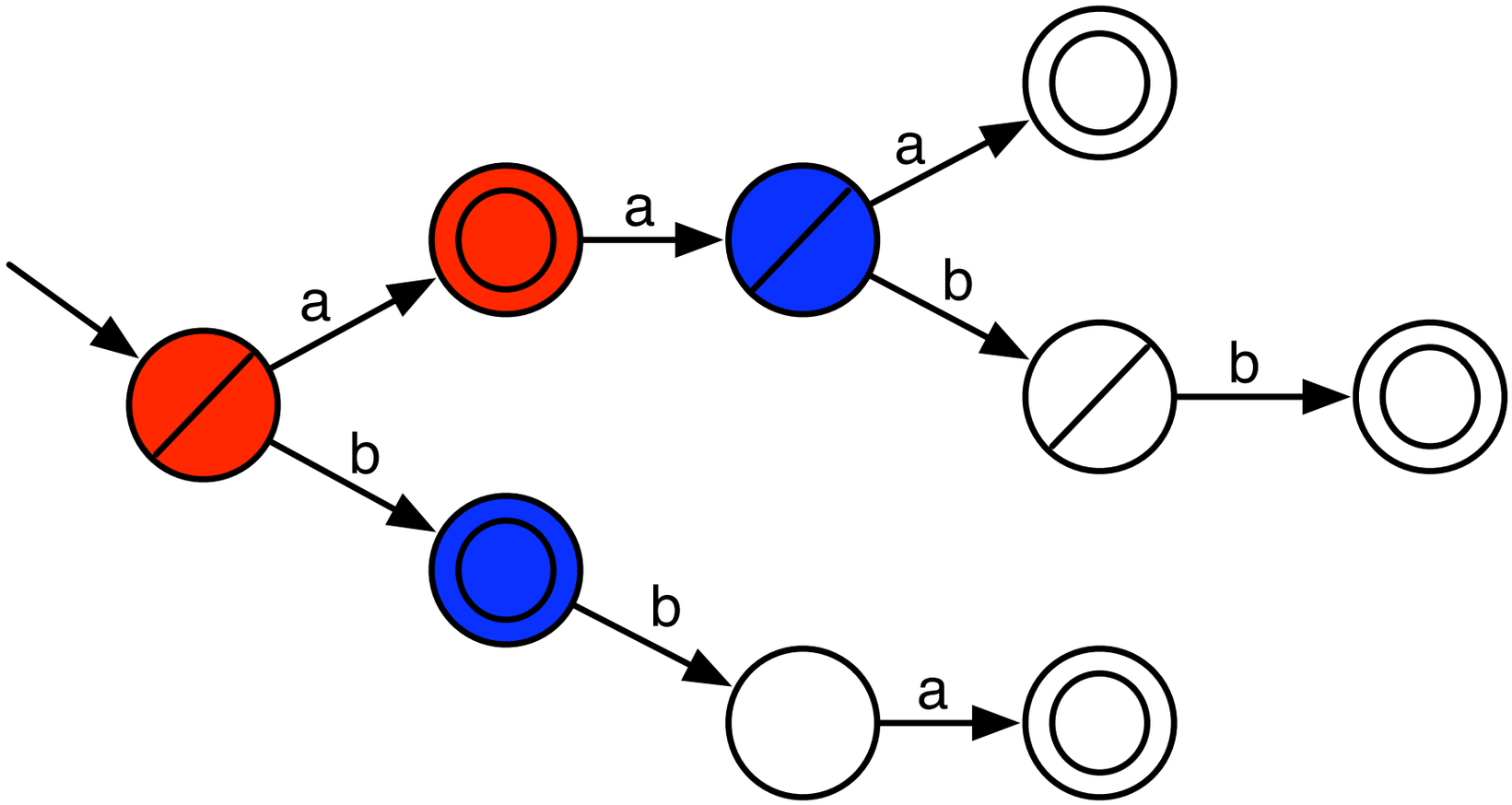
Labeled data

- Input: two sets S_+ and S_- of positive and negative timed strings
- Use the state merging and transition splitting algorithm, but
  - Allow merges between positive and negative states
  - Maintain a core of inferred states using the red-blue framework
  - Do not allow positive-negative merges in the core

Labeled data



A core or red states, with a fringe of blue states
only allow merges or red and blue states
only split transitions to blue states

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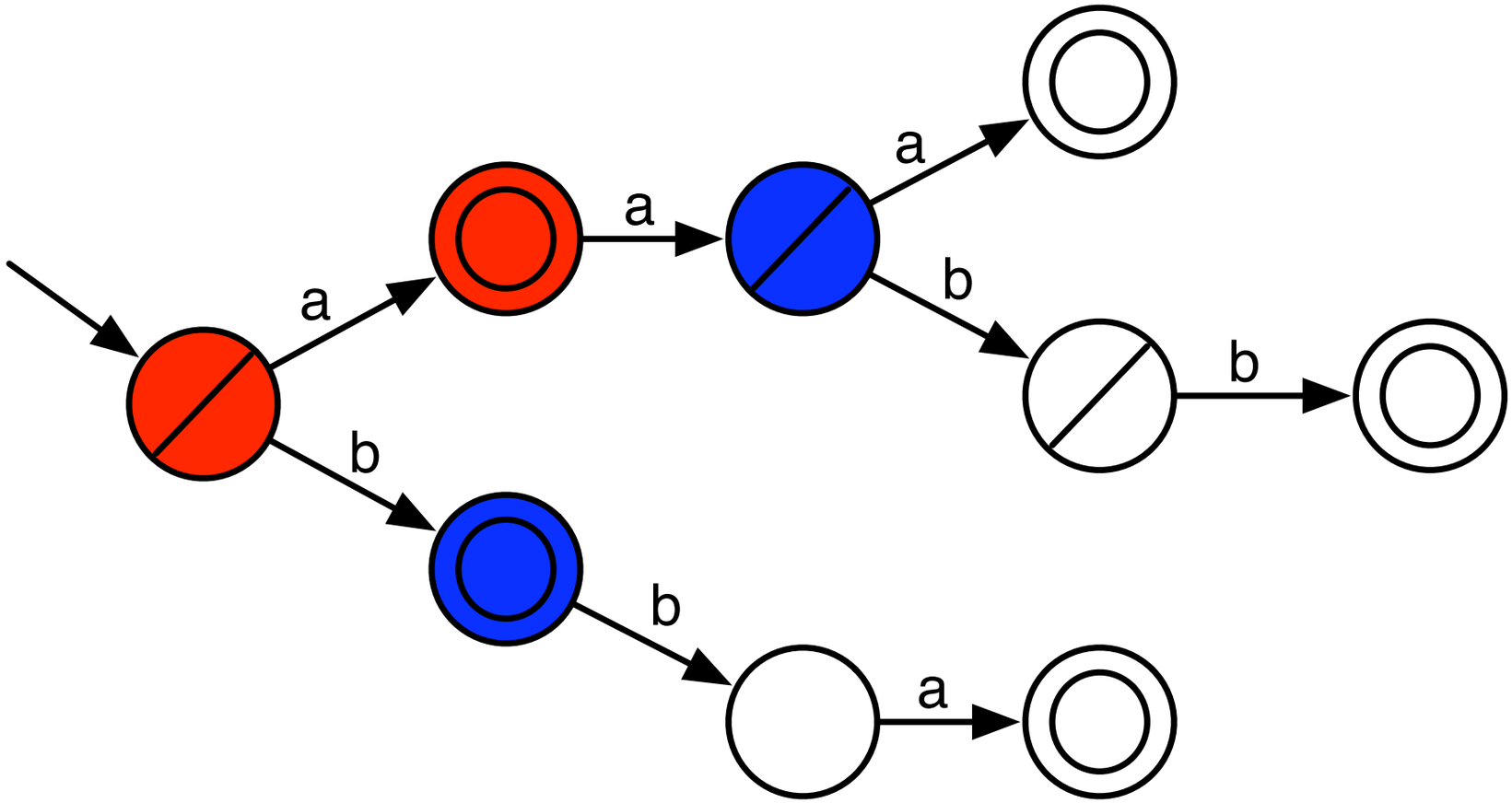
Labeled data



When a blue state cannot be merged or split,
this blue state is colored red

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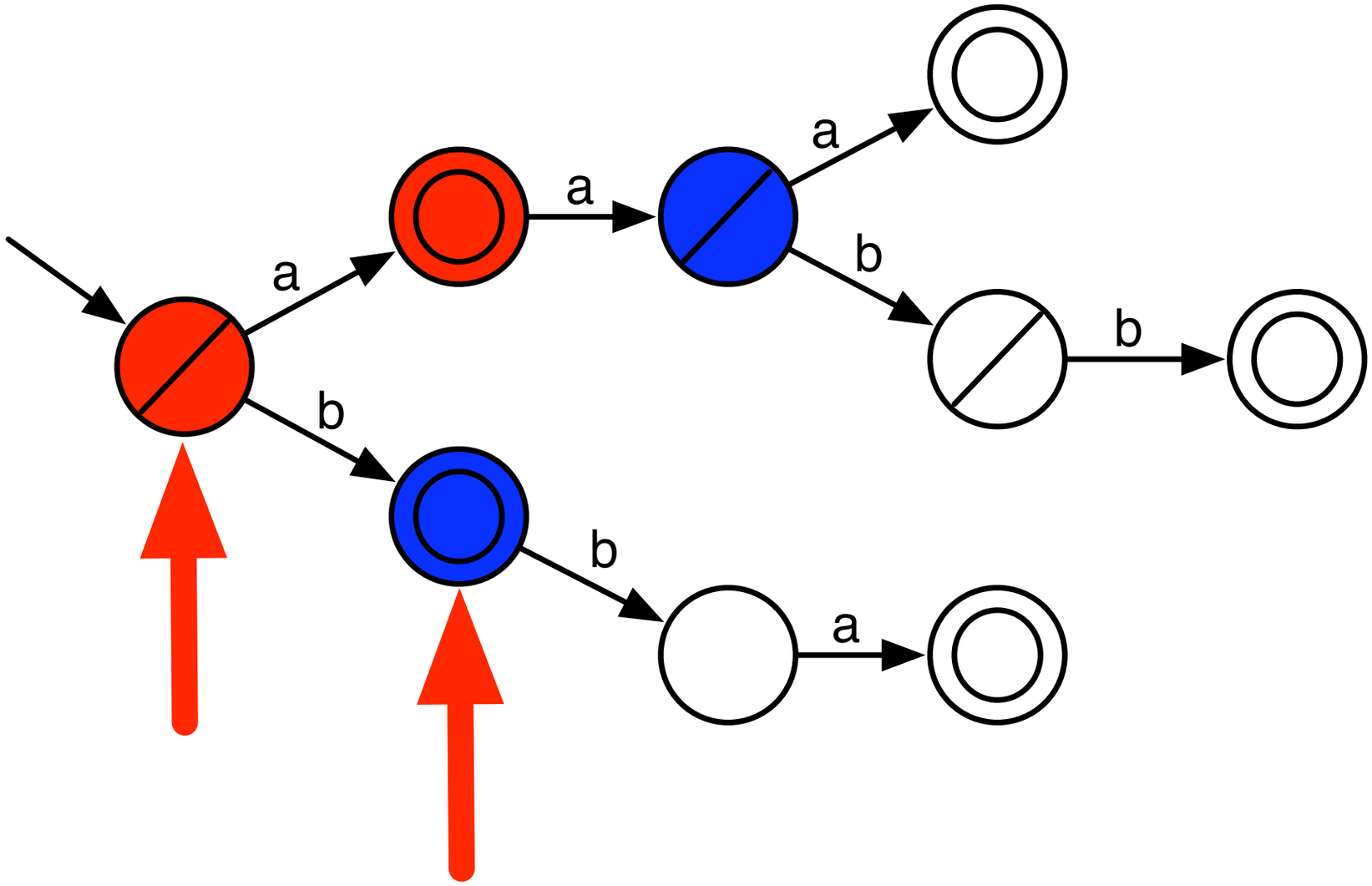
Labeled data



