Quantum Processes and Computation

Assignment 3, Monday, February 26, 2018

Exercise teachers:
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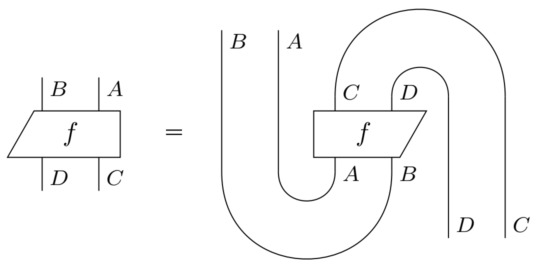
Handing in your answers: There are two options:

1. Deliver a hard copy to the mailbox of John van de Wetering. Mercator 1, 3rd floor.
2. E-mail a PDF to `wetering@cs.ru.nl`. Please include your name and the exercise number in the filename, e.g. `ACHTERNAAM-qpc-exercise1.pdf`.

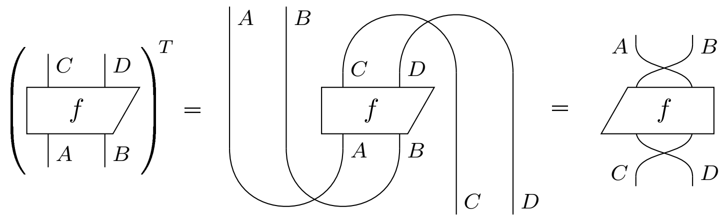
Deadline: Friday, March 2, 17:00

Goals: After completing these exercises you know how to reason with the transpose, adjoints, and the conjugate. You know about projections, unitaries and isometries. The total number of points is 100, distributed over 5 exercises.
Material covered in book: Chapter 4

In the lecture we saw the transpose. If a system has multiple inputs and outputs then there are two choices for the transpose. There is the *diagrammatic* transpose:



and the *algebraic* transpose:



(for more details see section 4.2.2 of the book)
Exercise 1 (4.39) (30 points): The adjoint is an operation which acts like a vertical reflection. Show that the algebraic transpose is in this regard an adjoint, i.e. show diagrammatically that:

1. It is an involution: $(f^T)^T = f$,
2. It preserves parallel composition: $(f \otimes g)^T = f^T \otimes g^T$,
3. It preserves sequential composition: $(f \circ g)^T = g^T \circ f^T$,
4. It preserves the identities: $\mathrm{id}^T = \mathrm{id}$,

5. It sends cups to caps: $\cup^T = \cap$,

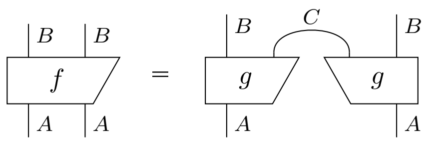
6. It reverse the direction of swaps: $\mathrm{SWAP}_{A,B}^T = \mathrm{SWAP}_{B,A}$

Note: It is sufficient to check the above properties for processes f and g with 2 in- and outputs.

Exercise 2 (4.25 & 4.59) (20 points): An *inverse* for a process $f : A \to B$ is a process $f^{-1} : B \to A$ such that $f^{-1} \circ f = \mathrm{id}_A$ and $f \circ f^{-1} = \mathrm{id}_B$. First show that if a process f has an inverse, that this inverse is unique, then show that for a process f the following are equivalent:

- f is unitary.
- f is an isometry and has an inverse.
- $f^\dagger$ is an isometry and has an inverse.

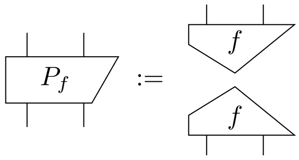
In the lecture we saw the notion of a positive process. There is also a notion of $\otimes$-positivity. A process f is $\otimes$-positive if there exists a process g such that



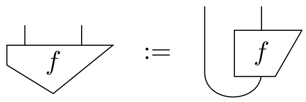
(see section 4.3.6 of the book for more information)

Exercise 3 (4.67) (10 points): Show that the sequential composition of two $\otimes$-positive processes is again a $\otimes$-positive process.

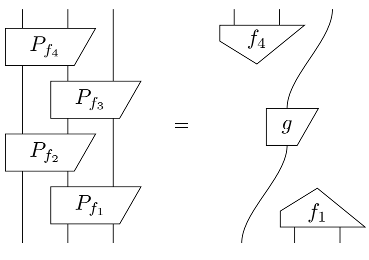
For a process $f : A \to A$ we define its *separable projector* by



where

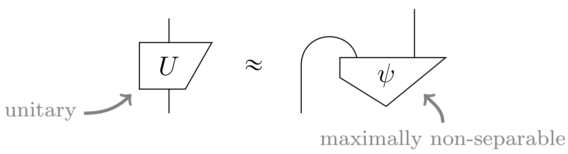


Exercise 4 (4.73) (20 points): Given processes $f_i : A \to A$ show that:

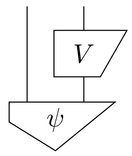


with $g \;:=\; f_3 \circ \bar{f}_4 \circ f_2^T \circ f_3^\dagger \circ f_1 \circ \bar{f}_2$.

Exercise 5 (4.82) (20 points): A state ψ is *maximally non-separable* if it corresponds to a unitary U by process-state duality, up to a number:



Show that if one applies a unitary V to one side of a maximally non-separable state:



that one again obtains a maximally non-separable state, and that this unitary can always be chosen such that the resulting state is the cup (up to a number).