

# Matrix Calculations

## Assignment 4, Tuesday, September 26, 2017

**Exercise teachers.** Recall the following split-up of students:

teacher	lecture room	email
John van de Wetering	HG00.114	wetering@cs.ru.nl
Justin Reniers	HG01.058	j.reniers@student.ru.nl
Justin Hende	HG02.028	justin.hende@gmail.com
Bart Gruppen	HG03.632	b.gruppen@student.ru.nl

The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

**Handing in your answers:** There are two options, *depending on your exercise class teacher*:

1. Delivery box (default): Put your solutions in the appropriate delivery box (see above). Before putting your solutions in the box make sure:
  - your name and student number are written clearly on the document.
2. E-mail (if your teacher agrees): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 4*'. This e-mail should only contain a single PDF document as attachment (unless explicitly stated otherwise). Before sending an e-mail make sure:
  - the file is a PDF document that is well readable
  - your name is part of the filename (for example MyName\_assignment-4.pdf)
  - your name and student number are included in the document (since they will be printed)

**Deadline:** Monday, October 2, 16:00 sharp!

**Goals:** After completing this assignment, you should be able to determine if a set of vectors forms a basis, translate linear maps to/from matrices, and do matrix/vector multiplication. The total number of points is 20.

1. **(6 points)** Determine if the following sets of vectors form a basis for the given vector space. If they do, prove that they are *linearly independent* and *spanning*. If they do not form a basis, explain why not.

(i)  $\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$  for  $\mathbb{R}^3$

(ii)  $\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$  for  $\mathbb{R}^3$

(iii)  $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix} \right\}$  for  $V := \{(x, y, 2y, 0) \mid x, y \in \mathbb{R}\} \subseteq \mathbb{R}^4$

2. **(4 points)** Prove explicitly that the following maps are linear by checking that they preserve addition and scalar multiplication.

(i)  $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  defined by  $f((x, y, z)) = (y + z, 2x + z, 3x - y + z)$ .

(ii)  $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  defined by  $f((x, y, z)) = (ax + by, cx + z, dx)$ , for  $a, b, c, d \in \mathbb{R}$ .

3. (2 points) Show that the following maps are *not* linear

(i)  $f: \mathbb{R}^3 \rightarrow \mathbb{R}^2$  defined by  $f((x, y, z)) = (x + z, y + xz)$ ;

(ii)  $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  defined by  $f((x, y)) = (x, y + 3)$ .

4. (4 points) Consider the following matrices and vectors:

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 9 & 8 \\ 6 & 5 \\ 3 & 2 \end{pmatrix} \quad \mathbf{v} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \quad \mathbf{w} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

Compute (i)  $\mathbf{A} \cdot \mathbf{v}$ , (ii)  $\mathbf{B} \cdot \mathbf{w}$ .

Give intermediate results.

5. (4 points)

This exercise is about transforming linear maps to/from matrices.

(a) Give the matrix corresponding the linear map:

$$f((x_1, x_2, x_3, x_4)) = (x_1 + x_2 + 2x_4, 2x_1 + 3x_2 + x_3 + 6x_4, x_1 + x_4).$$

in terms of the standard bases for  $\mathbb{R}^4$  and  $\mathbb{R}^3$ .

(b) Consider the following matrix, written in terms of the standard bases:

$$\mathbf{A} = \begin{pmatrix} 4 & 1 & 3 & -5 \\ 1 & 3 & 3 & 7 \end{pmatrix}$$

give the linear map associated to  $\mathbf{A}$ .