Results in a positive-negative merge in the core

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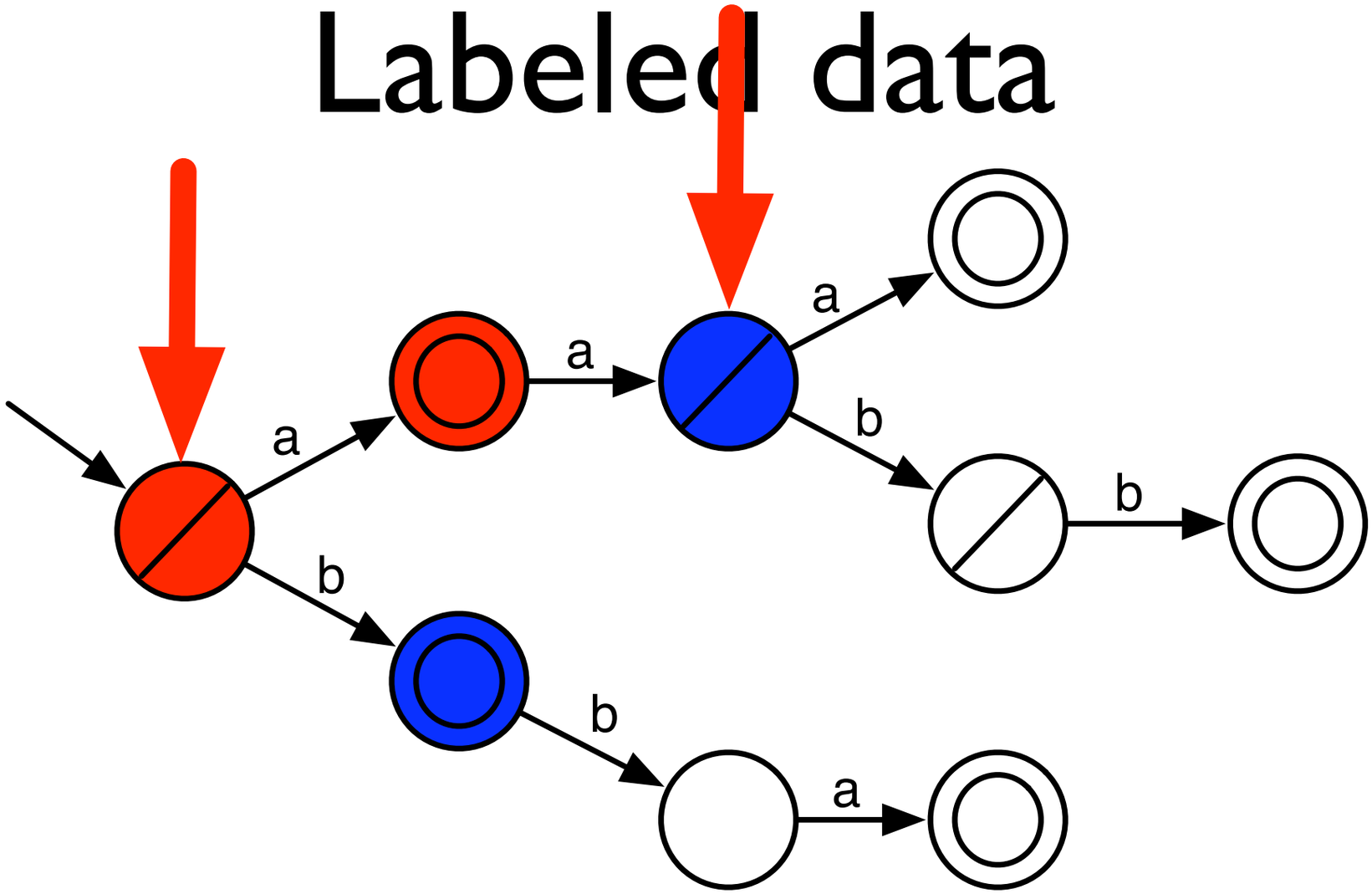
Labeled data



Does not results in a positive-negative merge in the core

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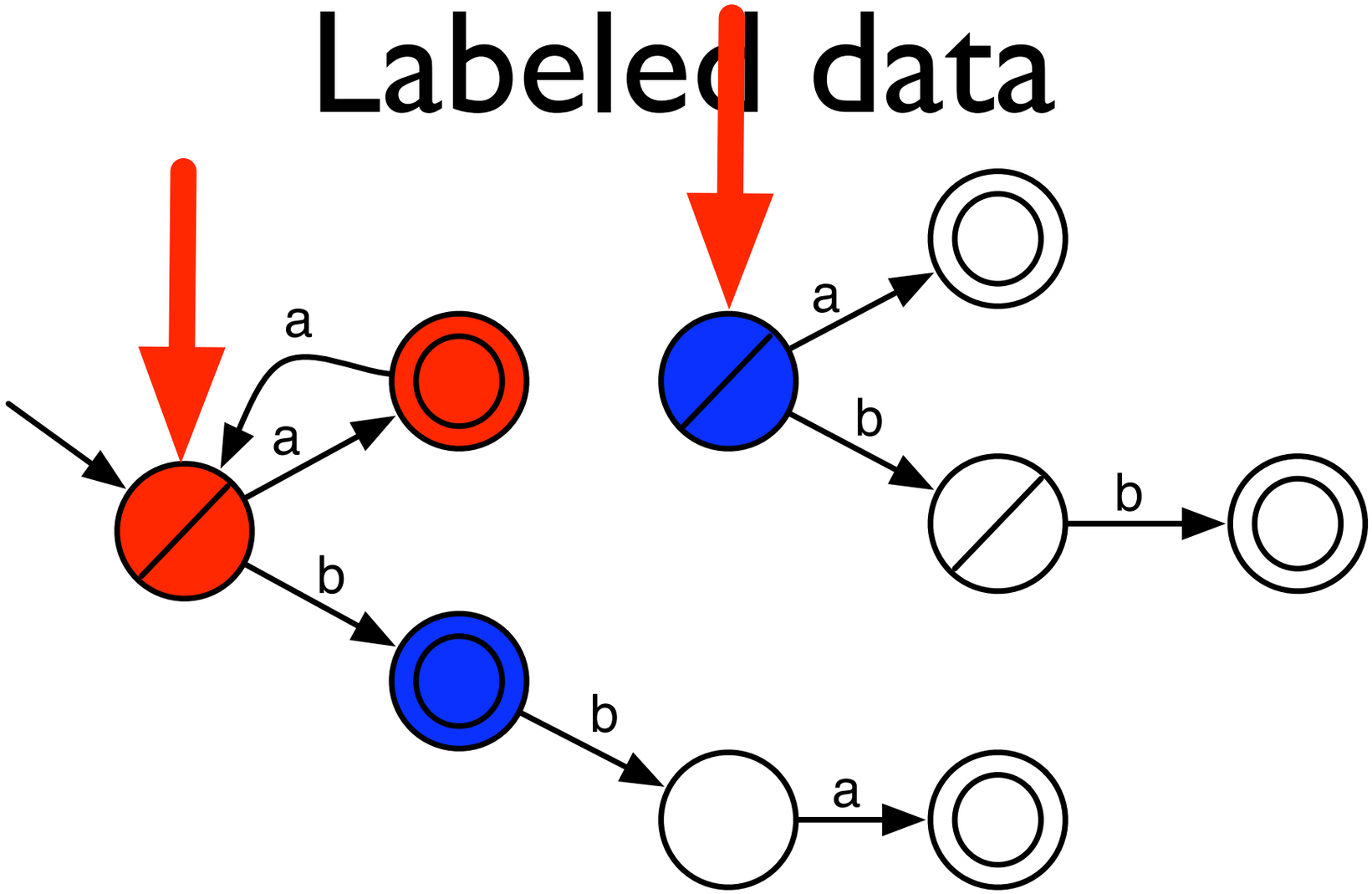
Labeled data



Does not results in a positive-negative merge in the core

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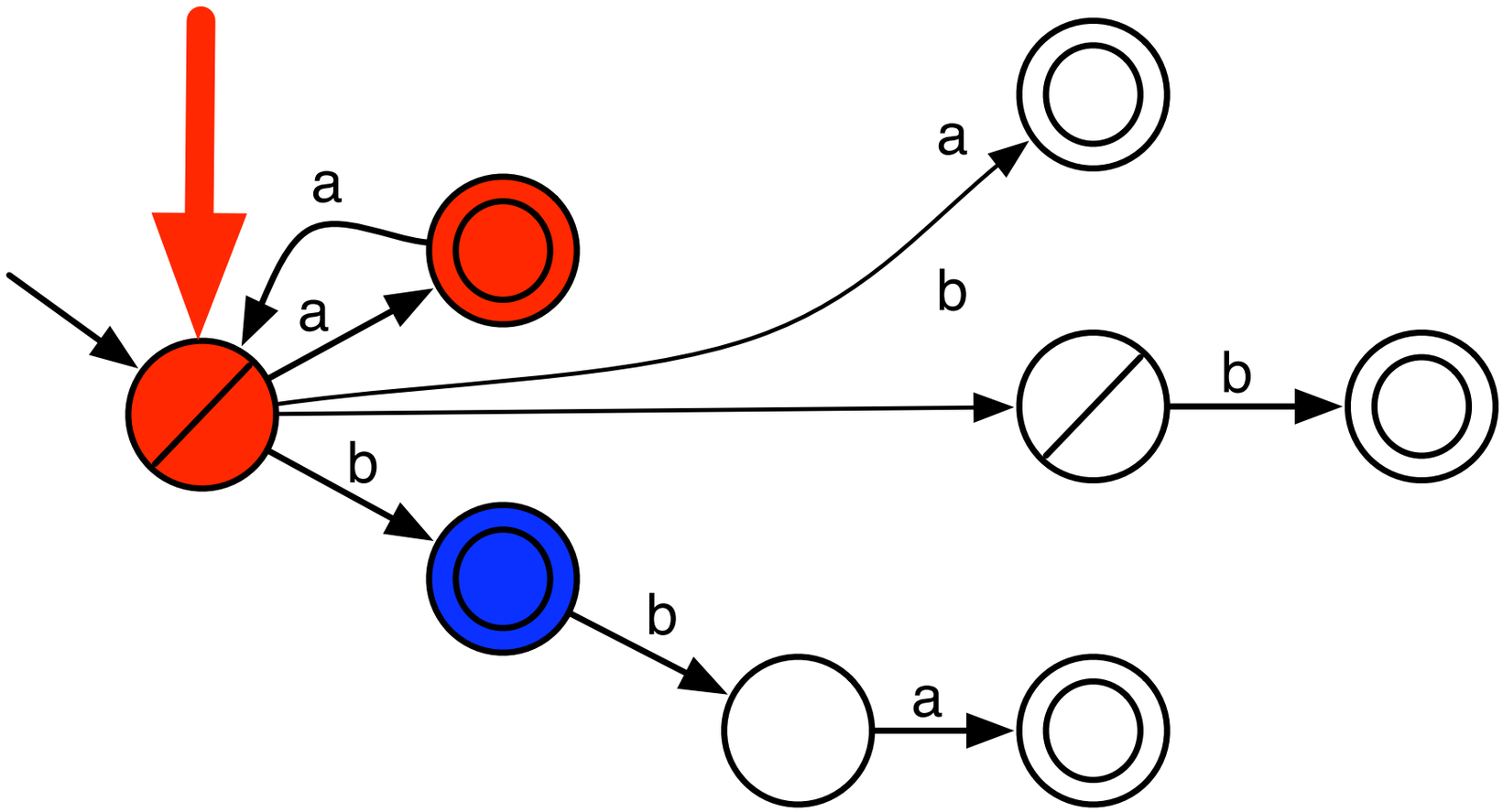

Does not results in a positive-negative merge in the core

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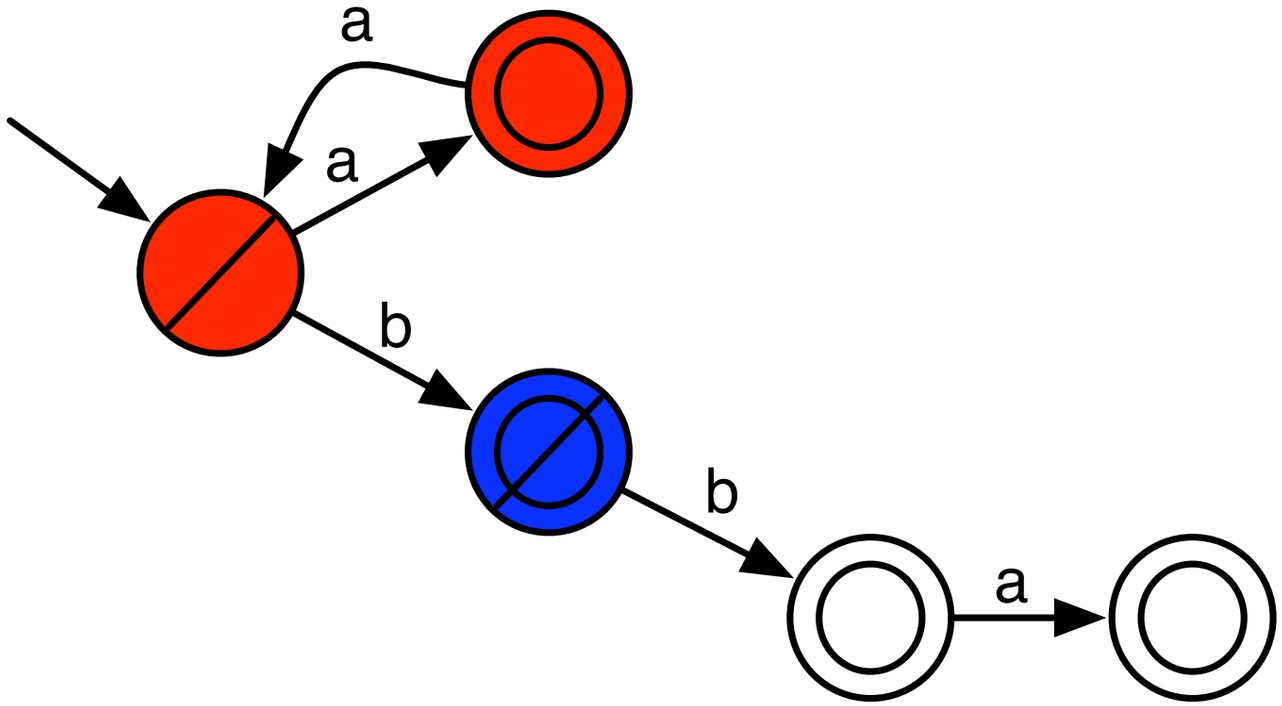

