Matrix Calculations

Assignment 7, Wednesday, October 17, 2018

Exercise teachers. Recall the following split-up of students:

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Justin Reniers	`j.reniers@student.ru.nl`	HG00.622 on 19 Oct
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The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

Handing in your answers: There are two options, *depending on your exercise class teacher*:

1. Delivery box (default): Put your solutions in the appropriate delivery box (see above). Before putting your solutions in the box make sure:
   - your name and student number are written clearly on the document.
2. E-mail (if your teacher agrees): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 7*'. This e-mail should only contain a single PDF document as attachment (unless explicitly stated otherwise). Before sending an e-mail make sure:
   - the file is a PDF document that is well readable
   - your name is part of the filename (for example MyName_assignment-7.pdf)
   - your name and student number are included in the document (since they will be printed)

Deadline: Tuesday, October 23, 16:00 sharp!

Goals: After completing these exercises, you should be able to compute eigenvalues, eigenvectors, the diagonalisation of a matrix, and various geometric properties of vectors involving inner products.

1. **(8 points)** Recall from last time that the matrix:

$$\boldsymbol{A} = \begin{pmatrix} 2 & -2 \\ -\frac{1}{2} & 2 \end{pmatrix}$$

   has eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 3$.

   (a) Compute the associated eigenvectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$.

   (b) Diagonalise $\boldsymbol{A}$. That is, find basis transformation matrices $\boldsymbol{T}_{\mathcal{B}\Rightarrow\mathcal{S}}$ and $\boldsymbol{T}_{\mathcal{S}\Rightarrow\mathcal{B}}$ and a diagonal matrix $\boldsymbol{D}$ such that:

$$\boldsymbol{A} = \boldsymbol{T}_{\mathcal{B}\Rightarrow\mathcal{S}} \cdot \boldsymbol{D} \cdot \boldsymbol{T}_{\mathcal{S}\Rightarrow\mathcal{B}}$$

   (c) Compute $\boldsymbol{A}^5$ and $\sqrt{\boldsymbol{A}}$ **using the diagonalisation of** $\boldsymbol{A}$. (n.b. for $\sqrt{\boldsymbol{A}}$, expressions involving square roots in the matrix, like '$\frac{1}{2} + 2\sqrt{2}$' are okay.)

2. **(6 points)** Consider the following "student transition matrix", denoting the fraction of RU students that will stay at / leave the RU and the fraction of non-RU students that will come to / not come to the RU:

$$\boldsymbol{S} = \begin{pmatrix} 0.7 & 0.1 \\ 0.3 & 0.9 \end{pmatrix}$$

   (a) Find eigenvalues and eigenvectors of $\boldsymbol{S}$.

   (b) Diagonalise $\boldsymbol{S}$.

   (c) Compute $\lim_{n\to\infty} \boldsymbol{S}^n$.

3. **(6 points)**

   Consider vectors:

$$\boldsymbol{u} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \qquad \boldsymbol{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad \boldsymbol{w} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

   compute the following quantities:

   (a) Inner products: $\langle \boldsymbol{u}, \boldsymbol{u} \rangle$, $\langle \boldsymbol{v}, \boldsymbol{v} \rangle$, $\langle \boldsymbol{w}, \boldsymbol{w} \rangle$, $\langle \boldsymbol{u}, \boldsymbol{v} \rangle$, $\langle \boldsymbol{v}, \boldsymbol{w} \rangle$, and $\langle \boldsymbol{u}, \boldsymbol{w} \rangle$.

   (b) Norms: $\|\boldsymbol{u}\|$, $\|\boldsymbol{v}\|$, $\|\boldsymbol{w}\|$

   (c) Distances: $d(\boldsymbol{u}, \boldsymbol{v})$, $d(\boldsymbol{v}, \boldsymbol{w})$

   (d) Normalised vector: $\boldsymbol{u}' = \frac{1}{\|\boldsymbol{u}\|}\boldsymbol{u}$. Show that $\|\boldsymbol{u}'\| = 1$.

   (e) Which pairs of vectors (if any) are orthogonal?