Matrix Calculations

Assignment 6, Wednesday, October 10, 2018

Exercise teachers. Recall the following split-up of students:

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Justin Reniers	j.reniers@student.ru.nl	E2.68 (E2.62 on 12 Oct)
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The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

Handing in your answers: There are two options, *depending on your exercise class teacher*:

1. Delivery box (default): Put your solutions in the appropriate delivery box (see above). Before putting your solutions in the box make sure:
   - your name and student number are written clearly on the document.
2. E-mail (if your teacher agrees): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 6*'. This e-mail should only contain a single PDF document as attachment (unless explicitly stated otherwise). Before sending an e-mail make sure:
   - the file is a PDF document that is well readable
   - your name is part of the filename (for example MyName_assignment-6.pdf)
   - your name and student number are included in the document (since they will be printed)

Deadline: Tuesday, October 16, 16:00 sharp!

Goals: After completing these exercises successfully you should be able to take the determinants of matrices, express vectors or matrices in different bases, and compute eigenvalues of a matrix. The total number of points is 20.

1. **(6 points)** Compute the determinants of the following matrices:

$$\boldsymbol{A} = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \qquad \boldsymbol{B} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 2 & 1 & 1 \end{pmatrix} \qquad \boldsymbol{C} = \begin{pmatrix} 1 & 2 & x \\ 0 & 0 & y \\ 2 & 1 & 1 \end{pmatrix}$$

   (where the determinant of $\boldsymbol{C}$ should be written in terms of x and y)

2. **(4 points)**
   Consider the following 2 bases for $\mathbb{R}^3$:

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\} \qquad \mathcal{C} = \left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

   Translate the following vectors into the $\mathcal{B}$ basis and the $\mathcal{C}$ basis:

$$\boldsymbol{v} := \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}_{\mathcal{S}} \qquad \boldsymbol{w} := \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}_{\mathcal{S}}$$

That is, write each vector in each of these two forms:

$$\begin{pmatrix} * \\ * \\ * \end{pmatrix}_{\mathcal{B}} \qquad \begin{pmatrix} * \\ * \\ * \end{pmatrix}_{\mathcal{C}}$$

3. **(6 points)** Consider the following two bases in $\mathbb{R}^2$:

$$\mathcal{B} = \left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ 3 \end{pmatrix} \right\} \qquad \mathcal{C} = \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \end{pmatrix} \right\}$$

   (a) Give the basis transformation matrices $\boldsymbol{T}_{\mathcal{B}\Rightarrow\mathcal{S}}$ and $\boldsymbol{T}_{\mathcal{C}\Rightarrow\mathcal{S}}$.

   (b) Give the basis transformation matrices $\boldsymbol{T}_{\mathcal{S}\Rightarrow\mathcal{B}}$ and $\boldsymbol{T}_{\mathcal{S}\Rightarrow\mathcal{C}}$.

   (c) Suppose $g : \mathbb{R}^2 \to \mathbb{R}^2$ is represented in the standard basis as:

$$\boldsymbol{A} = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}_{\mathcal{S}}$$

   Give a representation of g as matrices in the bases $\mathcal{B}$ and $\mathcal{C}$.

   (d) (BONUS) Suppose f is represented by the follow matrix in the $\mathcal{B}$ basis:

$$\boldsymbol{B} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_{\mathcal{B}}$$

   translate $\boldsymbol{B}$ into the $\mathcal{C}$ basis.

4. **(4 points)** For each of the two matrices:

$$\boldsymbol{A} = \begin{pmatrix} 2 & -2 \\ -\frac{1}{2} & 2 \end{pmatrix} \qquad \boldsymbol{B} = \begin{pmatrix} 1 & -2 \\ 0 & -1 \end{pmatrix}$$

   (a) What is the characteristic polynomial?

   (b) What are the eigenvalues?