Does not results in a positive-negative merge in the core

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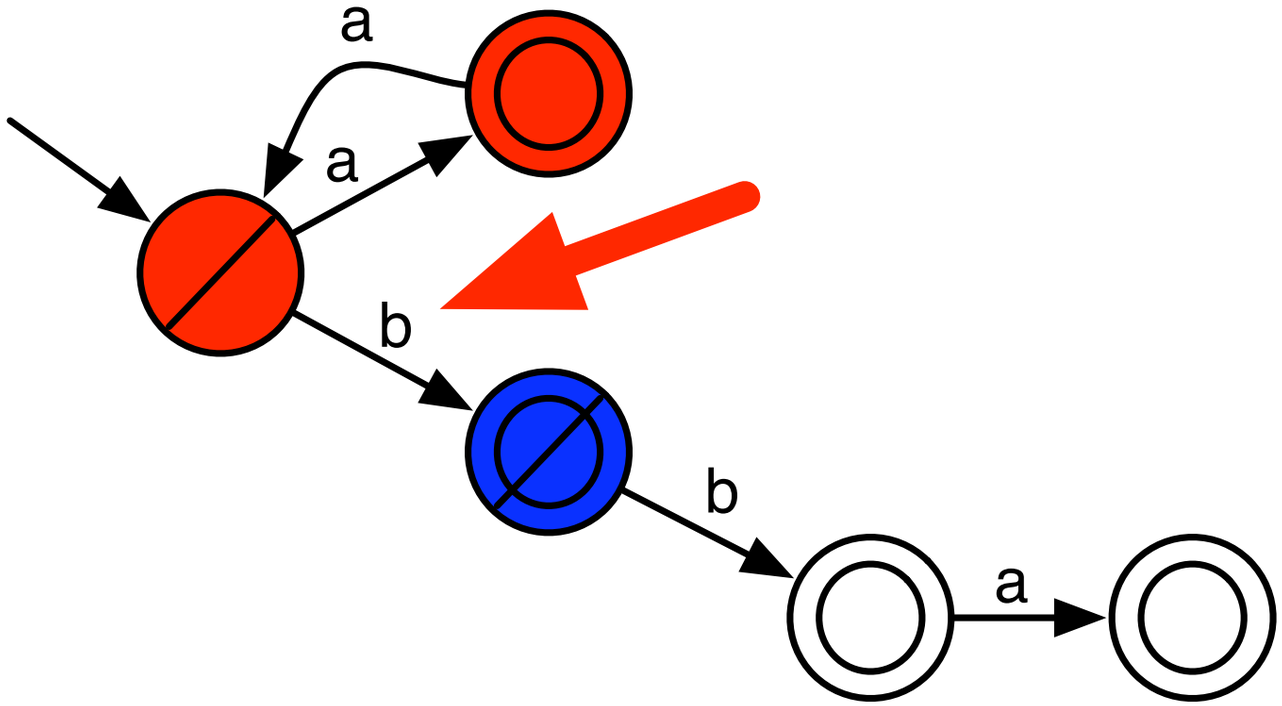

The positive-negative merge can be resolved using a split

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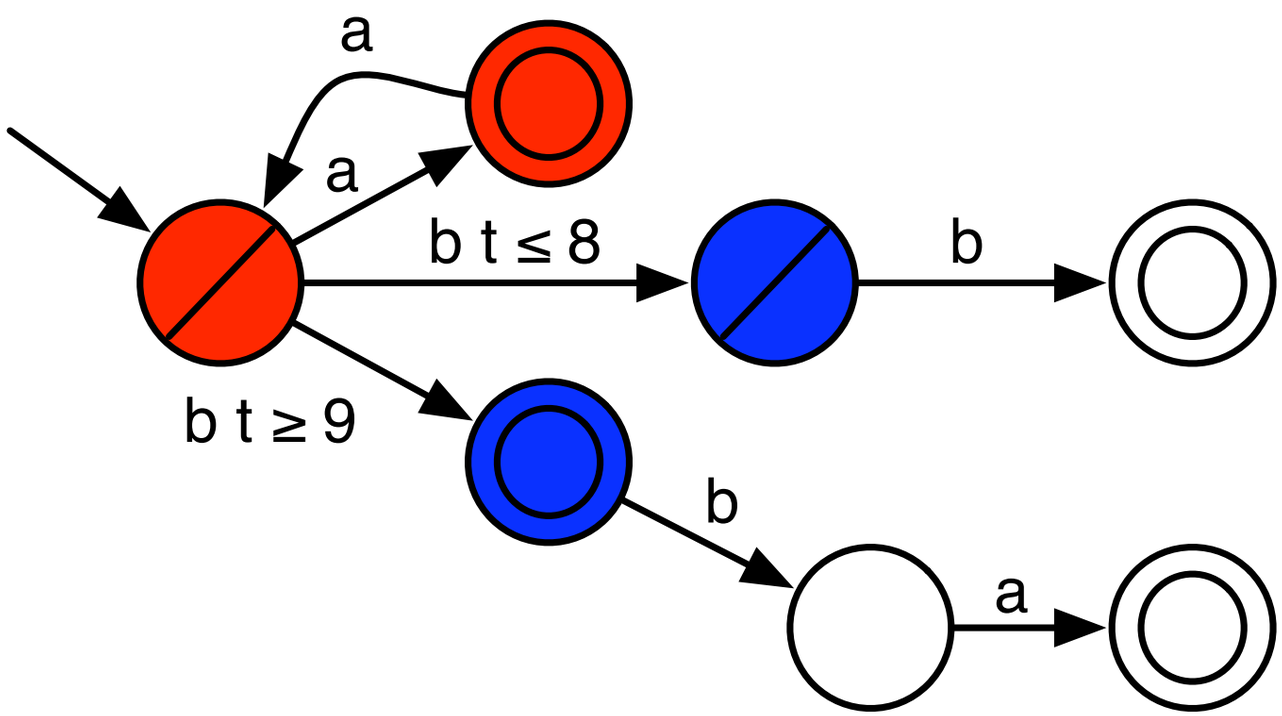
Labeled data



The positive-negative merge can be resolved using a split

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Labeled data

- Merges/splits can create/remove both consistent (pos-pos, neg-neg) and inconsistent (pos-neg) merges
- An inconsistent merge is only allowed if it can later be resolved using a split
- The inconsistent merges do not occur in the EDSM algorithm for DFAs, but provide useful information
- We adapt EDSM to make use this new information

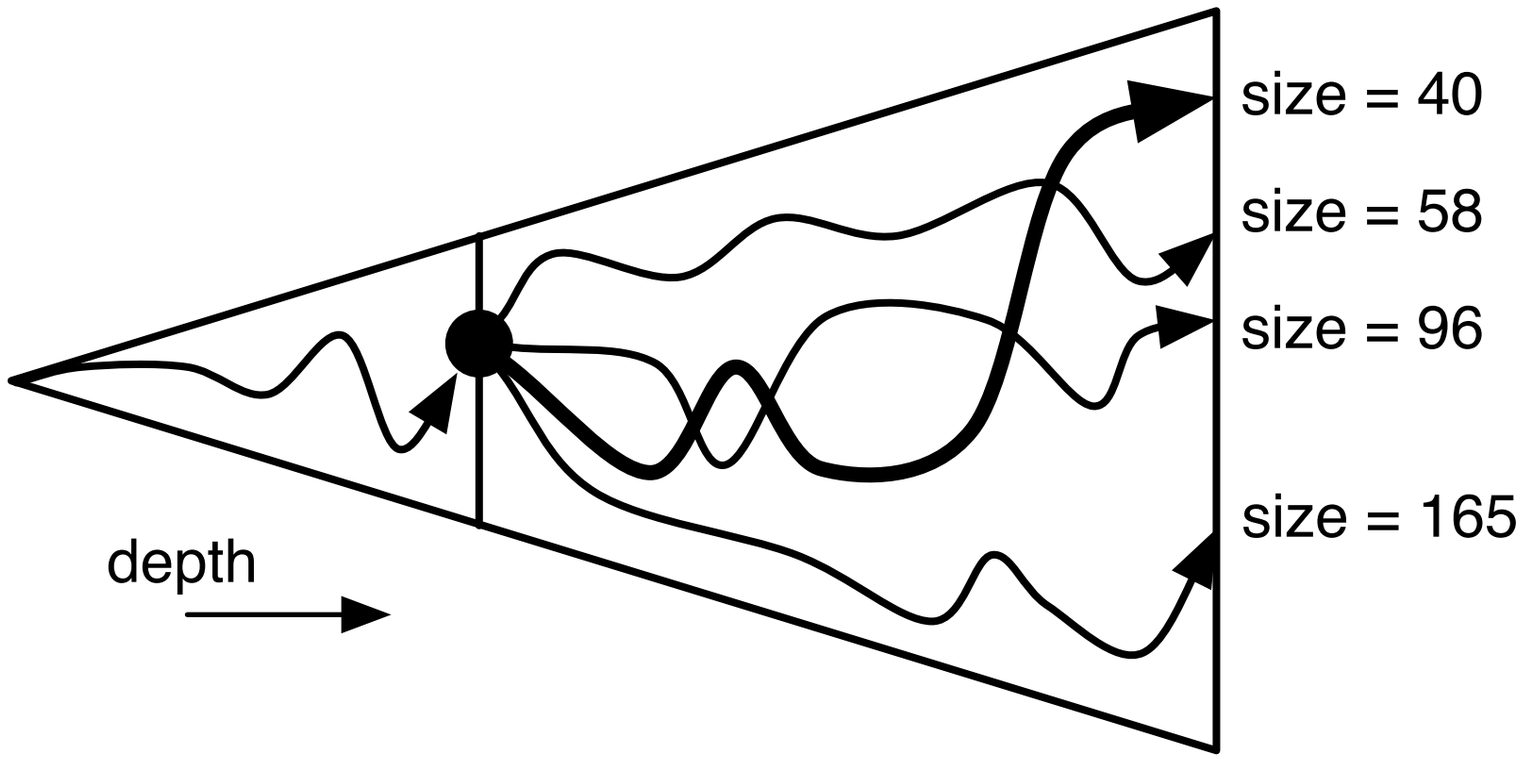
Labeled evidence values

- Consistent EDSM:
  - #consistent merges - # inconsistent merges
- Impact:
  - Weigh each merge by 1.0 - their resolving probability
  - ∑ consistent weights - ∑ inconsistent weights
- Split:
  - #consistent merges - #splits needed to resolve all inconsistent merges (approx.)

Sampled EDSM

- Replace every time value *t* in a timed string by *t*/*r* special time tick symbols, where *r* is a given constant
- Run the original EDSM algorithm on the transformed input, also using the red-blue framework
- The result is a DFA representation of the DRTA

Labeled search



Choose the split/merge that in the end (after a greedy run) results in the smallest PDRTA

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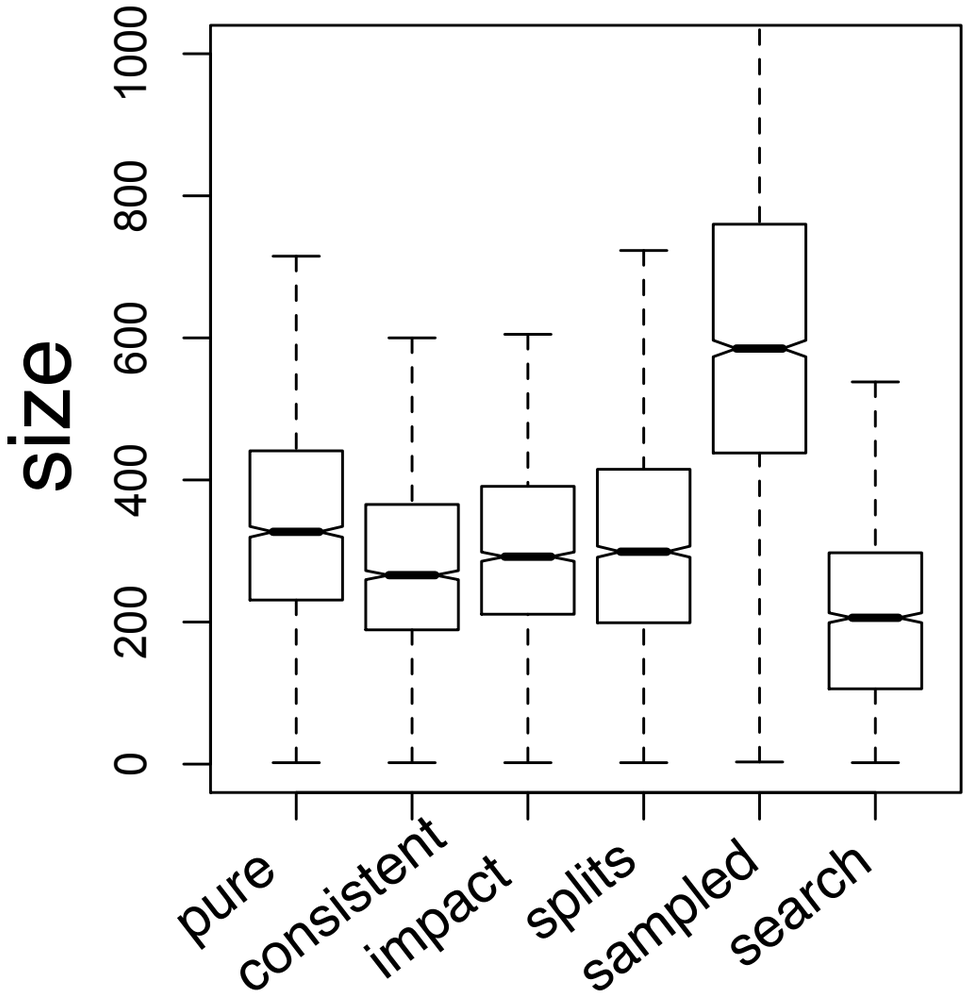
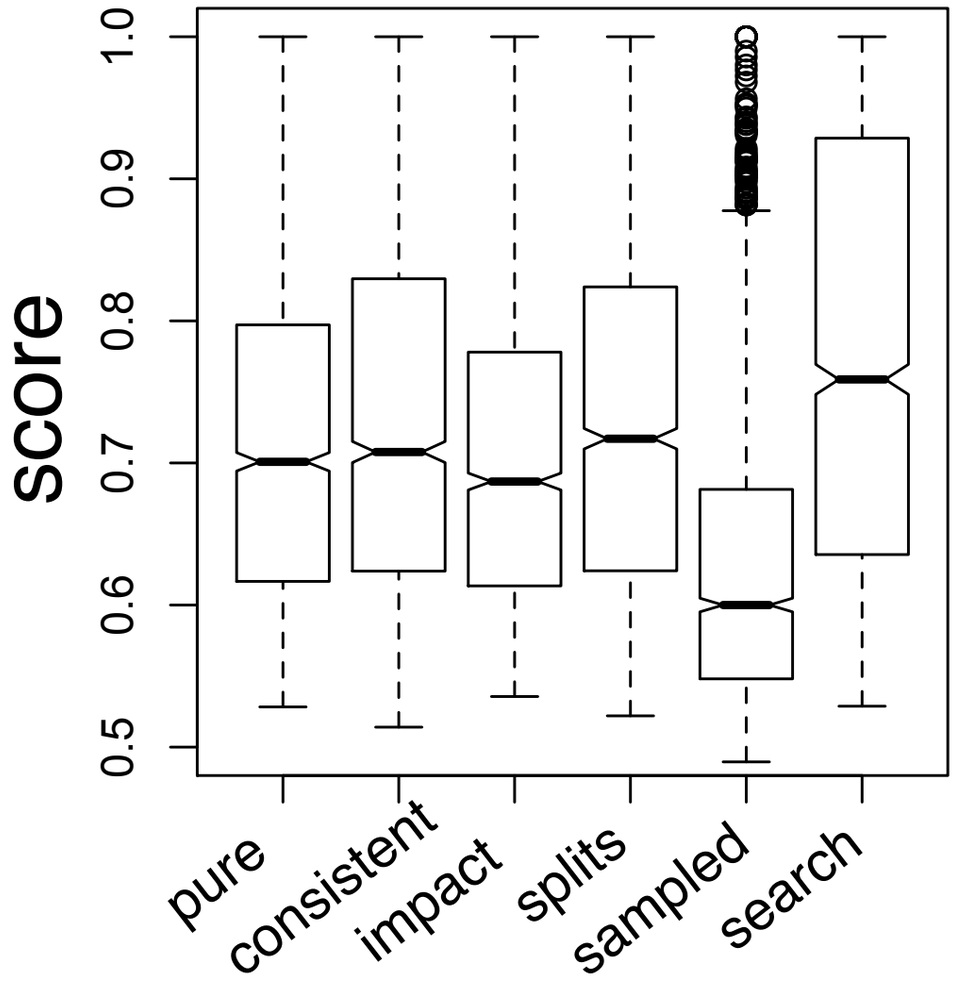
Results


