Matrix Calculations

Assignment 4, Tuesday, March 1, 2016

Exercise teachers. Recall the following split-up of students:

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All (blue) delivery boxes are located in the Mercator building on the ground floor where computing science is located.

Handing in your answers: There are two options, *depending on your exercise class teacher*:

1. Delivery box (default): Put your solutions in the appropriate delivery box. Before putting your solutions in the box make sure:
   - your name and student number are written clearly on the document.
2. E-mail (in case your exercise class teacher agrees): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 4*'. This e-mail should only contain a single PDF document as attachment. Before sending an e-mail make sure:
   - the file is a PDF document that is well readable
   - your name is part of the filename (for example MyName_assignment-4.pdf)
   - your name and student number are included in the document (since they may be printed).

Deadline: Monday, March 7, 12:00 sharp!

Goals: After completing these exercises successfully you should be able to compute with matrices i.e. to multiply them and find inverses. Also you should be able to compute the kernels and images and their dimension and you should understand a transition matrix of a Markovprocess. The total number of points is 20.

1. **(5 points)** Consider the following matrices and vectors:

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 3 & 4 \\ 1 & 1 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 3 & 1 \end{pmatrix} \qquad C = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad D = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

   Compute (i) ADD^T, (ii) $A + BCC^T - ACD^T$,(iii) AB^T, (iv) $B^T A$.

   Give intermediate results.

2. **(4 points)** Find the inverses of the following two matrices.

$$(i)\ A := \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \qquad (ii)\ B := \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{pmatrix}$$

   Hint: perform Gauss-Jordan elimination on $[A \mid I]$.

3. **(6 points)**

Consider the map $F : \mathbb{R}^4 \to \mathbb{R}^3$ given by, for $x_1, x_2, x_3, x_4 \in \mathbb{R}$,

$$F(x_1, x_2, x_3, x_4) = (x_1 + x_2 + 2x_4, 2x_1 + 3x_2 + x_3 + 6x_4, x_1 + x_4).$$

(a) Write down the matrix for F.

(b) Give the dimensions of the kernel and of the image of F.

(c) Find a basis for the kernel of F.

4. **(5 points)**

Consider the "RU student transition" matrix

$$T = \begin{pmatrix} 0.8 & a \\ 0.2 & 1-a \end{pmatrix}.$$

The matrix T denotes the fraction of a RU student cohort that remains at the university (80%) and the fraction that leaves (20%). It also denotes the fraction of the students outside the RU that enter the RU.

(i) Determine the value of the parameter $a \in [0, 1]$ for which T does **not** have an inverse.

(ii) Consider the transition matrix T with $a = 0.1$. Explain the student dynamics in terms of percentages of (non)-changes. What are the percentages after 3 cycles?