Matrix Calculations

Assignment 5, Wednesday, October 3, 2018

Exercise teachers. Recall the following split-up of students:

teacher	email	lecture room
Justin Reniers	`j.reniers@student.ru.nl`	E2.68 (E2.62 on 12 Oct)
Justin Hende	`J.Hende@gmail.com`	HG00.062
Iris Delhez	`iam.delhez@student.ru.nl`	HG00.108
Stefan Boneschanscher	`S.Boneschanscher@student.ru.nl`	HG01.028
Serena Rietbergen	`serena.rietbergen@student.ru.nl`	HG02.028
Jen Dusseljee	`j.dusseljee@student.ru.nl`	HFML0220

The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

Handing in your answers: There are two options, *depending on your exercise class teacher*:

1. Delivery box (default): Put your solutions in the appropriate delivery box (see above). Before putting your solutions in the box make sure:
   - your name and student number are written clearly on the document.
2. E-mail (if your teacher agrees): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 5*'. This e-mail should only contain a single PDF document as attachment (unless explicitly stated otherwise). Before sending an e-mail make sure:
   - the file is a PDF document that is well readable
   - your name is part of the filename (for example MyName_assignment-5.pdf)
   - your name and student number are included in the document (since they will be printed)

Deadline: Tuesday, October 9, 16:00 sharp!

Goals: After completing these exercises successfully you should be able to multiply matrices, perform certain transformations by matrix multiplication, and compute inverses. The total number of points is 20.

1. **(10 points)** For the following matrices:

$$\boldsymbol{A} := \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \boldsymbol{B} := \begin{pmatrix} 4 & -3 \\ 2 & 0 \end{pmatrix} \qquad \boldsymbol{C} := \begin{pmatrix} 1 & 3 & 3 \\ 2 & 0 & 1 \end{pmatrix}$$

   (a) Compute $\boldsymbol{A} \cdot \boldsymbol{B} \cdot \boldsymbol{C}$ and $\boldsymbol{C} \cdot \boldsymbol{A} \cdot \boldsymbol{B}$. Show intermediate calculations.

   (b) Find a 2×3 matrix $\boldsymbol{X}$ and a 3×2 matrix $\boldsymbol{Y}$ such that $\boldsymbol{B} = \boldsymbol{X} \cdot \boldsymbol{Y}$.

   (c) Find a matrix $\boldsymbol{D}_1$ such that $\boldsymbol{C} \cdot \boldsymbol{D}_1$ is the same as $\boldsymbol{C}$ but with its first column doubled. Similarly, find matrices $\boldsymbol{D}_2$ and $\boldsymbol{D}_3$ which double the second and third columns of $\boldsymbol{C}$.

2. **(5 points)** Consider the matrix $\boldsymbol{A}$:

$$\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

(a) Compute the inverse of $\boldsymbol{A}$.

(b) Use the inverse of $\boldsymbol{A}$ to solve the following system of linear equations:

$$\begin{aligned} x + 2y + 3z &= -5 \\ 2x + z &= 0 \\ 2x + y + z &= 10 \end{aligned}$$

3. **(5 points)** The sum of two matrices is just the sum of their elements, for example:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} a+w & b+x \\ c+y & d+z \end{pmatrix}$$

Give an example of a pair of 2×2 matrices $\boldsymbol{C}$ and $\boldsymbol{D}$ which are both invertible, but where $\boldsymbol{C} + \boldsymbol{D}$ is not invertible.