size
0
200
400
600
800
1000
pure
consistent
impact
splits
sampled
search


score
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0.6
0.7
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1.0
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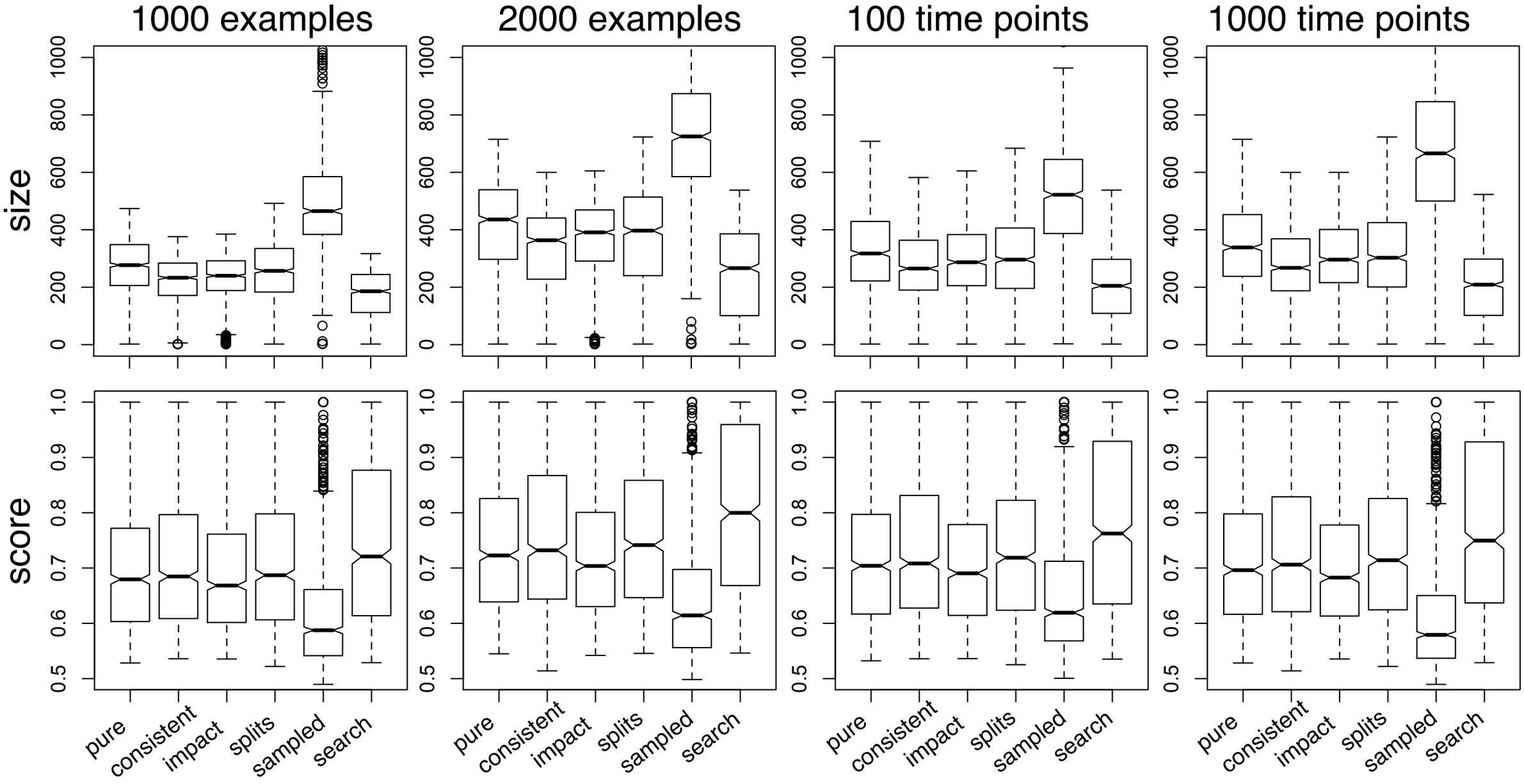
Results



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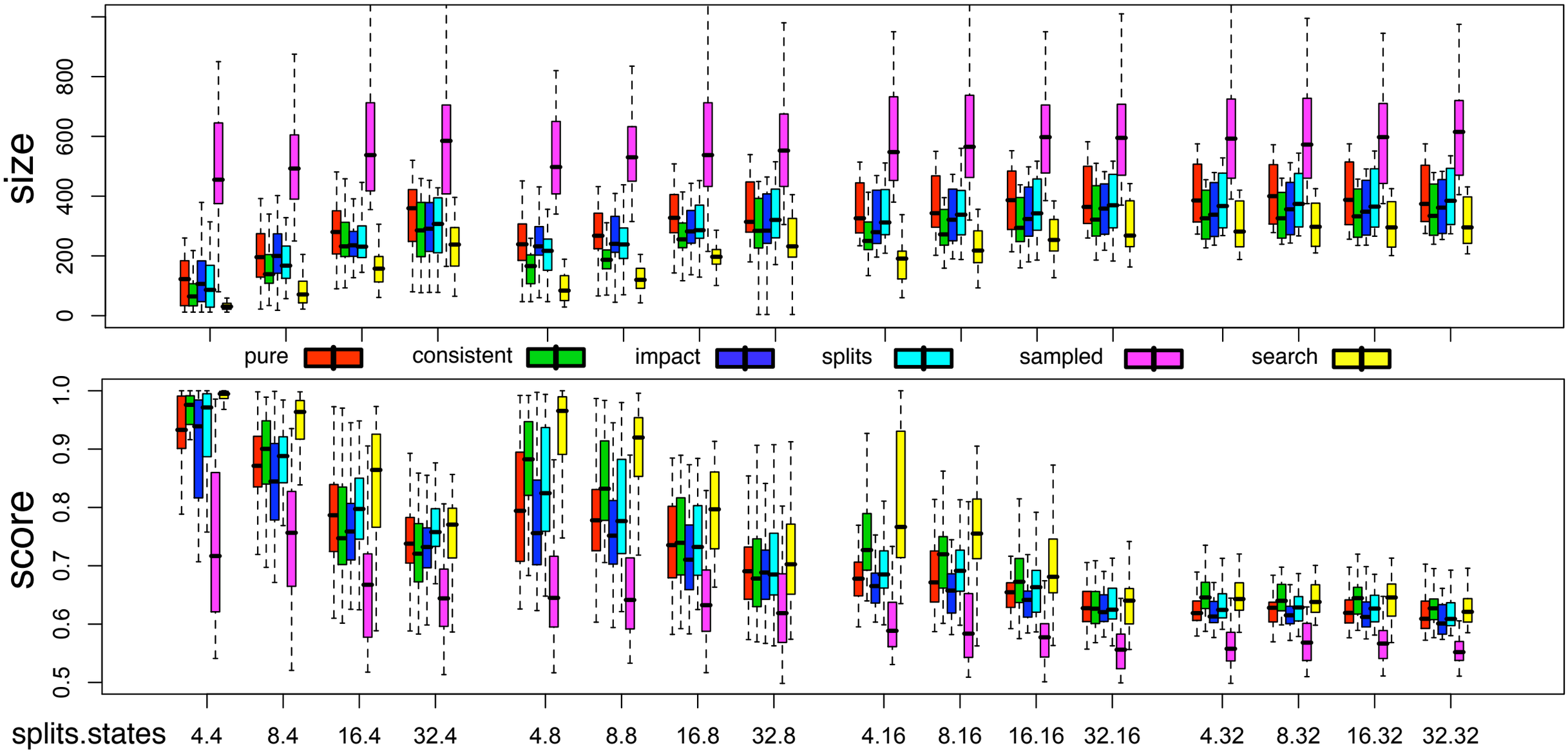
Results

alphabet size 4



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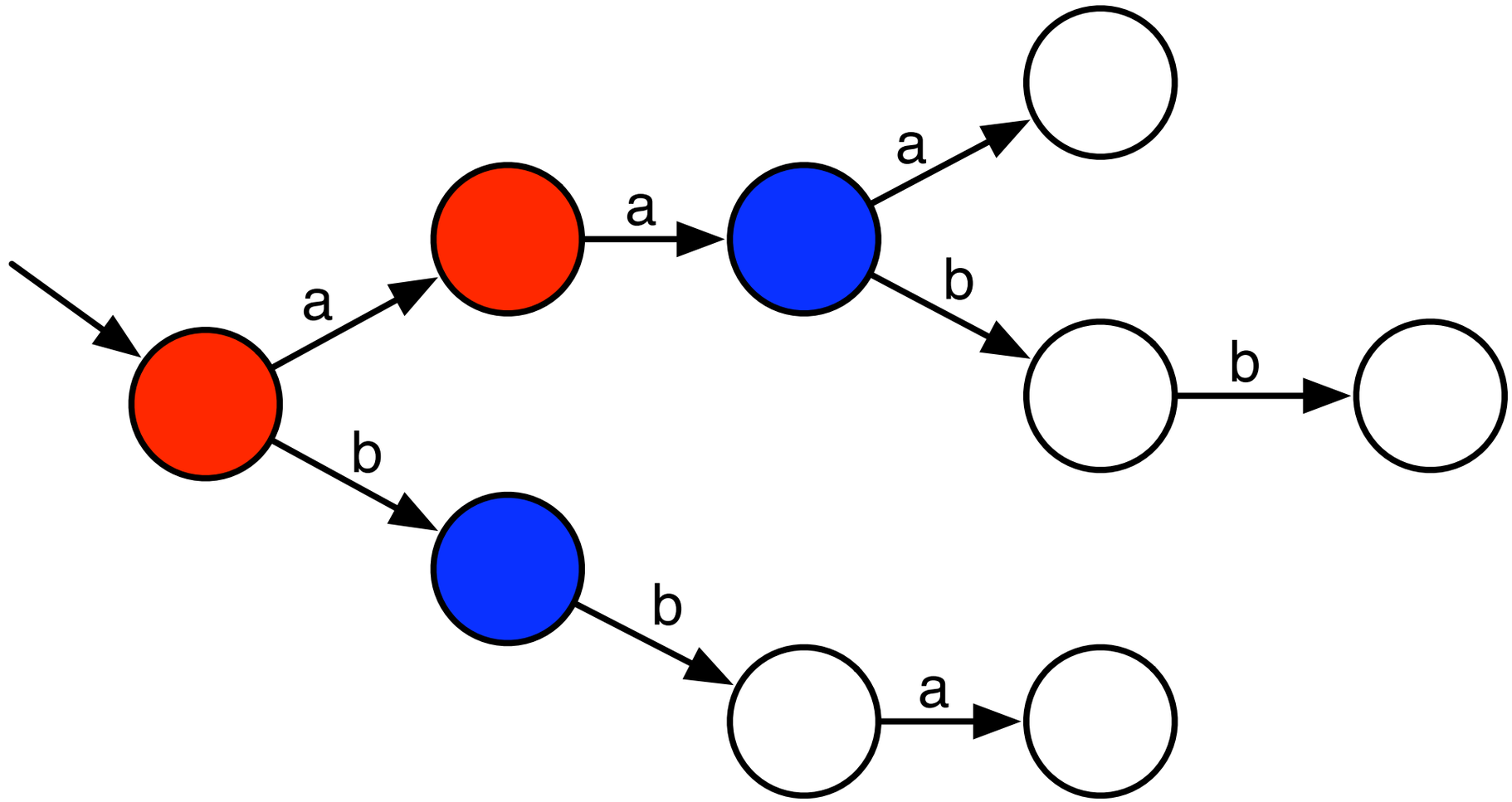
Unlabeled data

- Input: one sets S_+ of positive timed strings
- Occurs more often in practice
- Use the state merging and transition splitting algorithm, but
  - Maintain a core of inferred states using the red-blue framework
  - Only perform splits/merges of the transition/state that is visited by the most examples from S_+

