Tentamen Matrix Rekenen 2013

Thursday, April 11, 2013, 13:30 – 15:30

- **Put your student-card clearly visible at the top-right corner of your table.**
- Please write clearly, and put on **each page**: your name, your student number, your field (IC=informatica, IK=informatiekunde, KI=kunstmatige intelligentie, other), and your exercise class teacher (Silva, Lappenschaar, Verwer, Schwabe).
- The exam is closed book. You are NOT allowed to use a calculator, a computer or a mobile phone. You may answer in Dutch or in English.
- This exam consists of **3 questions**, printed on a single page. Each (sub)question indicates how many points it is worth. You can score a maximum of **100 points**.
- It is advised to explain your approach and to check your answers yourself.

1. **(42 points)** Consider the vector $u = (\frac{2}{3}, \frac{1}{3}, \frac{2}{3}) = \frac{1}{3}(2, 1, 2)$ in $\mathbb{R}^3$.
   (a) **(6 points)** Computer the length $\|u\|$.
   (b) **(6 points)** Compute the matrix product
   $$P_u = u \cdot u^T = \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{pmatrix} \cdot \begin{pmatrix} \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$
   where $(-)^T$ denotes the transpose of a matrix.
   (c) **(6 points)** Calculate the product matrix $P_u \cdot P_u$.
   (d) **(6 points)** Write down the matrix transpose $(P_u)^T$.
   (e) **(6 points)** Find a vector $v \in \mathbb{R}^3$ which is orthogonal to u (that is, with $v \perp u$).
   (f) **(6 points)** Calculate, as before, the matrix $P_v = v \cdot v^T$.
   (g) **(6 points)** Calculate the matrix product $P_u \cdot P_v$ — that is, the product of the two matrices associated with the two orthogonal vectors u, v.
2. **(12 points)** Compute the projection of the vector $v = (1, 1, 0)$ onto the plane $x + y - z = 0$.
3. **(46 points)** Consider the following matrix.
   $$M = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 2 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$
   (a) **(8 points)** Explain if M is a Markov chain or not. Draw the transition graph determined by M.
   (b) **(8 points)** Compute the inverse matrix M^{-1}, if it exists. If so, explain if M^{-1} is a Markov chain.
   (c) **(10 points)** What is the characteristic polynomial of M?
   (d) **(8 points)** Compute the solutions of the characteristic equation.
   (e) **(12 points)** Compute the eigenvectors corresponding to these solutions/eigenvalues.