Matrix Calculations

Sample Problems for Orthogonality

Goals: After completing these exercises successfully you should be able to use the Gram-Schmidt method to compute an orthogonal basis and you should be able to compute an orthonormal basis.

1. Consider vectors $a = (-1, 3, -7)$, $b = (2, -1, 5)$, $c = (0, 1, 5)$. Compute the following:

   (a) $(2\langle a, b\rangle - \langle c, b\rangle)(2\langle b, c\rangle + \langle b, a\rangle)$

   (b) Length of a and length of b

   (c) Distance between a and b

   (d) Angle between a and b (give the cosine of the angle)

2. (a) For $v = (2, 3, -6)$ in the direction of $w = (1, -5, 2)$, find a vector u that is orthogonal to both v and w in $\mathbb{R}^3$. Check your results!

3. The subspace $V \subseteq \mathbb{R}^4$ is spanned by the vectors:

$$v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \qquad v_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \qquad v_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}.$$

   (a) Transform the basis $\{v_1, v_2, v_3\}$ into an orthogonal one.

   (b) Get rid of the fractions in the orthogonal bases and demonstrate that the resulting vectors are indeed orthogonal. Why is it possible to do get rid of the fractions?

4. Let p be a parameter and let $U \subseteq \mathbb{R}^3$ be the subspace spanned by the independent vectors $v_1 = (4, -p, -p^2)$ and $v_2 = (2p, p^2, p)$.

   (a) Determine the values of p for which (v_1, v_2) forms an orthogonal basis of U. (Describe all possible values and argue that these are the only ones.)

   (b) Choose one basis that you've found in (a), and turn it into an orthonormal basis of U.