Unlabeled data



There are only positive states

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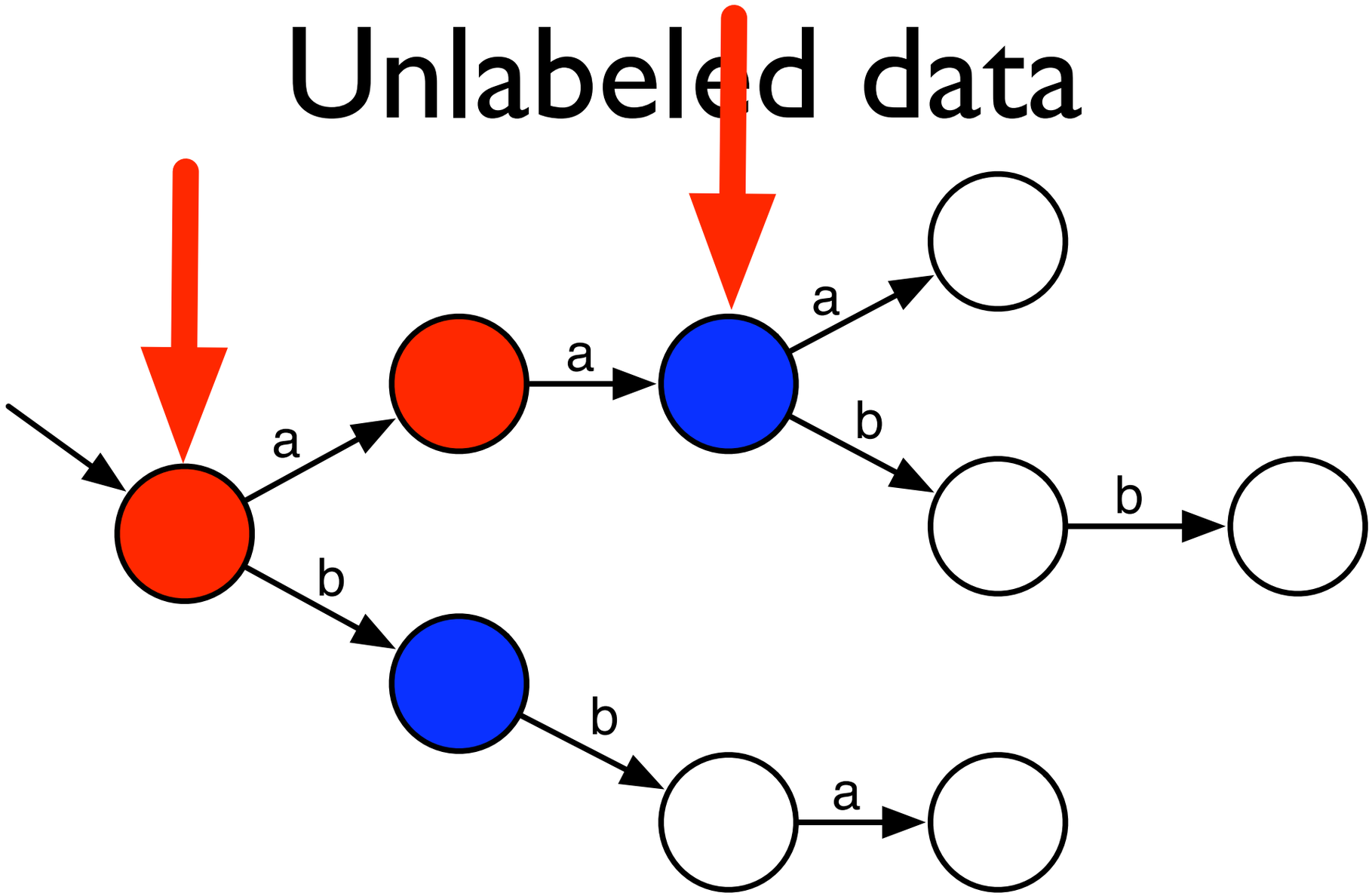
Unlabeled data



States should be merged if their future behavior is similar

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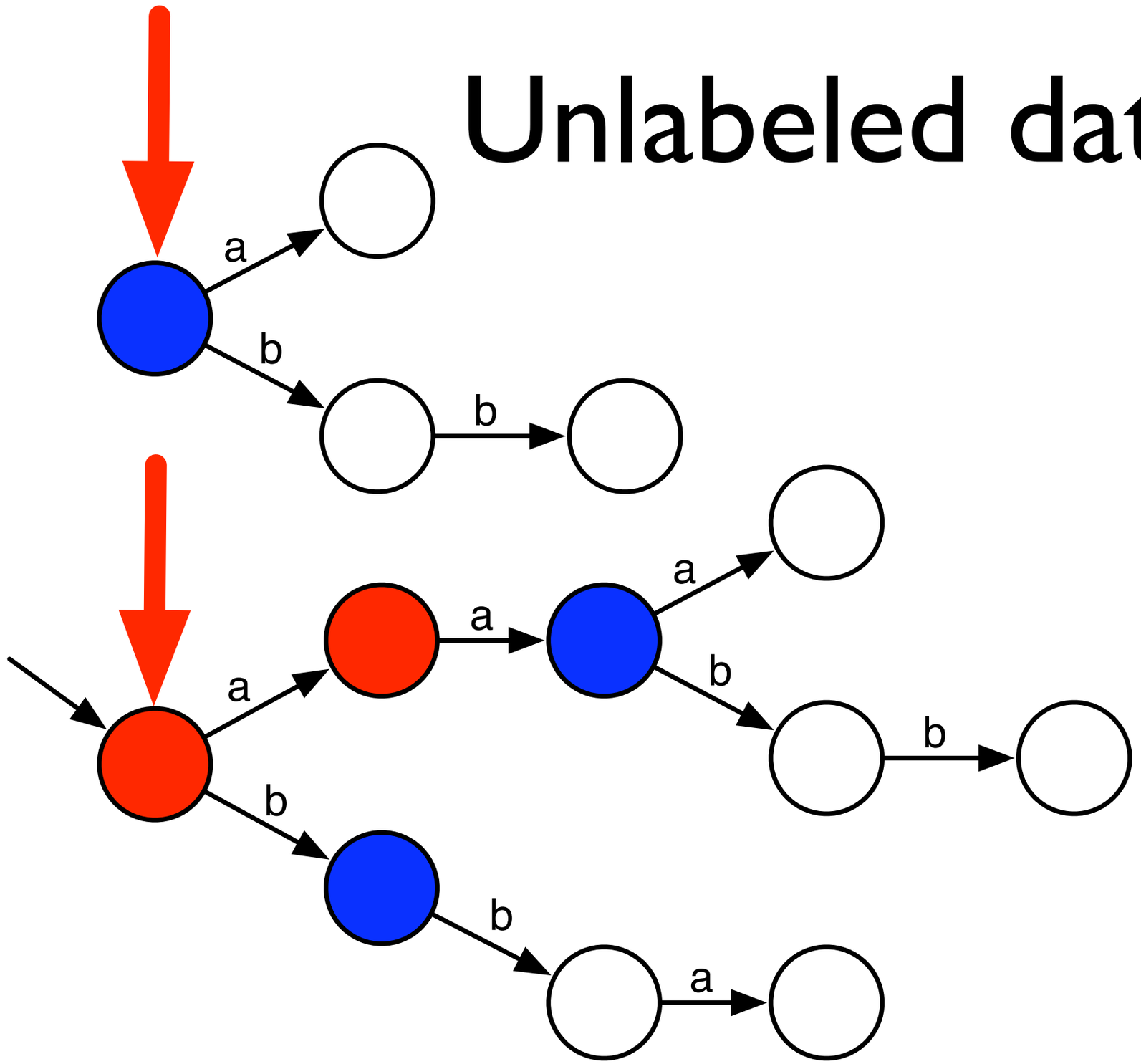
Unlabeled data



This behavior is determined by the trees rooted by the two states

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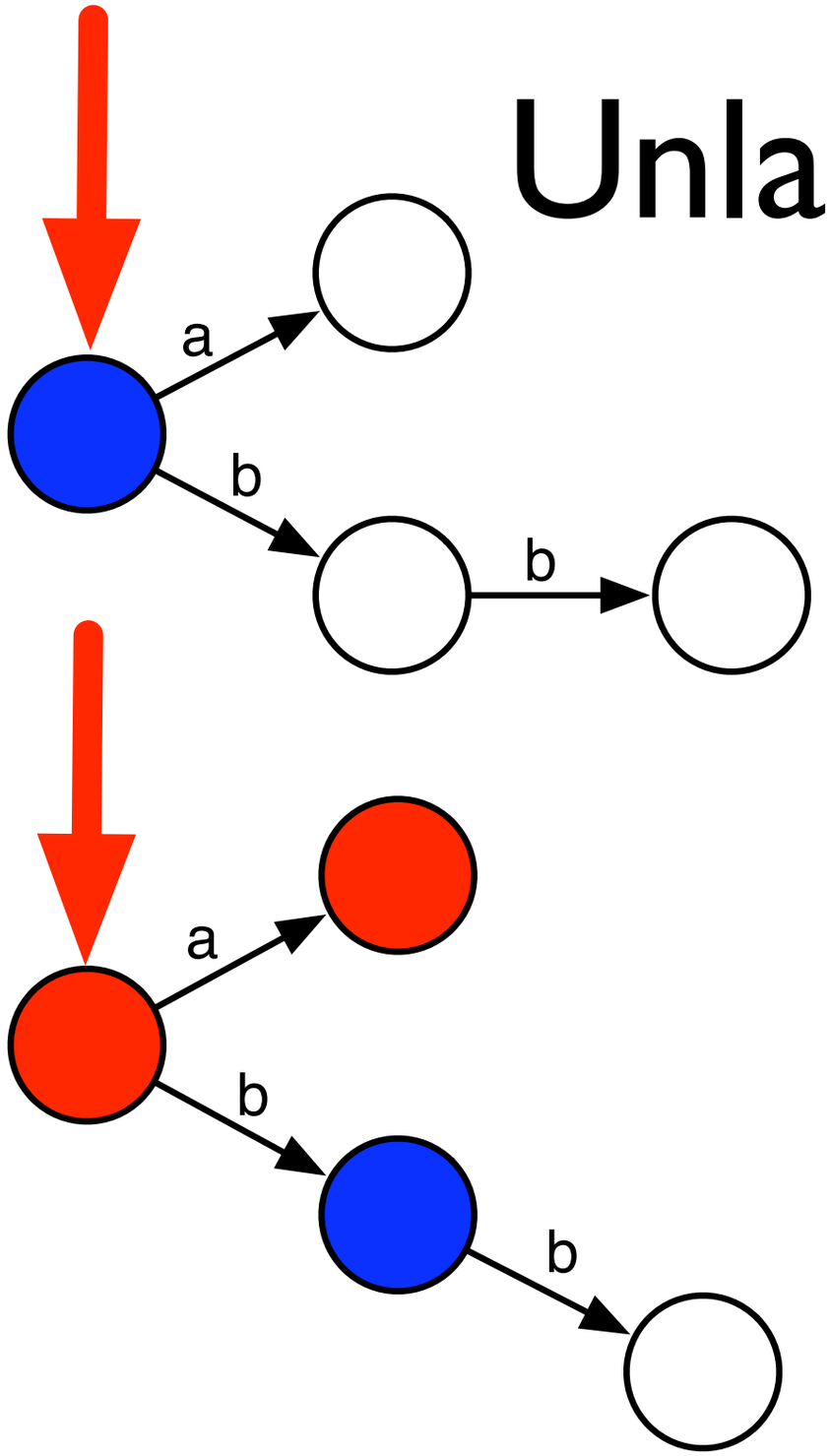
Unlabeled data



We are only interested in the overlapping behavior, i.e., the behavior for which we have data

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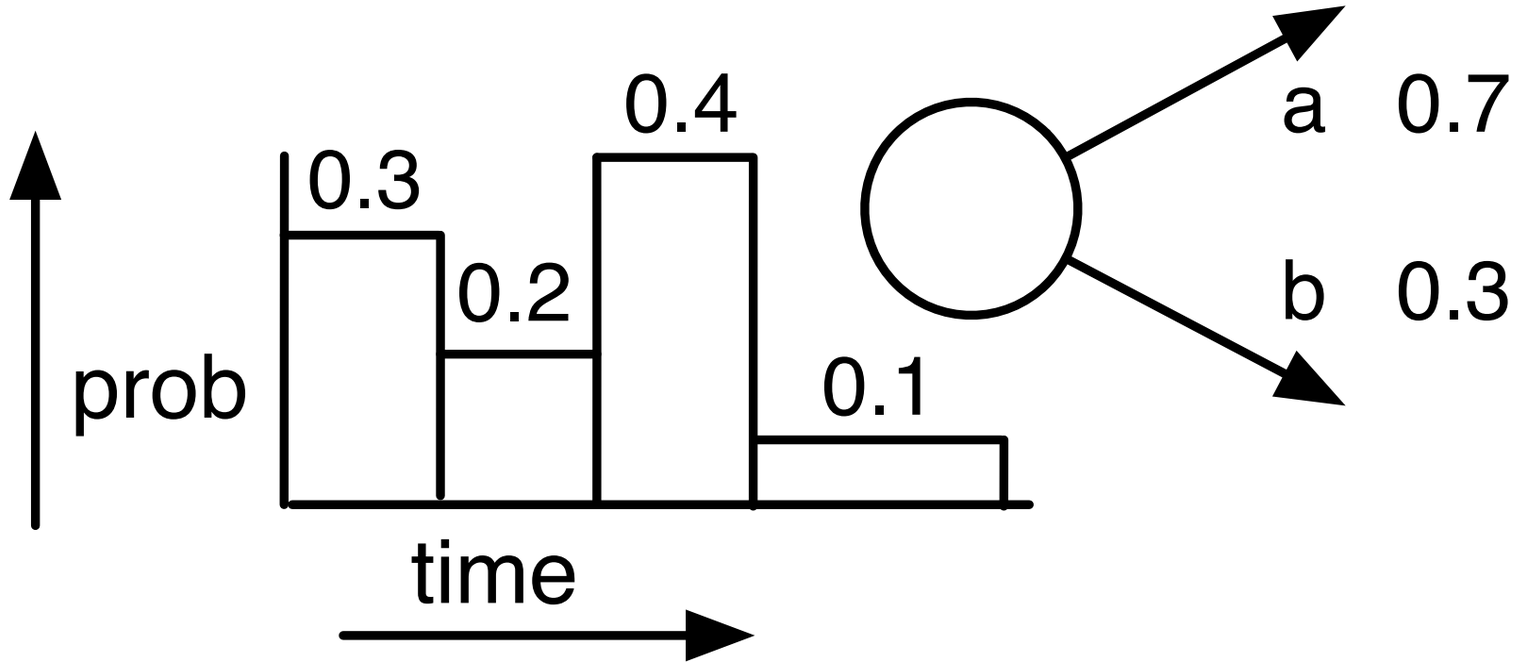
DRTA Distributions



