Quantum Processes and Computation

Assignment 1, Monday, February 5, 2018

Exercise teachers:
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The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

Handing in your answers: There are two options:

1. Deliver a hard copy to the mailbox of John van de Wetering. Mercator 1, 3rd floor.
2. E-mail a PDF to `wetering@cs.ru.nl`. Please include your name and the exercise number in the filename, e.g. `ACHTERNAAM-qpc-exercise1.pdf`.

Deadline: Friday, February 9, 17:00

Goals: After completing these exercises successfully you should be able to perform simple diagrammatic computations. The total number of points is 100, distributed over 6 exercises.

Exercise 1 (3.4) (20 points): Give two examples of a process theory. For each one answer the following questions:

1. What are the system-types?
2. What are the processes?
3. How do processes compose?
4. When should two processes be considered equal?

Exercise 2 (3.10) (20 points): Draw the diagrams corresponding to the following diagram formulas:

1. $f^{C_4}_{B_1C_2}g^{D_3}_{C_4}$
2. $f^{A_1}_{A_1}$
3. $g^{A_1}_{B_1}f^{B_1}_{A_1}$
4. $1^{A_6}_{A_1}1^{A_5}_{A_2}1^{A_4}_{A_3}$.

Use the convention that inputs and outputs are numbered from left-to-right.
Note this was not covered in the lectures. See section 3.1.3 in the book or lecture notes.

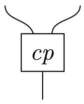
Exercise 3 (3.12) (20 points): Give the diagrammatic equations of a process $*$ taking two inputs and one output that express the algebraic properties of being

1. associative: $x * (y * z) = (x * y) * z$
2. commutative: $x * y = y * x$
3. having a unit: there exists a process e (with no inputs) such that $x * e = e * x = x$

Hint: x, y and z should not appear in your final diagrams. They are however useful in trying to figure out what the diagrammatic equation should be.

Can you also draw the diagram representing distributivity of two processes (e.g. $(x+y)*z = (x*z)+(y*z)$)? If not, what's the problem?

Exercise 4 (3.15) (10 points): Using the copy operation:

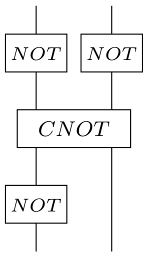


$$:: \quad n \mapsto (n, n)$$

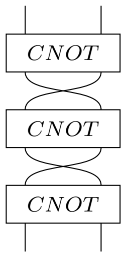
write down the diagram representing distributivity.

Exercise 5 (3.30) (10 points): First compute the values of the following functions, then give the commonly used name of these functions:

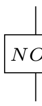
(a) :



(b) :

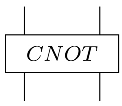


where:



$$:: \begin{cases} 0 \mapsto 1 \\ 1 \mapsto 0 \end{cases}$$

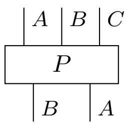
and



$$:: \begin{cases} (0,0) \mapsto (0,0) \\ (0,1) \mapsto (0,1) \\ (1,0) \mapsto (1,1) \\ (1,1) \mapsto (1,0) \end{cases}$$

Exercise 6 (3.31) (20 points): Suppose A, B, C, and D are sets and P is a relation given by:

$$\begin{aligned} A &= \{a_1, a_2, a_3\} \\ B &= \mathbb{B} \\ C &= \{\textbf{red}, \textbf{green}\} \\ D &= \mathbb{N} \end{aligned}$$



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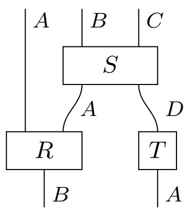

Compute P first for R, S, T given by:

$$R :: \begin{cases} 1 \mapsto (a_1, a_1) \\ 1 \mapsto (a_1, a_2) \end{cases} \qquad S :: \begin{cases} (a_1, 5) \mapsto (0, \textbf{red}) \\ (a_1, 5) \mapsto (1, \textbf{red}) \\ (a_2, 6) \mapsto (1, \textbf{green}) \end{cases} \qquad T :: \begin{cases} a_1 \mapsto 200 \\ a_3 \mapsto 5 \end{cases}$$

and then for R, S, T given by:

$$R :: \begin{cases} 0 \mapsto A \times \{a_2, a_3\} \\ 1 \mapsto A \times \{a_2, a_3\} \end{cases} \qquad S :: \begin{cases} (a_1, 0) \mapsto \mathbb{B} \times \{\textbf{red}, \textbf{green}\} \\ (a_1, 1) \mapsto \mathbb{B} \times \{\textbf{red}, \textbf{green}\} \\ (a_1, 2) \mapsto \mathbb{B} \times \{\textbf{red}, \textbf{green}\} \\ \vdots \end{cases} \qquad T :: \begin{cases} a_1 \mapsto \mathbb{N} \\ a_2 \mapsto \mathbb{N} \\ a_3 \mapsto \mathbb{N} \end{cases}$$