Quantum Processes and Computation

Assignment 8, Monday, April 23, 2018

Exercise teachers:
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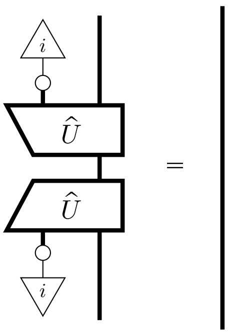
Handing in your answers: There are two options:

1. Deliver a hard copy to the mailbox of John van de Wetering. Mercator 1, 3rd floor.
2. E-mail a PDF to `wetering@cs.ru.nl`. Please include your name and the exercise number in the filename, e.g. `ACHTERNAAM-qpc-exercise1.pdf`.

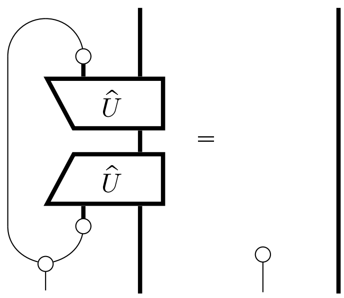
Deadline: Friday, May 4, 17:00

Goals: After completing these exercises you can work with spiders of the bastard, quantum and phased kind. The total number of points is 100, distributed over 4 exercises.
Material covered in book: sections 8.3, 8.4, 9.1.

A cq-map $\widehat{U}$ is a *controlled isometry* when

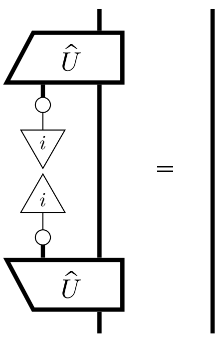


for all classical basis states $|i\rangle$. This is equivalent to the condition (see Proposition 8.88):

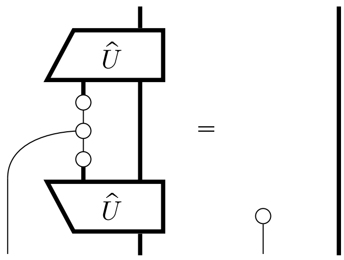


Exercise 1 (8.88 & 8.91) (30 points):

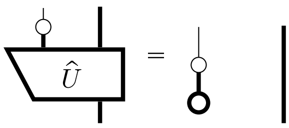
(i) A controlled isometry is a *controlled unitary* when it additionally satisfies:



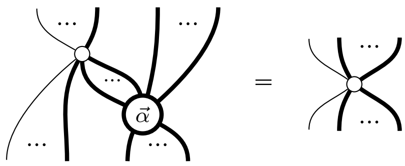
Show that this condition is equivalent to



(ii) By Proposition 8.90 a controlled isometry is also a regular isometry (up to a number). Show that however a controlled unitary is *not* necessarily a unitary (up to a number). Hint: let

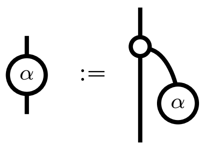


Exercise 2 (9.13) (10 points): Show that whenever a phase spider fuses with any bastard spider, the phase vanishes:



Exercise 3 (30 points):

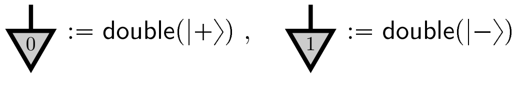
(i) A *phase gate* is a quantum process of the form



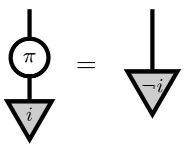
Show that a phase gate is always causal and unitary.

(ii) Show that the X-basis states $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$ are phase states (up to a number) for the standard Z-basis spider and find the phase they correspond with.

(iii) In exercise sheet 6 we introduced the notation

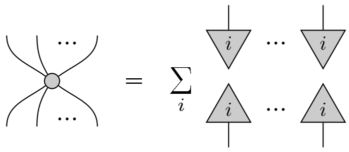


Show that the π-phase gate acts as a NOT-gate for the X-basis:



where $i = 0, 1$ and $\neg i$ is its negation.

Since the X-basis is an ONB, we can also use it to make (classical/quantum/bastard) spiders:

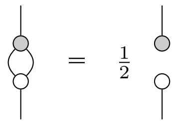


where we use the gray colour to denote it as an X-spider instead of a (white) Z-spider.

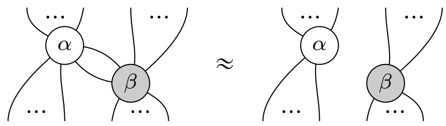
Exercise 4 (9.37 & 9.49) (40 points):

(i) Show that the (doubled) quantum states corresponding to the Z-basis elements $|0\rangle$ and $|1\rangle$ are phase states for the X-spider.

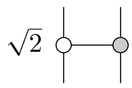
(ii) Show that



(iii) Use the previous equality to show that



(iv) Show that



acts as a CNOT-gate by plugging in Z-basis states and showing that $|i\rangle|j\rangle \mapsto |i\rangle|i \oplus j\rangle$.

(v) BONUS: Show that the above map acts as the CNOT gate with the input and control wire interchanged when instead of Z-basis states, X-basis states are inserted.