By counting the occurrences of symbols, and bins of time points (forming a histogram) in S_+

Every state defines a distribution over timed symbols

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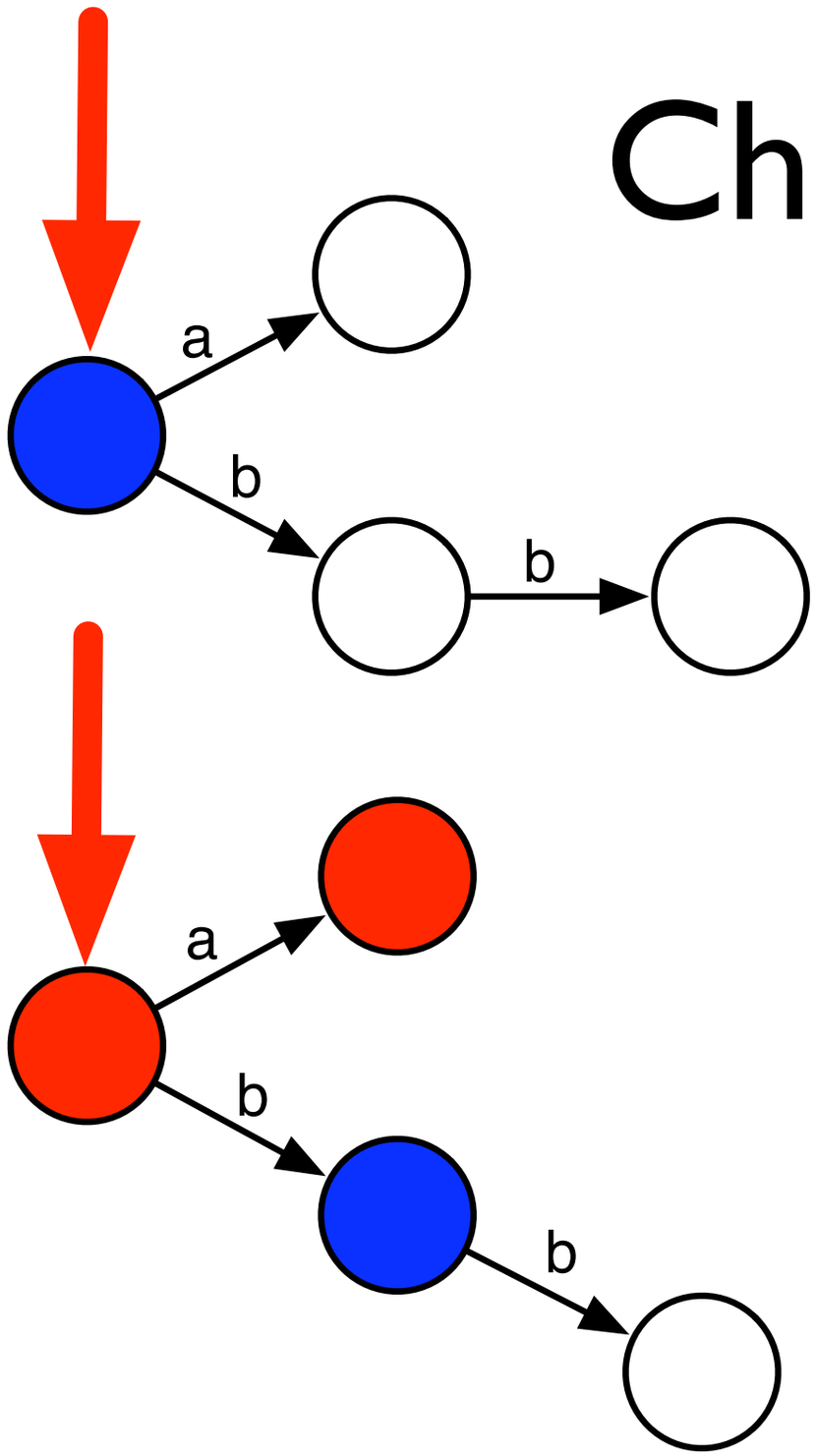
Three similarity values

- Two states and their future states define a distribution over timed strings
- Two states are similar if:

1. Chi squared, for S_+ their distributions are not statistically dissimilar
2. Likelihood ratio, merging them results in a significantly smaller model with respect to the likelihood of S_+
3. Akaike information criterium (AIC), merging them results in a smaller AIC (model selection criterium) for S_+

Chi squared



In the two states:

Compute a Chi-squared test for independence of the distributions over time values and symbols

This results in two p-values

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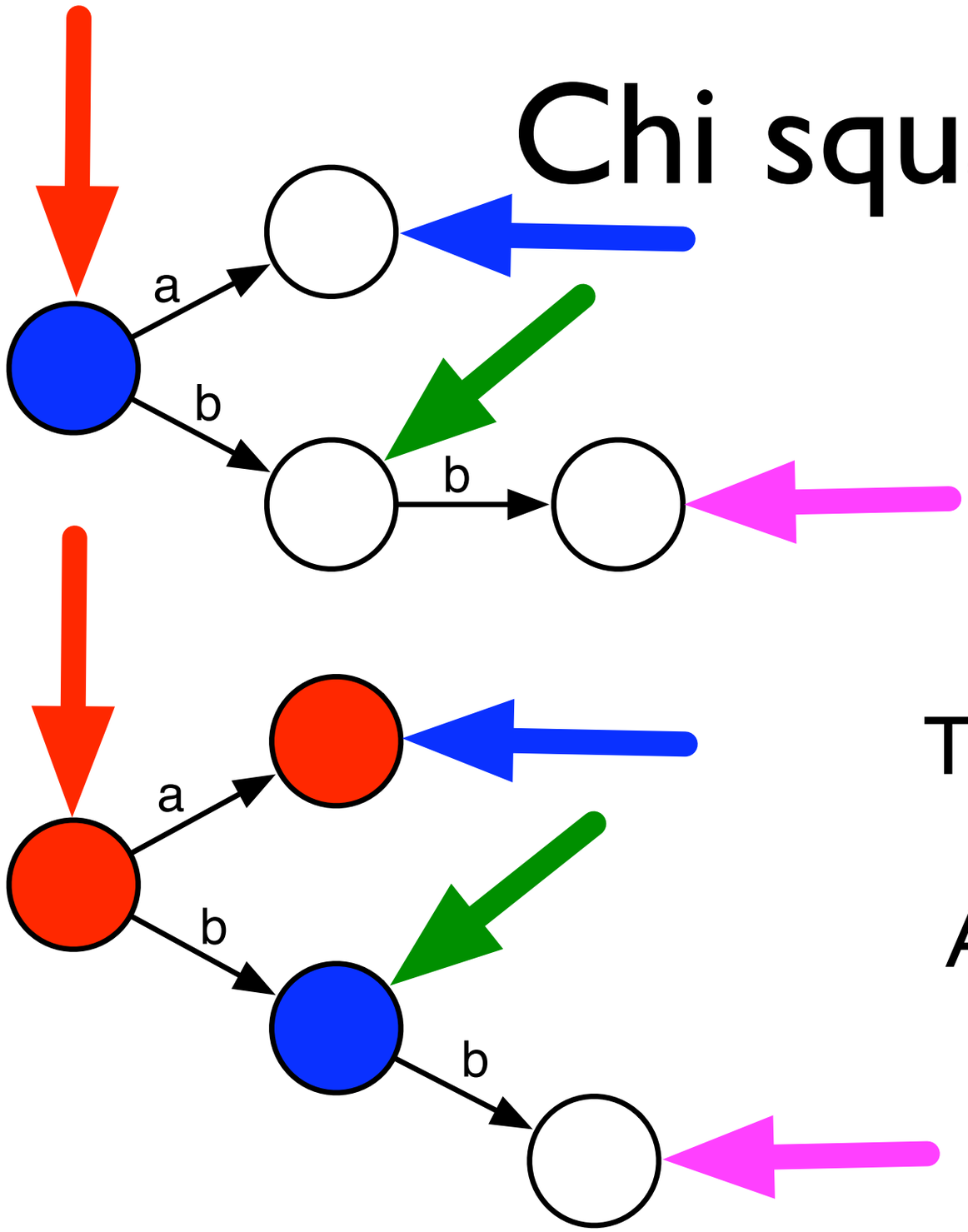
Chi squared



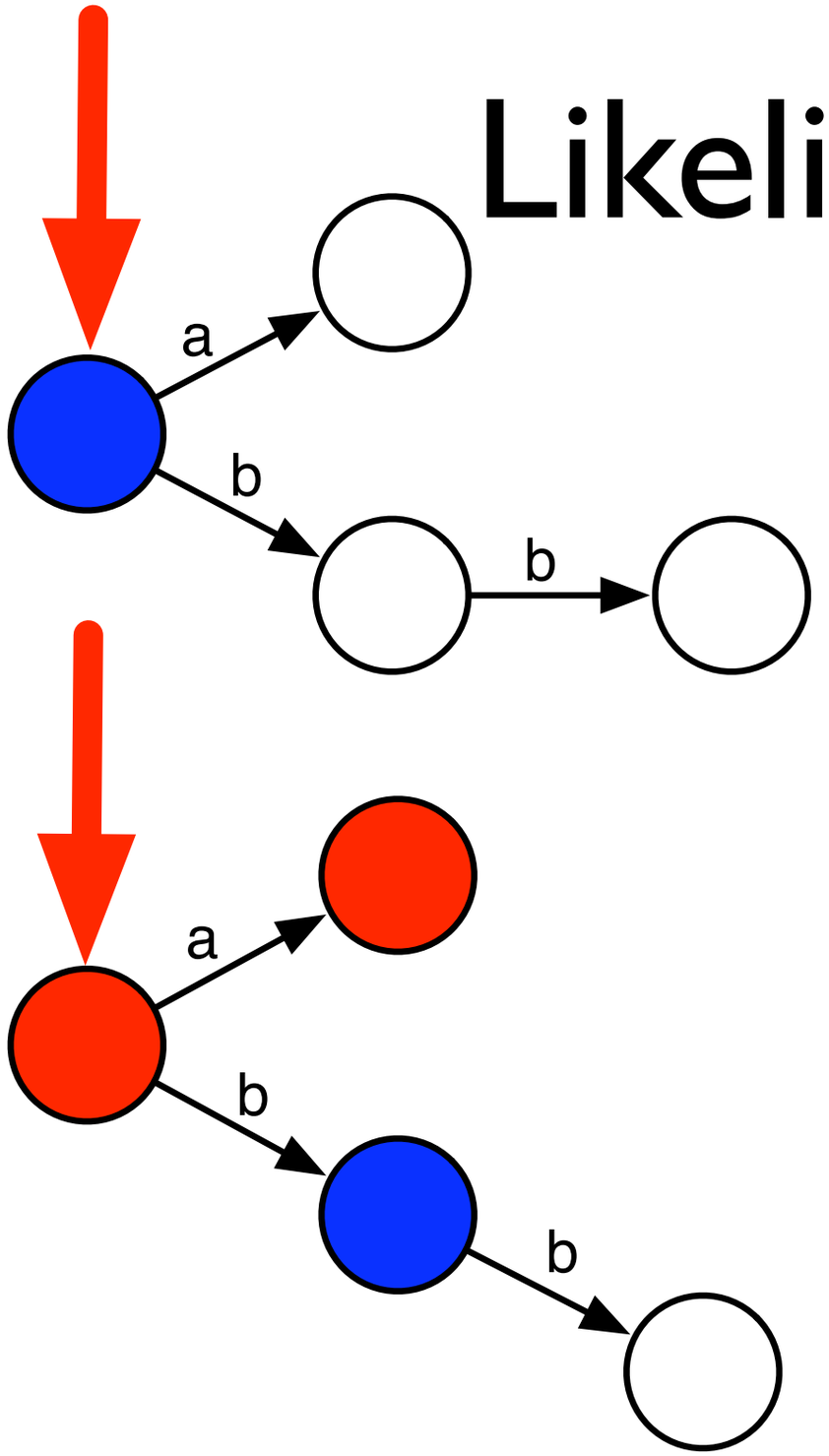
Do the same for every other overlapping future states

The result is many p-values

All of these are combined into a single test using a consensus test

TU Delft

Likelihood ratio



In the two states and the future states:

Determine the maximized likelihood of the data

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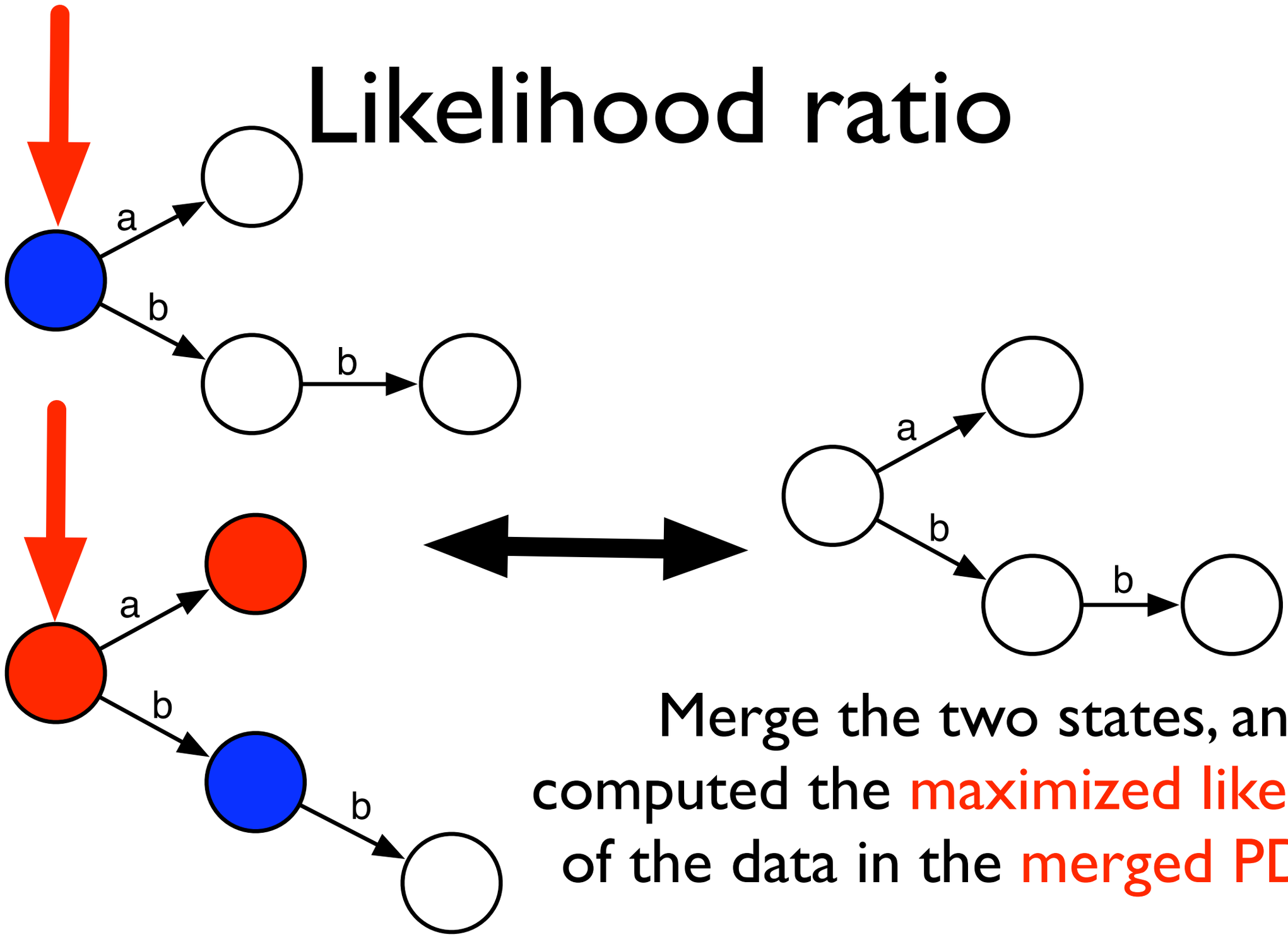
Likelihood ratio



Merge the two states, and computed the maximized likelihood of the data in the merged PDRTA

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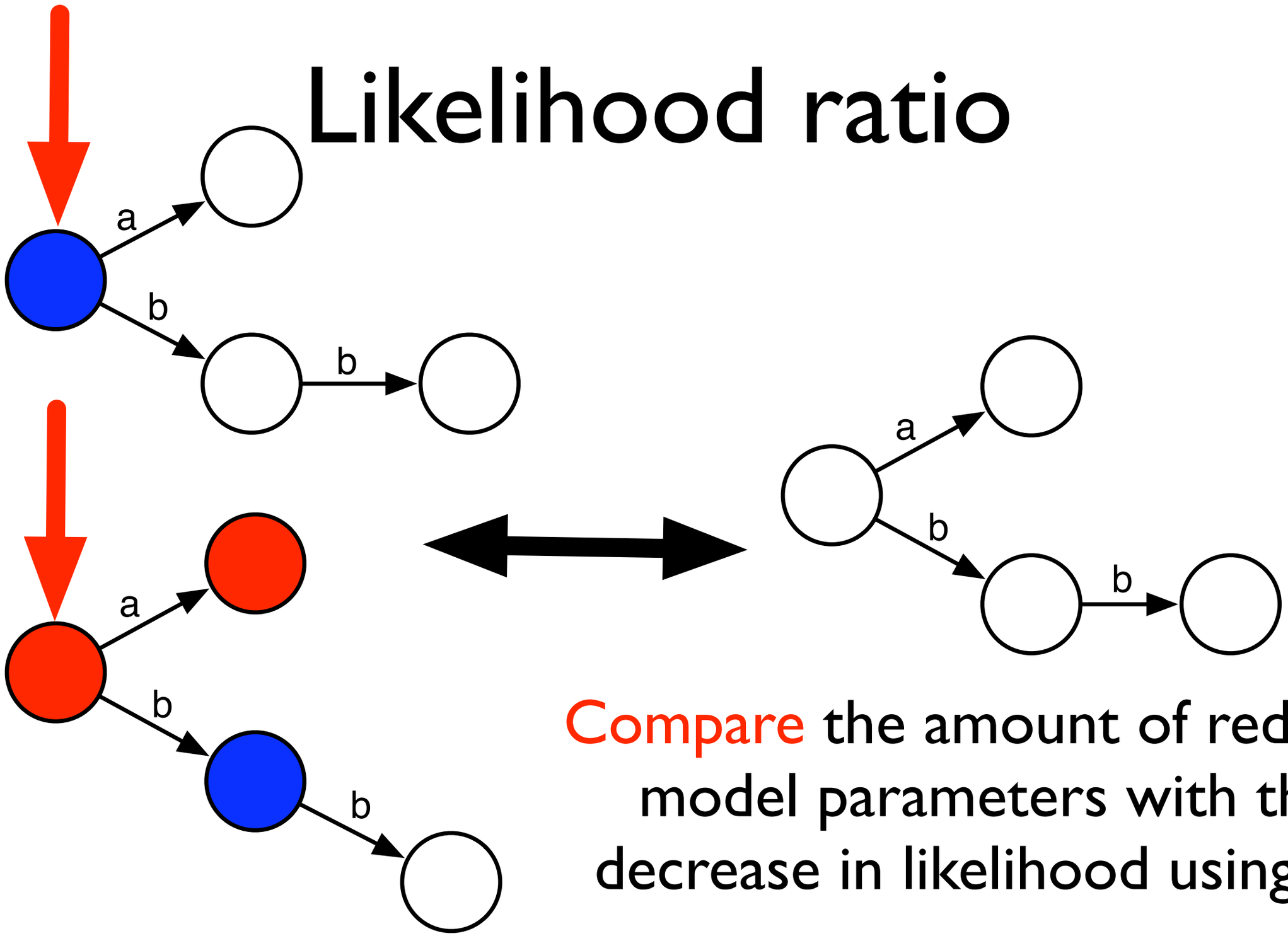
Likelihood ratio



Compare the amount of reduced model parameters with the decrease in likelihood using LR

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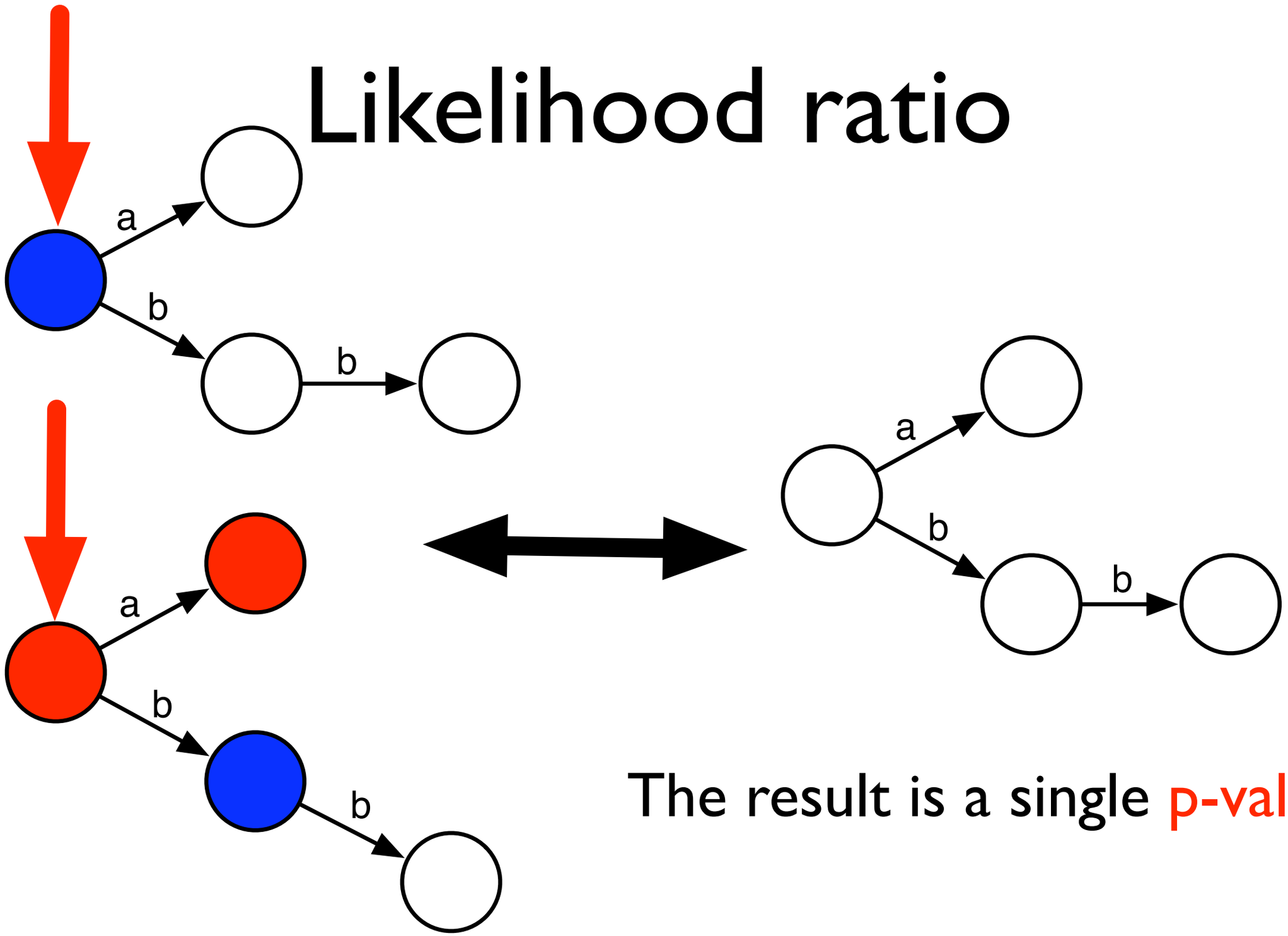
Likelihood ratio



The result is a single p-value

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Using similarity values

- Try all possible merges and splits of the transition to a blue state, compute their similarity p-values
- If the smallest split p-value is less than 0.05:
  - perform the split with the smallest p-value
- Else if the largest merge p-value is greater then 0.05:
  - perform the merge with the largest p-value
- Else color the blue state red

Using the AIC

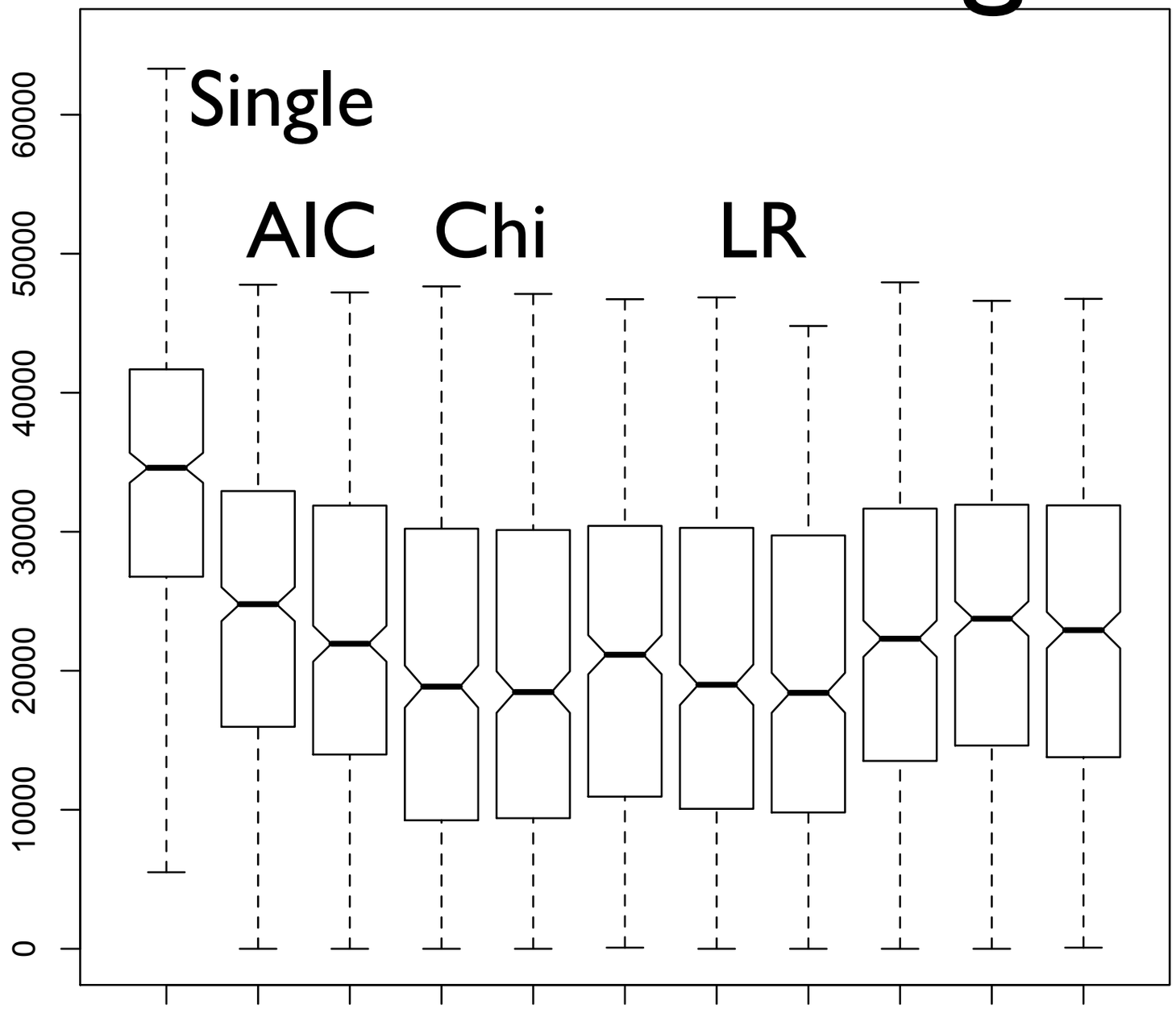
- Try all possible merges and splits of the transition to a blue state, compute their AIC values
- If the AIC of a DRTA resulting from a merge or split is less than the AIC of the current DRTA:
  - Perform this merge or split
- Else
  - Color the blue state red

Results 1000 strings

AIC result

-

AIC optimal



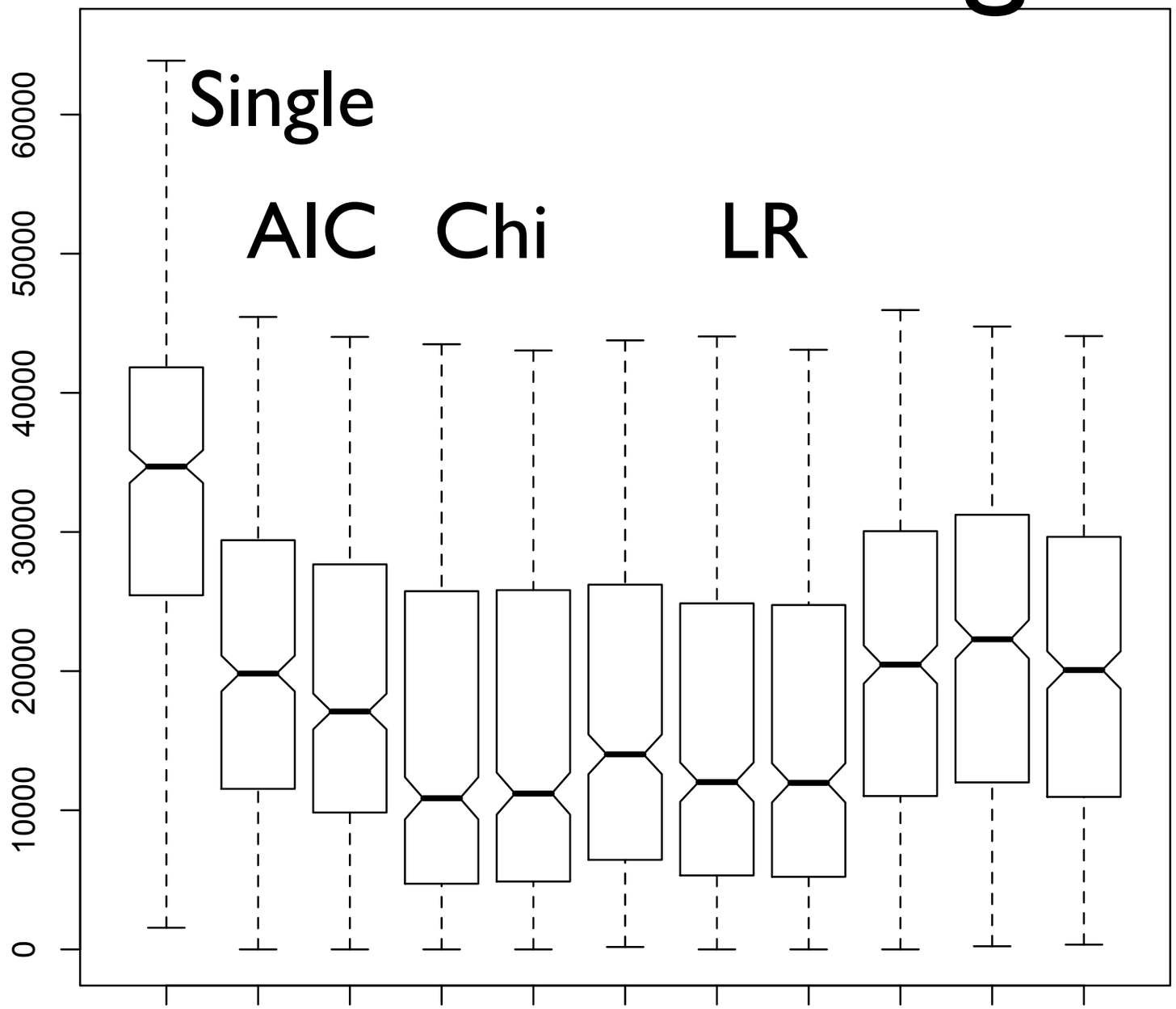
TU Delft

Results 2000 strings

AIC result

-

AIC optimal



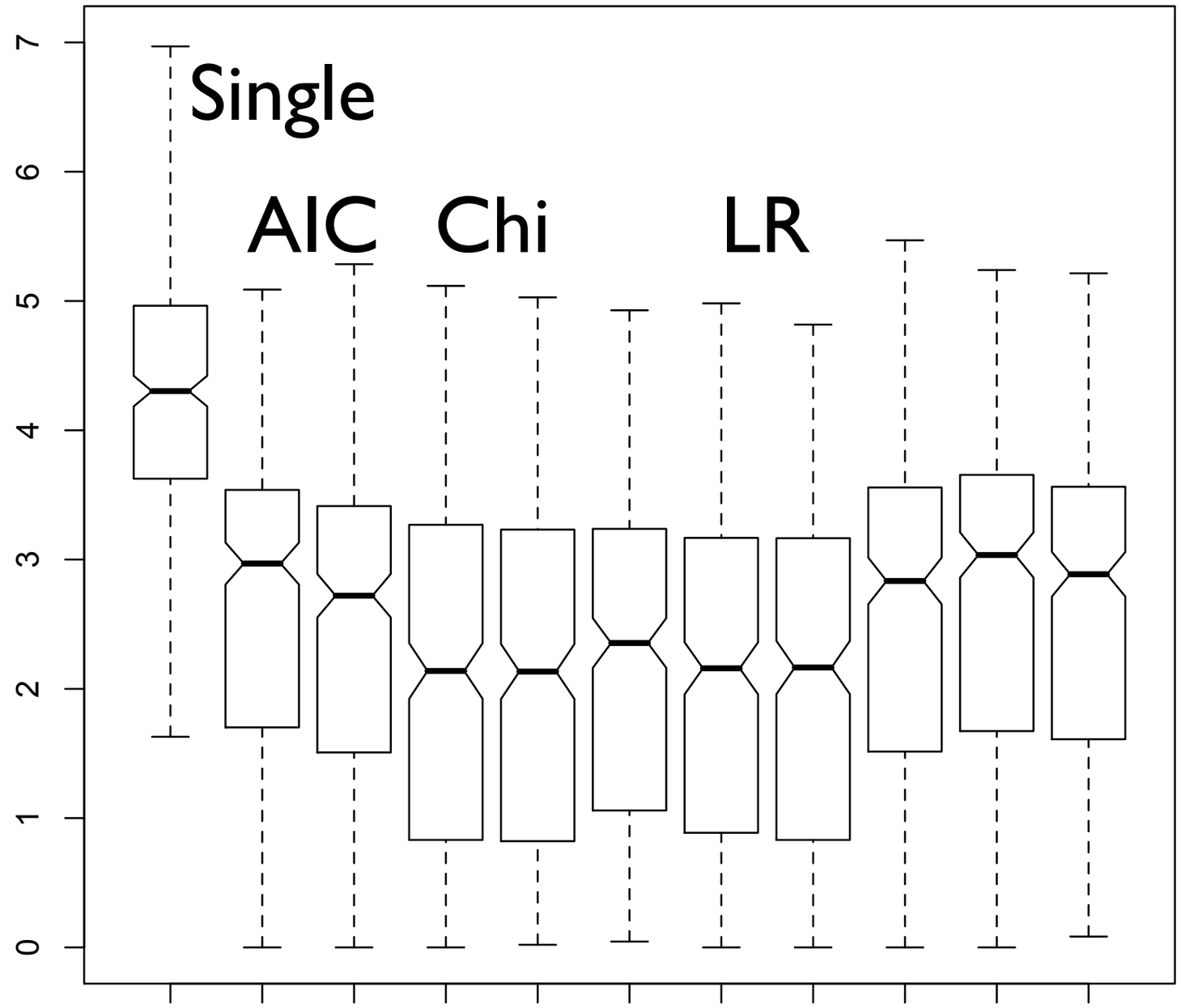
Results

Alphabet size = 4

Perplexity result

-

Perplexity optimal



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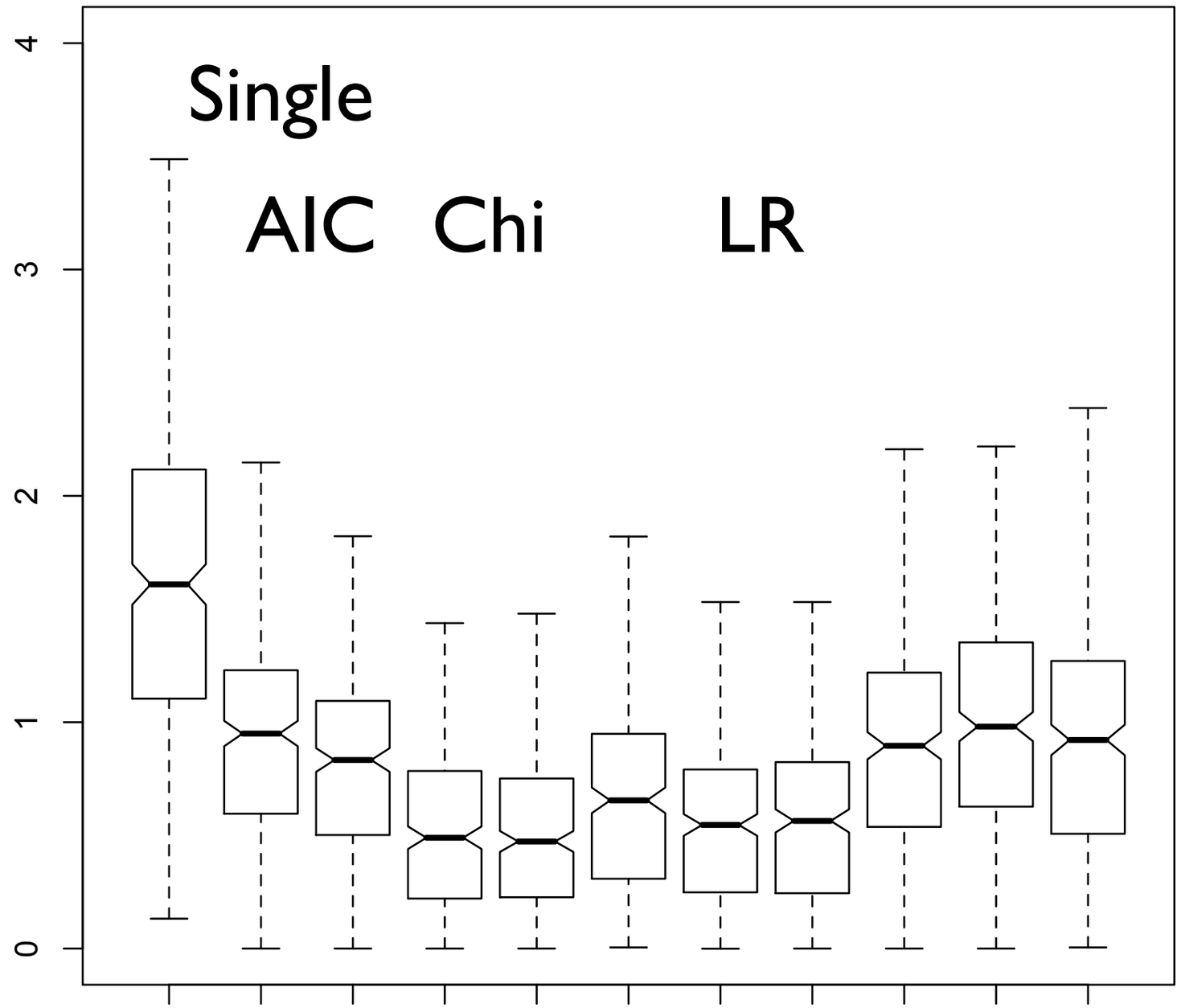
Results

Alphabet size = 2

Perplexity result - Perplexity optimal



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Conclusions

- We have an efficient algorithm for learning deterministic real-time automata (DRTA)s
- The algorithm works both on labeled and unlabeled data
- The algorithm is based on the best known algorithm for learning DFAs, state-merging
- It shows promising results on artificial data

Contact

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- DFA learning references, see:
  - Colin de la Higuera, A bibliographical study of grammatical inference, Pattern Recognition, Volume 38, Issue 9, Pages 1332 1348
- DRTA (and TA) learning references, see my homepage